PRIMITIVE NUMBERS

BY

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PLACE OF NUMBERS IN THE GROWTH OF KNOWLEDGE

The gateway to knowledge of aboriginal character is found in aboriginal conduct; for among primitive folk, habits of action are more trenchant than systems of thought. Yet full knowledge of aboriginal character may be gained only through study of both the activital habits and the intellectual systems of the aborigines; for in every stage of human development, action and thought are concomitant and complementary.

In dealing with aboriginal customs connected with numbers (simple counting, numeration, calendar systems, etc.), the working ethnologist is confronted by the elusive yet ever-present fact that primitive folk commonly see in numbers qualities or potencies not customarily recognized by peoples of more advanced culture. Accordingly it seems especially desirable to trace the thoughts, as well as the customs, of primitive number-users, and this may be done with a fair degree of confidence in the light of homologies with the early stages of mathematics and related knowledge among peoples of advanced culture.

Fairly close homologies with the numbers of primitive peoples are afforded by the early stages of chemistry and astronomy. Chemistry grew slowly out of alchemy as natural experience waxed and primeval mysticism waned; and in earlier time astronomy grew out of astrology in similar fashion. The growth of chemistry is fairly written, and that of astronomy less fully recorded in early literature; and in the history of both sciences the records are corroborated and the sequence established by vestigial features—for such features are no less useful in defining mental development than are vestigial organs and functions in outlining vital evolution.

Now on scanning the long way over which modern knowledge came up, it becomes clear that the beginning of chemistry marked the third step in the development of science, and that the beginning of astronomy marked an earlier step; and it also becomes clear that another

step, taken amid the mists of unwritten antiquity, was marked by the beginning of mathematics. In the absence of records, the rise of mathematics may be traced partly (like the growth of the next younger sciences) by vestigial features and functions; and these vestiges indicate that, just as scientific chemistry came out of mystical alchemy and as scientific astronomy sprang from mystical astrology, so rational mathematics grew out of a mystical system which long dominated the minds of men and slowly waned under the light of natural experience concentrated among the Arabs of past millenniums. In Arabia this mystical system preceded the simple and essentially natural, though happily conventional, system of enumeration and notation long known as algorithm (or algorism)—i. e., that inchoate form of arithmetic which permitted numerical treatment of quantities, and thus gave a foundation for science. The mystical system is even more clearly represented in algebra, in which the conventional symbols now used to express natural values were originally employed as indices of magical potencies, like the characters inscribed on amulets and talismans; indeed the literature of science yields definite records of that long-abandoned side of algebra known as almacabala (sometimes written almachabel) from the Arabic word for learning and the Hebraic (or older) term for mystical or magical attainment of purpose,[1] the whole constituting a jumble of occult or semi-occult redintegration such as appeals strongly to the ill-developed mind. Accordingly the stepping stones to modern science may be enumerated as (1) almacabala, (2) astrology, (3) alchemy, leading respectively to mathematics and astronomy and chemistry, the oldest branches of definite knowledge.

While the transition from almacabala to mathematics is indicated somewhat vaguely by the records and more clearly by vestiges among the peoples influenced by Arabic culture (including all the Aryans and their associates, who make up the intellectual world), the sequence is established by parallel developments displayed by other lines of culture. The import of these parallelisms becomes clear in the light of principles pertaining both to science in general and to anthropology in particular; and some of these principles are worthy of enumeration:

1. In all science it is necessarily (albeit often implicitly) postulated that knowledge grows by successive increments through experience and its assimilation, through observation and comparison (or generalization), through discovery and invention, or, in short, through natural processes. In the natural (or chiefly inductive) sciences and in recent decades this postulate is commonly made consciously and deliberately; in the more abstract (or chiefly deductive) sciences the postulate is less frequently made consciously, though a notable example of recognition

[1] "Cabala, or 'practical cabala,' as described by Hebraic authors, is the art of employing the knowledge of the hidden world in order to attain one's purpose in accordance with the mysticism expounded in the 'Sefer Yezirah' (Book of Creation), in which the creation of the world is ascribed to a combination and permutation of letters of the alphabet."—The Jewish Encyclopedia, Vol. I, 1891, p. 548.

of the experiential basis of mathematics was recently afforded by the president of the American Mathematical Society.[1]

2. In all departments of definite knowledge, but especially in the several branches of anthropology, it is implicitly, if not explicitly, postulated that knowledge is diffused and its acquisition stimulated through association and interchange among individuals and peoples: indeed, this postulate affords the warrant, and forms the basis, for education.

3. In anthropology as in other sciences it is necessary to recognize a volume or body of knowledge proper to each people, made up of the combined intellectual possessions of all the individuals, increasing with successive experiences, decreasing only through disuse or neglect, and in greater part perpetuated by record and tradition if not by direct heritage.

4. In ethnologic research, as measurably in other lines of inquiry, it is desirable and fair to assume that (*a*) mental capacity and (*b*) the sum of knowledge, either in the individual or in the group, are in the long run practically equivalent.

5. In ethnologic inquiry it is convenient to assume that the course of development is approximately uniform (or about as nearly similar as are environmental conditions) in each separate or independent group of men. This assumption, which was recognized first by Powell under the law of activital similarities, and later by Brinton under the formula "unity of mind," is rapidly crystallizing in the minds of anthropologists; it is, indeed, but a corollary of the primary postulate on which all science rests, namely, that knowledge grows by natural means; and latterly the postulate (which is but a generalization of invariable experience), with its corollaries and applications, has been formulated as one of the cardinal principles of science, namely, the responsivity of mind.[2]

The recognition of the foregoing principles yields a means of outlining intellectual development in general, and hence of defining the grades, or growth-stages, of given intellectual stocks (or peoples); for when once the general scheme of development indicated by the several examples is perceived clearly, the relative positions of each of the examples are evident. The relations of the natural stages in intellectual development may be illustrated by comparison with the growth-stages of aged sequoia groves of prehistoric birth, whose beginnings no man recorded and no living man saw, but whose history may be read clearly in terms of younger groves in other counties; for the towering groves of the big-tree species and the upshooting forests of human ideas may well be likened in individual and collective growth, save that the vegetal species is decadent and shrunk into scattered

[1] "Even pure mathematics, though long held apart from the other sciences, must be founded, I think, in the last analysis, on observation and experiment."—R. S. Woodward, Science, new ser., vol. XIII, 1901, p. 522.

[2] Proc. Washington Academy of Sciences, vol. II, 1900, pp. 1–12.

patches, while the mental growth is luxuriant and spreading exuberantly from province to province throughout the lands of the earth. In both cases the interpretation in terms of growth-stages is established by conformity with natural law; did the grove receive extranatural impulse at any stage, or did knowledge arise otherwise than through interactions of nature, the interpretation would fail; but in the absence of evidence against the uniformity of nature, the equivalence of corresponding stages must be recognized alike for the figurative forests of ideas and the material forests of wood and leafage.

Now the acceptance of these principles, and the recognition of the general course of intellectual development, afford a means of tracing the unrecorded history of Aryan culture and of interpreting the meager records of Arabia's mathematical pioneering in terms of the culture of other peoples still below, or just rising above, the plane marked by the birth of writing—i. e., the beginning of scriptorial culture. Especially useful for comparison are various practically independent Amerind peoples, some low in prescriptorial culture, others grappling with the rudiments of definite graphic art, and still others just within that phase of scriptorial culture marked by conventional calendric and numeral systems; hardly less useful are several African peoples representing various early stages of development; of much significance, too, are the Australian tribes, of culture so low that numerical knowledge is inchoate only, together with different Polynesian tribes whose culture curiously reflects their distinctive environment; while useful suggestions as to the origin of numerical concepts may be drawn from various subhuman animals. True, the lines of mental growth maturing in mathematical systems must vary with environmental conditions, and doubtless with hereditary traits persistently reflecting both ancestral and proto-environmental factors; yet, if knowledge be not an extranatural product rather than a reflex of nature (as brilliantly conceived by Bacon) the lines must be so far conformable as to render the comparisons trustworthy and sufficiently accurate for practical purposes—just as the retracing of the history of an isolated grove by comparison with the growth-lines of other groves must be inexact in detail, though trustworthy in general and sufficiently accurate to meet practical needs.

CHARACTERISTICS OF PRIMITIVE THOUGHT

In tracing the lines of intellectual growth maturing in modern enlightenment, it is needful to note certain habits of mind characteristic of all primitive men, yet measurably distinct (in degree if not in kind) from those common to civilized and enlightened men; and for present purposes, as for practically all others, it will suffice to define primitive peoples as those who have not yet acquired and assimilated

the art of writing, i. e., as those who remain in prescriptorial culture: for the longest single step in the development of mind and the widest chasm dividing humanity is that marking the transition from the lowly stage of unaided thinking to the stage of mechanically extended memory and mentation.

Mysticism of primitive thought—All primitive men are mystics. Believers in extranatural potencies, inexpert observers, and inconstant reasoners, their vague faith veils or counterfeits realities and clothes its own figments with all manner of attributes, oftener incongruous than germane. In their simple (and presumptively primeval) aspect, the fear-born figments are grotesque shadows or fantastic duplicates of actual things moved by capricious or malicious motives, like those of human kind: in somewhat advanced thought the figments are more complex, and are incarnated chiefly in self-moving things and invested with enlarged and intensified autonomy: while in the higher stages of primitive culture the figments are idealized into mystical potencies conceived to actuate the objects and powers of the universe in accordance with impulses and motives such as those observed to control human action. And this lowly faith, with its imputation of animistic impulses and agencies to all nature, is far more than mere abstraction; in all its aspects the belief is profound and paramount: it is an ever-present possession, passing often into complete obsession, whereby action and thought are habitually and wholly controlled.

In every phase of primitive culture the mystical potencies imputed to natural things are held to be the chief factors of failure or success in the ceaseless strife for existence. So these potencies are invoked by fasting, propitiated by sacrifice, celebrated by feasting, and expatiated and glorified by individual and collective ceremony, as well as by the marvelously persistent tradition of prescriptorial culture. The first effect of recurrent ceremony is to crystallize the animistic concepts and concentrate the imputation of potency on the more conspicuous objects of current experience, and hence to lead to the deification of strong and swift beasts, venomous serpents, rapacious birds, turbulent waters, destructive volcanoes, and other impressive things: though since the successful men and tribes give more thought to joyous glorification and less to anxious propitiation than their unsuccessful contemporaries, the beneficent potencies tend to survive and the maleficent mysteries tend to die out of the darksome—but ever brightening—faith of primitive men. Yet throughout the whole domain of lowly culture the mystical potencies are dominant factors of thought.

In all aspects of primitive faith the controlling mysteries are conceived as associated with symbolic objects and actions: and by reason of this notion both mysteries and symbols are zealously enshrouded in ever deeper mysticism. So, fetishism and shamanism grow apace; not only ceremonial objects, but places and persons and forms of utter-

ance become secret or sacred, as when the plaza is forbidden to all save priests, and when the Word is deemed a symbol of the Life of the speaker. So, too, esoteric observances, impressive insignia, and imposing formalities are established, and systems of rank or caste grow up as tangible expressions of the intangible structures of controlling subjectivity. Cumulatively strengthened by reaction of symbol on mystery and of mystery again on symbol, the pervading mysticism is exalted above all other motives in primitive thought; and the artistic concepts, the industrial devices, the social relations, and the themes and forms of speech all pass under the control of the unreal potencies which shadow the primitive thinker.

Throughout primitive culture invocation habitually carries a reverse of incantation, so that the normal course of fiducial development is attended by persistent magic, sortilege, thaumaturgy; while in the higher stages necromancy and soothsaying, spells and enchantments, conjury and exorcism, oracles and ordeals, and divination by lot or chance become characteristic. In the higher strata, too, expressions supplement or supplant the objective symbols of lower plane, and the jargon of jugglers and the farrago of fakirs take the place of fetiches and idols; and it is particularly significant that words and verbal formulas come to be regarded as superpotent expressions of mystical power, and that even the letters of early times were credited with creative powers in practical cabala. Some savage tribes regard their language as sacred, some have hieratic languages, and among all known tribes personal names are considered magical or tabu in one way or another; while just within the lower strata of scriptorial sculpture (as illustrated by the Arabs and Hindoos and other Eurasians of a few centuries ago, and attested by literary and linguistic and objective vestiges), shibboleths and numerical formulas become rife, and the inscribed talisman and abracadabra and mystical number, and eventually the magic square, form favorite symbols of occult power.

The growth of writing and the attendant decadence of tradition sounded the knell of primitive mysticism; for one of the leading functions of lowly faith in the actual economy of thought was the maintenance of long series of mnemonic associations, and when this function was assumed (and better performed) by mechanical devices the strongest support of the crude philosophy fell away. Yet the mode of thought crystallized by uncounted generations of habit was too firmly fixed for easy dropping, and innumerable vestiges in the line of Aryan culture, as well as the examples afforded by other lines, demonstrate the potency of primeval mysticism and the tenacity of its hold on the human mind even beyond the verge of modern enlightenment.

Egoism of primitive thought—All primitive men are egoists. Knowing little of the external world, tribesmen erect themselves or

their groups into centers about which all other things revolve according to the caprice of their all-potent mysteries; they act and think in terms of a dominant personality, always reducible to the Ego, and an Ego drawn so large as to stand for person, place, time, mode of action, and perhaps for raison d'être—it is Self. Here. Now. Thus, and Because. Science shows that the solar system hurtles through space, presumably about an unknown center; it showed before that the sun is the center of our system; but the heliocentric system was expanded out of an antecedent geocentric system, itself the offspring of a demo-centric system, which sprang from an earlier ethnocentric system born of the primeval egocentric cosmos of inchoate thinking. In higher culture the recognized cosmos lies in the background of thought, at least among the great majority, but in primitive culture the egocentric and ethnocentric views are ever-present and always-dominant factors of both mentation and action.

The prominence of self-centred thinking in lowly life is exemplified by kinship organization, the universal basis of primitive society. In the lowest of the great culture stages, the recognized kinship is maternal, and in the next higher (but still prescriptorial) stage it is nominally paternal, though increasingly modified by adoption and other conventional devices; yet the organization is maintained by bonds and interrelations which can not better be illustrated than by analogy with the planetary assemblage: Each individual rotates independently, may be attended by satellites, and revolves primarily about the head of the family yet ultimately about the patriarch of the group, and each exerts a definite attractional influence (albeit proportional to individuality—or perhaps intellectuality—rather than mass) on all his associates. The relative social positions are expressed and kept in mind by habitual conduct and form of speech; each member of a family, each family of a clan, and each clan of a tribe has a fixed place in the group to which he or she is kept by thon's own memory and constrained by the consensus of associates; and among most primitive peoples no individual can speak to or of a companion without reference to the currently accepted view of his circumscribed cosmos—a man can not say "brother," but must say "my elder brother," or use some other term implying the relative position of several individuals to himself, and among each other as reckoned through himself; and in many tribes the terms of relationship used by women differ from those employed by men.

The ever-present view of a self-centered cosmos finds expression throughout primitive language, as well as in the lowly faith with which it is bound up and in the social organization by which it is maintained. Primitive speech is essentially associative, abounding in numbers and genders, persons and cases, moods and tenses, in a complex structure reflecting the egocentric habit of thought. This structure

is crystallized in a characteristically and often chaotically elaborate grammar, well suited to the formulation and utterance of a limited number of ideas representing a few main classes (or lines) of thought, and well adapted to maintaining the associative thought habit; so that primitive languages are essentially structural or morphologic, only incidentally lexic. With the multiplication of ideas accompanying cultural advance, the bonds of linguistic association break under their own weight, and discrete vocables multiply at the expense of unwieldy collocations; and with the attainment of writing, the function of linguistic association largely disappears, and speech becomes essentially lexic, only incidentally morphologic.

Concordantly with self-centered language, primitive arts and industries are conspicuously egoistic. The most strikingly inchoate esthetic thus far critically studied is the totemic face-paint borne by the matrons of clans, apparently as beacon-signals analogous to the face-marks of various animals,[1] while the tattoo-marks denoting marriage among the women of many Amerind tribes are clear vestiges of the more primitive beacons; and the autobiographic winter count of the warrior and the closely related calendar of the shaman are commonly egocentric, never more than ethnocentric—for if the motives of the primitive scribe perchance transcend self, they never outpass the clan or tribe, or at most the confederacy. Similarly the industrial devices of early culture are held to absorb and retain a part of the personality of, and indeed to become subjective appendages to, their makers and users; while in advancing culture the subjective personality of the device passes over into the industries in such wise as to engender guilds and crafts, and ultimately to grow into the "art and mystery" of conventional apprenticeship.

Concordantly, too, egocentric thought finds expression in primitive belief; for the individual long retains his personal tutelary or fetish, endowing it with characters revealing his own subjectivity; and it is with exceeding slowness that he rises first to the recognition of family fetishes and clan totems, and eventually to the inheritance, or perhaps as among the Kwakiutl Indians to the conjugal acquisition, of those symbols of potency, and much later that he rises to that recognition of alien tutelaries which expands with piratical and amicable acculturation, and ends in pantheism.

So in every line of human activity self-centered thinking is crystallized by custom, and the thought and custom interact with cumulative effect in dominating the primitive mind well into the upper strata of prescriptorial life. The persistence of the cumulative effect is clearly indicated by numberless vestiges of egocentric cosmology clinging often to the higher phases of Aryan culture.

[1] Cf. The Seri Indians: Seventeenth Annual Report of the Bureau of American Ethnology, 1898, part 1, p. 168.

In short, it can not be too often stated or too strongly emphasized that primitive thought is unlike the finer product of contemporary intellectuality. While the differences are many, the most conspicuous are those connected with the pervading mysticism and prevailing egoism of primitive thinkers, both magnified in their influence by the fewness of concurrent intellectual stimuli and motives; so that prescriptorial culture may justly be regarded as the outgrowth and outshowing of that mysticism-egoism which arose early in the unwritten past, which began to decline with the birth of writing, but which still retains some hold on the minds of men.

PRIMITIVE COUNTING AND NUMBER SYSTEMS

Numeration

Simple counting is an accomplishment common to men and many lower animals. The special appreciation of numbers sometimes displayed by horses, dogs, and pigs may be due to human association, while the geometric sense of the bee may be considered mechanical merely; yet the well-known ability of the crow to count (or at least to discriminate units) up to six or seven, the similar faculty of the fox, and the habits of wasps in providing fixed numbers of spiders for their unborn progeny, as well as various other examples, demonstrate a native capacity for numerical concepts on the part of birds and mammals and insects.

Apparently similar is the numerical capacity of various lowly tribes of different continents: Numerous Australian tribes are described as counting laboriously up to two, three, four, or six, sometimes doubling two to make four or three to make six, and in other ways revealing a quasi-binary system; though both Curr and Conant opine that "no Australian in his wild state could ever count intelligently to seven."[1] Certain Brazilian tribes are also described as counting only to two, three, or four, usually with an additional term for many; while the Tasmanians counted commonly to two and sometimes to four, and were able to reach five by the addition of one to the limital number.[2]

The analogy between the counting of the tribesmen and that of the animals is not so close as the bare records suggest, since the descriptions of the tribal reckoning relate to systems of vocal numeration rather than to actual ability in discrimination and enumeration; moreover, most of the tribesmen reveal the germ of notation in the use of sticks, notches, knotted cords, and the like to make tangible the numerical values—something which lower animals never do so far as is known. Actually the savages, even those of lowliest culture,

[1] The Number Concept, by L. L. Conant, 1896, p. 27; The Australian Race, by E. M. Curr, 1886, vol. I, p. 32.

[2] The Aborigines of Tasmania, by H. Ling Roth, 1890, p. 147.

habitually think numerically up to or above three, as is shown by the plurality of plurals and by other features of their speech; and the meagerness of their numeration no more negates numerical capacity than does the absence of such systems among counting crows and foxes and wasps. Nevertheless, the comparison is instructive. In the first place, it indicates roughly corresponding ability to count on the part of higher animals and lower men; it also defines the origin of vocal numeration at the bottom of the scale of human development; and it is especially significant in demonstrating that neither the animals nor the men (1) either cognize quinary and decimal systems, or (2) use their own external organs (toes, fingers, etc.) as mechanical adjuncts to nascent notation—unless the binary numeration of certain Australian tribes is really binanual, as W. E. Roth implies.[1] Many primitive peoples count by fingers and hands, sometimes with the addition of toes and feet, and thereby fix quinary, decimal, and vigesimal systems; but the burden of the evidence derived from animal counting and from the numeration of lower savagery seems to demonstrate that these systems are far from primeval.

Simple number systems of mystical or symbolic character abound among the better-studied tribes of middle-primitive culture, including the aborigines of North America. The most widespread of the mystical numbers is four. It finds expression in Cults of the Quarters in North America, South America, Asia, and Africa, and is suggested by certain customs in Australia;[2] it is crystallized in the swastika or fylfot and other cruciform symbols on every continent, save perhaps Australia; and it is established and perpetuated by associations with colors, with social organization, and with various customs among numerous tribes. In much of primitive culture the hold of the quatern concept is so strong as to dominate thought and action —so strong as to seem wholly inexplicable save through the interwoven mysticism and egoism of the lowly mind. The devotee of the Cult of the Quarters is unable to think or speak without habitual reference to the cardinal points; and when the quadrature is extended from space to time, as among the Papago Indians, the concept is so strong as to enthrall thought and enchain action beyond all realistic motives. To most of the devotees of the quatern concept —forming probably the majority of the middle primitive tribes of the earth—the mystical number four is sacred, perfect, and all potent, of a perfection and potency far exceeding that of six to the Pythagoreans and of the hexagram to Paracelsus and his disciples; they are unconscious or only vaguely conscious of any other numerical concept; and many investigators fail to discover the reverse of the quartered shield and so trace the mystical figure to the subconscious self which it invariably reflects. Yet careful inquiry shows

[1] Ethnological Studies among the North West-Central Queensland Aborigines, 1897, p. 2.
[2] Curr, The Australian Race, vol. I, pp. 339, 340.

that the cardinal points are never conceived apart from the ego in the center: that the subjectively prepotent part of the swastika is the intersection or common origin of the arms; that the four colors of brightening sunrise and boreal cold and blushing sunset and zephyr-borne warmth must have a complementary all-color in the middle; that the four winds are balanced against some mythic storm king (able to paralyze their powers in response to suitable sacrament) in or near the middle of the world; that the sky falls off in all directions from above the central home of the real men; that the four termini of Papago time relate to the end of the period conceived always with respect to the beginning; that the four worlds of widespread Amerindian mythology comprise two above and two below the fate-shadowed one on which the shamans have their half-apperceived existence; that the four phratries or societies are arranged about the real tribal center; and that in all cases the exoterically mystical number carries an esoteric complement in the form of a simple unity reflecting the egoistic personality or subjectivity of the thinker. It is easier to represent the quatern concept graphically than verbally—indeed it has been represented graphically by unnumbered thousands of primitive thinkers in the cruciform symbols dotting the whole of human history and diffused in nearly every human province, or in the form of the equally widespread but less conspicuous quincunx.

The exoterically quatern and esoterically quincuncial concept appears to mark a fairly definite phase of human development; a somewhat higher stage is marked by the use of six as a mystical or sacred number. In this stage the mythology remains a Cult of the Quarters, though the cardinal points are augmented by the addition of zenith and nadir, while a third upperworld and a third underworld may be added to the tribal cosmology. The ramifications of the concept are still more extended than those of the quatern idea, and lead to even more patent incongruities—particularly when the attempt is made to graphically depict the essentially tridimensional concept on a plane. Now the senary concept, like its simpler analogue, is always incomplete in itself: the six cardinal points must be reckoned from a common center, the three underworlds and the three upperworlds are reckoned from the middle world of actuality, and the six colors (for example, of corn, as among the Zuñi, according to Cushing and others) are habitually supplemented by a central all-color; so that, in this case, as in that of the quasi-quaternary system, the exoterically perfect number is esoterically perfected through the unity of subjective personality, i. e., the ever-present ego.[1] It is significant that the six-cult is much

[1] The perfecting of the mystical numbers four and six by the addition of unity has been recognized by many investigators, notably by Powell (On Regimentation, in the Fifteenth Annual Report of the Bureau of Ethnology, 1893-94, 1897, p. cxvii and elsewhere), Morris (Relation of the Pentagonal Dodecahedron . . . to Shamanism: Proceedings of the American Philosophical Society, vol. XXXVI, 1897, pp. 179-183), and Cushing (ibid., p. 185 and elsewhere).

less extensively distributed through history and throughout the world than the four-cult, though it may be traced in different continents; and it is peculiarly meaningful in establishing that marvelous prepotency of the number cult which, among many tribes, carried the nascent numeral system past the point at which nature strove, through the obvious organic structure of the hand and through simple algorithmic order, to implant the quinary system. Indeed, if further evidence than that of bestial and savage counting were required to show that finger numeration and the quinary system were not primeval, it would be afforded by the development of the senary-septenary system in so many lands.

The quaternary and senary cults illumine the binary systems prevailing among tribes still lower in the scale of intellectual development. Especially helpful is the light on the Australian aborigines, who are found thereby to exemplify what might be called a Cult of the Halves; for they are controlled by a binary concept of things expressed not only by their numeration, but even more clearly by their social and fiducial systems, which, in turn, shape their everyday conduct and speech. "The fundamental feature in the organization of the central Australian, as in that of other Australian tribes, is the division of the tribe into two exogamous intermarrying groups," say Spencer and Gillen;[1] and all other students of native Australian society have either been overwhelmed by an apparently irresolvable nebula of overlapping classes and subclasses and superclasses, or have been led to a related conclusion. Indeed the Gordian knot of entangled relationships constituting Australian society is easily cut by the student who places himself in the position of an individual blackfellow, and projects from self dichotomous class-lines occasionally uniting and bifurcating in other individuals, after the manner of the dichotomous lines of Aristotelian classification and the Tree of Porphyry; for the social classes, and the conduct involved in their maintenance, are fixed by a bifurcate series of ordinances, ostensibly descended from the mystical olden time, and put in the form of tabus and equally mystical mandates by the shamans. In like manner the obscure pantheon of the Australians seems to be arranged in nearly symmetric pairs; and even the individual shade (or mystical double of the person) is conceived as bipartite, as among the Arunta, who designate the ghostly attendants Iruntarinia and Arumbaringa, respectively.[2]

Although typically developed among the Australian aborigines, the binary philosophy is by no means confined to the Austral continent and primeval culture; it existed among the Tasmanians, it reappears in Africa, persists in China and Mongolia, and may clearly be traced in America, e. g., in the "sides" forming the primary basis of society in the Seneca and other Amerind tribes; while no fiducial system is

[1] The Native Tribes of Central Australia, by Baldwin Spencer and F. J. Gillen, 1899, p. 33.
[2] Op. cit., p. 513.

wholly free from the persistent dualism springing from binary interpretations of nature. Yet the mystical Two is no more complete in itself than the mystical Four and Six of higher culture; the primary classes or "sides" are perfected in the tribe both in Australia and in America, the Iruntarinia and Arunmbaringa are conjoined in and nonexistent apart from the personality they are held to shadow, and the mandates and prohibitions of Australian (and indeed of most other) laws are perfected in permissive, or normal, conduct; in Australia indeed the central factor is so well developed that Lumholtz was led to note a ternary concept as expressing a definite "idea of the Trinity" among the southeastern tribes;[1] so that the exoterically binary system of thought is esoterically, or in subconscious fact, ternary.

The dichotomous fiducial and social structure clarifies the Australian numeral system. The abundant numerations recorded by Curr and others strongly suggest the simple binary system traced by Conant. A common form is *goona, barkoola, barkoola-goona, barkoola-barkoola* (1, 2, 2-1, 2-2) sometimes followed by "many" or "plenty" and more rarely by *barkoola-barkoola-goona* (2-2-1), though usually the table does not go beyond the fourth term, which may itself be replaced by "many." Now, examination of the numerous records shows (1) that none of the terms correspond with fingers; (2) that a very few of the terms correspond with the word for hand, such terms being three, four, one, and two in (approximate) order of frequency; (3) that a somewhat larger number of terms, chiefly three, one, and two, correspond with the words for man; (4) that a considerable number of threes and ones, with a few fours and twos, suggest affinities with obscure roots used chiefly in terms for man, tribe, wild dog, I, yes, etc.; and (5) that there is a strong tendency to limit the formal numeration to three. It is particularly noticeable, too, that certain persistent number-terms are used sometimes for two and sometimes for three among numerous slightly related tribes—i. e., the term is more definitely crystallized than the concept, which oscillates indiscriminately between two and three, betraying a confusion impossible to arithmetic thought. Similarly the Tasmanian numerations are binary, and without reference to finger or hand, though five sometimes appears to connote man. These features clearly indicate that the Australasians do not count on their fingers, and are without realistic notion as to the number of fingers—indeed the Pitta-Pitta of Queensland are able to count their fingers and toes only by the aid of marks in the sand,[2] while the abundant Australian pictographs reveal habitual uncertainty as to the number of fingers in the human hand (save where the picture is developed from a direct impression).

Suggestively analogous in form and meaning are certain South

[1] Among Cannibals, 1889, p. 129.

[2] Ethnological Studies, by Walter E. Roth, p. 26.

American number-systems—e. g., that of the Toba, whose ordinary numeration ends with six (the term meaning also "many" or "plenty"), though Bárcena has traced it to ten. The terms are somewhat variable, and of such form as to imply actual or vestigial connotive character; as recorded by Quevedo[1] they are *natha dae*, *cacayni* or *niroca*, *cacayni'lia*, *nalotapegat*, *niroca cacainilia* (2+3), *cacayni cacayni'lia* (2×3), *natha dae cacayni cacayni'lia* (1+2×3), *niroca nalotapegat* (2×4), *niroca nalotapegat natha dae* (2×4+1), *cacayni niroca nalotapegat* (2×4+2). Now, it is noteworthy (1) that none of the terms connotes finger, hand, or man; (2) that there are alternative terms for two in both simple and composite uses; (3) that two is the most prominent factor in the composite part of the series; (4) that one of the terms for two and the term for three are closely similar, and distinguished only by inflection; (5) that the term for four apparently connotes equality (*nalotath*=equal) and declaration (*na-pega*=they say; *sena-pega*=I say, etc.); and (6) that the system is definitively not quinary or decimal. There are suggestions, both in the combinations and connotations of the terms, of two threes of ill-defined numeric character, corresponding respectively to the numeric two and three; and that four is an essentially mechanical square. There are also many indications that the system is inchoate so far as the strictly numerical aspect is concerned.

In the dearth of knowledge concerning the original or collateral meanings of the Australian and South American number-terms, it is difficult to formulate the fundamental concept or to give it graphic expression; but a suggestion of great inherent interest is found in the Shahaptian numeration, in which, according to Hewitt, the first two integer-terms are denotive or arbitrary merely, while the term for three means Middle or Middle ONE—not middle finger or middle of the hand, but apparently a general (or semi-abstract) Middle like that of the Zuñi ritual; and the suggestion is enforced by corresponding expressions in Serian, Iroquoian, and some other Amerindian tongues. The Zuñi expression for the middle finger, as rendered by Cushing, is particularly suggestive, viz, "Counter-equally-itself-which-does";[2] and the persistent tendency to double as well as to divide is illustrated by the Hai-it terms (incorporated by Dr Thomas, postea, p. 871) for two, four, and eight, viz, *pun*, *tsoo'-ik*, and *pun'-tsoo-ik* (2×4), and still more clearly by the absence of the numeral nine—indeed this brief vocabulary displays a curious combination of the binary and quinary systems.

In the light of these analogies the Australian thought-mode, with its numerical and social and fiducial expressions, and measurably also that

[1] Arte de la Lengua Toba, por el Padre Alonso Bárcena * * * con Vocabularios * * * por Samuel A. Lafone Quevedo, Biblioteca Linguistica del Museo de la Plata, vol. II, 1898, p. II.

[2] Manual Concepts, Am. Anthropologist, vol. v, 1892, p. 293.

of the Toba and perhaps other South American tribes, assume definite and harmonious shape in a binary-ternary system, in which things are conceived in pairs related subconsciously to an initial or central interpretative nucleus—that is, to the dominating Ego of primitive ideation.

The three number-systems pertaining to prescriptorial culture are essentially distinct from modern Aryan numeration, and indeed from the whole of Arabic algorithm and arithmetic, in motive as well as in mechanism. Primarily, they are devices for divination or for connecting the real world with the supernal, and it is only later or in minor way that they are prostituted to practical uses; yet by reason of the magical potency imputed to them they dominate thought and action in the culture-stages to which they belong and profoundly affect the course of intellectual development—indeed, like other figments (or pure abstractions, dissevered from the actualities of nature), their office is first to stimulate and later to enchain mentation.

In mechanism the three systems correspond substantially, even if they are not actually correlative, for each rests on an exoteric base in the form of a small even number, and each is really controlled and perfected by a half-apperceived unity, itself the reflection of the Ego, whereby the base is raised esoterically to the next higher odd number. The systems differ only in the value of the exoteric base, which is a measure of the intellectual capacity normal to the culture-stage to which it pertains. The two higher systems have graphic equivalents which shape and intensify their mystical potency (for the mechanical conditions attending graphic representation always interact with primary concepts in primitive thought); but the lowest and presumptively primeval system is without known graphic symbol.

Notation and Augmentation

Resting as they do on inconstant and largely subjective bases, and pertaining as they do to prescriptorial culture (or at the best to inchoate ideographic representation), the primitive number systems are not susceptible of algorithmic notation. Concordantly they are insusceptible of treatment by the methods of rational arithmetic; though the two higher systems (and probably the lowest also) lend themselves to combinations made in accordance with a method or law which may be styled *augmentation*—a process tending to perpetuate itself, and, while neither addition nor multiplication, tending to generate both. This curious law of augmentation is of much significance; in the first place, it represents a process apparently lost (along with the observational basis of arithmetic) from the recorded history of mathematics; and, in the second place, it seems to explain the interrelations and evolution of the magical number-systems; again, it would seem to constitute the germ of the fundamental arithmetic processes, and hence to explain the transition from magical to rational numbers; and finally

it is of no small interest as a source of those vestigial features of almacabala still persisting in Aryan culture, still cropping out in "lucky numbers" and in other fantastic forms.

The augmentation of the widely diffused quaternary-quinary system is made clear by aid of its mechanical symbolism, which combined with the egoistic concept to shape the system. The commonest (and nearly world-wide) symbol is the cruciform figure ✛, or the quincunx, ⁙ Now, magnification of the peripheral powers or objects is readily and intuitively represented by adding a line or dot to each of the four extremities of the symbol, whereby it is converted into the simple swastika in its prevailing forms, ⊕, or ✢ Actually the figure is sometimes developed (as among some Pueblo peoples, according to Cushing) by laying down four billets or arrows radiating from a fetishistic Middle toward the east, north, west, and south, and then adding, as the ritual proceeds, shorter transverse sticks touching the extremities of the four cardinal billets, the whole being done in such a manner as to harmonize ritual and symbol, and impress the former by the objective representation in the latter. In any case, the symbol is raised from its original value of 4+1 to 8+1; and the graphic representation accords with the shadowy concept lying behind the number system in which the mystical Middle is persistent, and can be counted but once howsoever the value be augmented. Similarly the peripheral potencies may be multiplied by the addition of dots, as in a common form of the swastika noted by Wilson, 卐 or 卍,[1] or by the development of the "meander," 回, which thus represent, respectively, 12+1, 20+1, and 20+1; and the augmentation may proceed indefinitely, by either mechanical or mental addition, though always in accordance with the primary principle that the Middle is reckoned but once.

The mechanical conditions accompanying the development of the figure tend to maintain its symmetry, i. e., the supplementary transverse billets, or sticks, are naturally so laid as to form counterparts in relation to the primary billets and to the center; but, as pointed out by Wilson (after Max Müller and Burnouf), the additional billets completing the swastika proper may be turned either to right or to left, i. e., the development of the figure may be either clockwise or counterclockwise. The question has even been raised whether distinct names should be applied to the alternative forms; but in view of the fact that the habitual motions of primitive peoples are predominantly centripetal, or toward the body, while the predominant motions of advanced peoples are centrifugal, it seems safe to infer that the clockwise swastika represents the higher cultural plane (just as writing toward the right represents a higher plane than the archaic mode of writing

[1] The Swastika, Report of the United States National Museum for 1894, p. 767.

toward the left), and accordingly that this form would be normal if the form itself were normal to advanced culture; but that since the symbol pertains in all essential respects to the lowly culture characterized by centripetal hand-movement, the counter-clockwise form would seem to be more properly considered the normal one—and it is drawn herein.

While the concept of the senary-septenary system is much more complex than that of the quaternary-quinary system, the law of augmentation is similar; and it is significant that the similarity accompanies (and presumptively results from) analogous efforts at graphic representation. Commonly the concept is directional, as in that form of the Cult of the Quarters in which zenith and nadir are reckoned as cardinal points; and the mechanical symbol is complicated, and eventually modified, through the difficulty of depicting tridimensional relations on the bidimensional surface. Among the pueblo peoples this difficulty is overcome by bisecting two of the quadrants in a simple cruciform symbol in such manner as to produce the asymmetric figure  ; but the ever-acting mechanical tendency operates to produce the regular figure as the applications of the systems are extended. In either case, augmentation is effected by doubling or further increasing the peripheral extremities in such manner as to produce simple hexagrams, at first irregular,  , and eventually regular,  , or  . The value of successive augmentations is expressed by the figures 6+1, 12+1, 18+1, etc., i. e., by successive additions (mechanical or mental) to a once-reckoned Middle.

Now, comparison of these two number systems, especially as illumined by the Pueblo method of depicting the fifth and sixth directions, indicates that the higher is produced from the lower simply by the superposition of a binary system on the quaternary system; and the inference, coupled with the patent fact that the higher base is the measure of increased intellectual capacity, seems to define the course of development of both systems. True, it is difficult for the arithmetical thinker to see how the mathematical pioneer missed the now-plain road from the indefinite quaternary-quinary notion to the definite quinary concept; but the fact can not be gainsaid that the road *was* missed by many primitive tribes of especially mystical cast of mind, and that it was found and followed only by the ancestors of the practical Arabs with their decimal system, the barefoot Mexicans with their vigesimal system, and a few other peoples of exceptionally vigorous mind. The failure to find so plain a way may be ascribed largely to the complete domination of primitive thought by mystical concepts; and it would seem to repeat the demonstration by other facts that throughout much of prescriptorial culture little if any use

was made of nature's abacus, the ever present human hand—for a habit of finger-counting could hardly fail to fix the quinary system in the minds of counters able to grasp so high a number as five without aid of extraneous symbols.

The growth of the senary-septenary system out of the quaternary-quinary arrangement forcibly suggests the genesis of the latter; for just as the hexagram of the higher system represents the swastika of the lower system plus a trigram of the binary-ternary system superposed by almacabalic augmentation, so the swastika itself merely represents two superposed trigrams. This view of the growth of the three systems in the order of passage from the simple to the complex is supported by all that is known of the relative intellectual capacity of their users; and it would seem to be established by the occasional advances from the binary-ternary system to the quaternary-quinary plane by some of the Australian numerations, as well as by various vestiges of the binary-ternary system along various culture lines, notably the Mongolian and Aryan.

The presumptively primeval system apparently arose spontaneously (perhaps along lines noted later) and became fixed through habitual mental effort shaped less by purpose-wrought symbols than by personal or subjective associations. Analogy with the higher systems would indicate that the number-concept outlined vaguely through the dull mentation of the Australian blackfellows might be symbolized by any regular trigram uniting the perceived pair of objects and the unapperceived Ego, i. e., connecting the objective impression with its subjective reflex; but the inequality of all social pairs in the tribal organization, the ever-varying relative potencies of the good and evil mysteries, the unequal rank of the two ghostly Doppel-ichen, and divers other indications, would suggest that a better figure for the concept would be an irregular trigram. Yet howsoever the system be represented graphically by the student (for apparently the blackfellow had no notion of notation), the law of augmentation common to the two higher systems prevailed, as is shown both by certain of the Australian number-terms and by the Mongolian vestiges—i. e., the augmentation proceeded by successive additions to a once-reckoned middle, yielding the values 2+1, 4+1, 6+1.

It is questionable whether any enlightened student will ever enter sufficiently into the prescriptorial thought represented by any considerable number of distinct primitive peoples to grasp and record all the stages and substages in the growth of number systems; yet the records already extant would seem to indicate the lines of growth in fairly adequate fashion. The records are consistent in indicating that primitive peoples used integral numbers rather as symbols of extranatural potencies than as tokens for natural values; that they combined the symbols through mechanical devices by aid of a simple rule

tending to develop into algorithmic processes; and that the mechanical arrangements employed to represent the numerical combinations tended to develop into geometric forms and symbols—the several processes being characterized by the method of reckoning from an ill-defined unity counted but once in each combination.

GERMS OF THE NUMBER-CONCEPT

The course of intellectual development defined by the three prescriptorial number-systems (2–3, 4–5, 6–7) naturally leads interest toward the inception of the number idea among lower men—something which must always remain obscure, save as illumined by analogies with lowest men and higher animals. Now, the more intelligent feral animals and the lowest known savages are fairly comparable in their capacity for counting; they are also alike in another respect of such consequence as to shape the character of both—their lives (as Ernest Seton-Thompson so well shows for the animals) are lived in the shadow of tragedies unto often early and always tragic death. This great fact of inevitable tragedy overlays all other facts woven in the web of nascent mind; the most firmly fixed habit of lowly life is that of eternal vigilance; the everpresent thought is that of ever-present danger; the dominant motive is that of mortal fear.

No line of intellectual development can be fairly traced without full recognition of the ceaseless terrors of feral life; and the primeval interpretations of environment by animals and men alike manifestly reflect their tragic experiences: The fear-born cunning of the fox engenders that care for a way of escape without which he ventures on no advance; his every intuition is molded by living realization of a two-side universe—the danger side in van, the safety side in rear—with self as the all-important center; and only religious adherence to experience-shaped instincts enables him to survive and permits his tribe to increase. The sagacious crow, even in semidomestication, constantly betrays his notion of a two-side cosmos in frequent backward glances as he surveys the novel or forbidden field in front; and he is an arrant mystic, crazed with abject terror by night, replete with flippant joy by day, and given to the formless fetishism of hoarding uncanny things in well-hidden shrines.[1] In like manner nearly all animals, from the fiercest carnivores to the timidest herbivores, manifest constant realization of three overshadowing factors in nature as they know it—factors expressed by Danger, Safety, Self, i. e., by Death and Life to Self, or in general terms, the evil of the largely unknown and the good of the fully known coordinated in the vaguely defined subject of the badness and the goodness; and the chief social activities of animal mates and parents are exercised in gathering their

[1] Wild Animals I have Known, by Ernest Seton-Thompson, 1898, pp. 72, 83.

kind into the brightness of the known, and educating their native dread of all outer darkness. So, too, the more timid tribesmen of different continents betray, in conduct and speech, a dominant intuition of a terrible Unknown opposed through self to a small but kindly Known. This intuition is not born of intertribal strife, since it is strongest in those innately amicable family groups who (despite an implication of their designation) typify lower savagery, and since it is slowly modified with the rise of self-confidence among vigorous and aggressive tribes in whose minds the good grows large with the wax of conscious power; it is merely the subjective reflection of implacable environment—yet it is vaguely personified as a grisly and horrent bestial power, flaunting specters of death by tooth and claw, by serpent venom and swallowed poison, by pitiless famine and insidious disease, by wracking storm and whelming flood, by hydra-headed chance against half-felt helplessness; and over against this appalling evil there is a less completely personified good reflecting the small nucleus of confident knowledge with its far-reaching penumbra of faith. Accordingly, the lowest men and the higher animals seem much alike in their interpretation of nature—both rest their deepest convictions on a two-side cosmos connected in and through a largely passive Self.

A vague yet persistent placement of the two ever-present sides with respect to Self is clearly displayed in the conduct of animals and men—the evil side is outward, the good side at the place or domicile of the individual and especially of the group, as is shown by the homing instinct of the wounded carnivore, by the haste of the fire-crazed horse to meet the flames in his familiar stall, by human and equine nostalgia, and by the barbarian longing for burial in native soil. Moreover, both animals and men reveal indications of instinctive placement of the sides in the individual organism; and the indications consistently point to persistent intuition of face and back as the essential factors of self. Yet there is a significant diversity in the assignment of the sides of the organism to the sides of the good-bad cosmos: In general it appears that among the lower and the more timid the back stands for or toward the evil, the face toward the good, and that among the higher and more aggressive the face is set toward the danger; thus, defenseless birds and sheep huddle with heads together, savages sleep with heads toward the fire, and timid tribesmen tattoo talismans on their backs, while litters of young carnivores lie facing in two or more directions, self-confident campers sleep with feet to the fire, and higher soldiery think only of facing the foe. The interesting and significant growth of self-confidence need not be followed; it suffices to note that the primeval concept of the organic ego, as revealed in the conduct of animals and men, appears to be that of a face-back (and not bilateral) unity, with the two sides set toward the two aspects of a cosmos conceived in fear-born philosophy.

The passage of the primeval concept of a Face-Back Ego into that notion of two cardinal points suggesting a Cult of the Halves is happily represented among those Polynesian tribes who, according to Churchill,[1] have a system of geographic coordinates dominated by two cardinal directions, primarily seaward and landward, and secondarily northward and southward, respectively; while the language and customs connote a corresponding pantheon, capriciously malevolent on the sea side and steadily benevolent on the land side. This system of orientation is especially significant as a link in the chain of conceptual evolution, and equally as an explanation of the persistence of quasi-binary systems throughout Polynesia and Australasia with their shorelands of antithetic potencies; and no less significant are the facts in their bearing on the question of the habitat of primeval man, or of the orarian prototype already inferred from other facts.[2] Although varying from tribe to tribe in its relation to the meridian, this nascent orientation is no fleeting figment, but a deep-laid instinct so firmly rooted as to control every serious thought and direct every vital industry; indeed the Samoans and related navigators have developed their orientations into one of the most marvelous instincts in the whole range of animal and human life, viz, a cognition of definite albeit invisible sailing paths, whereby they are able to traverse the open Pacific, far beyond sight of land, with a degree of safety nearly equal to that afforded by chart and compass.

The Polynesian orientation at once illumines the unformulated Cult of the Halves, and opens the way to an explanation of the Cult of the Quarters; for each point of the shore is necessarily defined by sea in front and land in rear, and also by strands stretching toward the right and toward the left. Moreover, assemblages of Polynesians and Australasians, like the Iroquoian tribal councils, find it convenient to arrange themselves in coordinate groups or "sides," so placed laterally as to face a speaker at the end of the plaza or prytaneum; and there is good reason for opining that the collective habit was soon strengthened, even if it was not initiated, by the slight asymmetry of the human body whereby the left brain receives blood a little more directly than the right and gives proportional excess of strength and cunning to the right hand. The initial inequality was doubtless too slight to yield more than barely perceptible physiologic advantage to the dextral forelimb, as Brinton and Mason and others have shown; yet it may well have sufficed to set in operation a chain of demotic interactions leading to the survival of the right-handed and the extinction of the left-handed

[1] Personal communication. While United States consul at Samoa, Mr Churchill collected voluminous linguistic and other data well worthy of publication, though not yet issued. Conformably, Lesson and Martinet note that in Tahiti north and south are distinguished by denotive terms bearing a suggestive relation to tempestuous and milder winds, while east and west are without denotive designations, and are indicated only by descriptive phrases (Les Polynésiens, vol. II, 1881, p. 314.)

[2] The Trend of Human Progress: American Anthropologist, new series, vol. I, 1899, p. 423.

throughout the earlier eons of human development. A clue to the demotic process is easily found in widespread horror of left-handedness, especially among primitive peoples; the clue becomes definite in the light of systematic infanticide among many tribes, whereby all manner of natal deformity is eliminated; it becomes conclusive in the light of the customs of those American tribes who habitually eliminate the sinistral offspring as monsters betokening the wrath of the powers. So, apparently initiated by slight physiologic difference and unquestionably intensified by demotic selection, right-handedness became even more predominant among primitive men than among their less superstitious descendants; the dexter and dextrous hand came to be exalted in scores of languages as "The One That Knows How" or "The Wise One," while the sinister hand was degraded by linguistic opprobrium unto a symbol of evil and outer darkness. Naturally and necessarily the bilaterally symmetric division of the Ego into Right and Left fell into superposition with the antecedent Face-Back concept, and produced a quatern notion such as that expressed in the Cult of the Quarters. Happily this transition is crystallized in the language of the Pitta-Pitta of Queensland, which possesses directional inflections indicating Front and Back reckoned from the Ego; and it is especially significant (in connection with the bimanual count inferred by W. E. Roth) that the inflection for Front applies also to (right?) Side.[1]

It is evident that the passage from the Cult of the Halves to the Cult of the Quarters marked a considerable intellectual advance, both in extension and in intension; and it is evident, too, that the transition must have introduced novel and distinctive thought-modes, susceptible of growth into habits and hence of crystallization into instincts. Concordantly, men in several stages of culture as well as certain higher animals are found to display habits and instincts reflecting some such system of coordinates as that formulated in the Cult of the Quarters. The habits are especially prominent among the many primitive folks who ceremoniously venerate the cardinal points, systematically orient the doorways and other structural features of their houses, and maintain social relations in terms of direction. The instincts are particularly conspicuous among horses and kine and swine with their remarkable direction-sense, and most notable of all in the mule with its curiously concentrated hereditary intelligence, and the carrier-pigeon with its carefully cultivated homing-sense. In the present state of knowledge it would be impracticable to trace confidently the entire course of development of the direction-sense in animals and men, partly because so few naturalists have sought, like Ernest Seton-Thompson, to interpret the habits and instincts of lower animals, partly because so few anthropologists have really entered the esoteric life of primitive peoples; yet it is easy to perceive the general trend

[1] Ethnological Studies, p. 2.

of the developmental lines from an obscure beginning in higher animality to a conspicuous culmination somewhere in that lower humanity in which the direction-sense is fixed by generation on generation of direction-worship. And it is not to be forgotten that the quatern concept, born of unrecorded myriads of experiences and nurtured by unwritten eons of ceremonies, is much more than an idle fancy of kiva and camp-fire. Intensified by the strongest motives of primitive life, it doubtless attained maximum strength before writing arose to divide its functions; yet despite the decadence of millenniums, it still survives in one, if not both, of the two strongest instincts of higher humanity—the instinct of orientation, with the correlative instinct of right-handedness.

On the whole, it would seem safe provisionally to trace the beginnings of the number-concept in the light of common attributes of animals and men, and especially in the strong light afforded by the late-studied workings of primitive minds; and when this is done, the lines of natural development seem clearly to define a crude philosophy, or rather a series of intuitive thought-modes, whence all almacabalic and mathematical systems must necessarily have sprung.

MODERN VESTIGES OF ALMACABALA

The character of almacabala, and the strength of its hold on the human mind, are illustrated by numberless vestiges, mainly mystical numbers and cognate graphic symbols. The entire series of mystical numbers may readily be ascertained by juxtaposing the three almacabalic number systems and the products of their augmentation under the almacabalic rule. They are as follow (the super-mystical numbers accentuated):

2–3—3, 5, 7, 9, etc.
4–5— 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69, 73, etc.
6–7— 7, 13, 19, 25, 31, 37, 43, 49, 55, 61, 67, 73, etc.

The vestigial uses of the binary-ternary system are innumerable. Two persists as the basis of the semi-mystical Aristotelian classification, which still exerts strong influence on Aryan thought; 2 is the basis, also, of the largely-mystical Chinese philosophy in which the complementary cosmologic elements, Yang and Yin, are developed into the Book of Changes[1]; and it finds expression, either alone or in its normal union, in most Aryan cults. The mystical 3 pervades nine-tenths of modern literature and all modern folklore; it finds classic expression in the Graces and the Fates; it is particularly strong in Germanic and Celtic literature, cropping out in the conventional Three Wishes and Three Tests (a survival of the ordeal), and also as a customary charm number; and in these or related ways it persits in half

[1] Chinese Philosophy, by Paul Carus, 1898, p. 3 et seq.

the families and most of the child-groups even of this country and of today. The concept survives, also, in all manner of trigrams—triangles, triskelions, hearts, etc.—of mystic or symbolic character.

The quaternary-quinary system survives conspicuously in the form of graphic devices, especially the world-wide cruciform symbol, which has taken on meanings of constantly increasing nobility and refinement with the growth of intelligence. Hardly less conspicuous are the classic and later literary survivals in the Four Elements—air, earth, fire, water—of alchemistic philosophy, the Four Winds of astrology and medieval cartography, the Four Iddhis of Buddha, and the Four Beasts of Revelation, with their reflections in the ecclesiastic writing of two millenniums; while the survivals in lighter lore are innumerable. The system persists significantly also in its augmentals, especially 9, 13, 25, 49, and 61. The numerical vestiges are naturally for the most part quaternary, since the quinary aspect is merged and largely lost in algorithm.

The senary-septenary system survives as the bridge connecting almacabala and mathematics. In the graphic form it became Pythagoras's hexagram of two superposed triangles, the equally mystical hexagram of Brianchon, with which Paracelsus wrought his marvels, and the subrational hexagram of Pascal, while the current hexagram of the Chinese is apparently a composite of this and the binary as well as algorithmic systems. In the numerical form, 6 and more especially 7 play large rôles in lore and in the classic and sacred literature revived during the Elizabethan period; even so recently as the middle of the century the hold of the astrologic 7 was so strong as to retard general acceptance of the double discovery of the eighth planet, Neptune; and equally strong is the hold on the average mind of certain senary-septenary augmentals, particularly those coinciding with the augmentals of the lower systems. In idealized (or reified) form, the number 7 has exerted marvellous influence on thought and conduct, especially in the medial stages of human development; according to Addis, "The common Hebrew word for 'swear' meant originally 'to come under the influence of the number 7'"[1]; and this is but a typical example of reverence for the magical number among various peoples.

In tracing vestiges in the form of augmentals, it is clearly to be borne in mind that their significance, like that of the primary numbers, is mystical rather than quantitative, so that certain augmental numbers possess greater vitality than others of corresponding arithmetic grade. This is especially true of the almacabalic doubles, notably 9 as the first augmental of 5, and 13 as that of 7; for in these and other cases the first augmental is commonly of opposite sign, in almacabalic sense, from its basis—thus, 5 and 7 are beneficent or "lucky,"

[1] The Documents of the Hexateuch, part 1, 1893, p. 35.

while 9 and especially 13 are maleficent or "unlucky" numbers. Moreover, there is a further mystical intensification in squares of the bases (perhaps growing out of mechanical or arithmetical superpositions on the mystical notions); and the charm seems to be still further augmented by coincidences between the several systems. It is partly through this mystical accentuation of the always mystical augmentals that such numbers as 9, 13, 49, and 61 become conspicuous as factors and vestiges of almacabala.

Nine survives as a mystical number in the Muses of classical mythology, in Anglo-Saxon aphorisms emphasizing the vitality of the cat and the effeminacy of the tailor, and as a recurring tale in all of the superabundant Celtic lore such as that currently recorded by Seumas MacManus: it even survived in the schoolbooks of the early part of the century in the more curious than useful arithmetic process of "casting out the nines:" and throughout the last decade of the nineteenth century the newspaper-writing jugglers with nines found (and diffused) much mystery-tinged amusement in almacabalic analyses of the numbers 1890–1899.

Glaringly prominent in the mythology of recent centuries is the bode clustering about the ill-omened first augmental of "lucky" 7—indeed it is probable that nearly half of the enlightened citizens of the world's most intelligent country habitually carry the number 13 in their minds as a messenger or harbinger of evil. The almacabalic double of 13 (which is at the same time an augmental of 5) has largely lost its mystical meaning in Europe and America, apparently through friction with practical arithmetic; but it retains no little hold on the oriental mind, and finds expression in twenty-five-fold collectives in India and China, and in a rather frequent organization of Tibetan tribes into 25 septs or formal social units. Eminently conspicuous in Europe and America is the mystical number 49, especially when expressed as 7×7; for, in the belief of a large element of European population, the seventh son of a seventh son needs no training to fit himself for medical craft, while scanners of advertising columns of American newspapers may daily read anew that the seventh daughter of a seventh daughter is a predestined seeress.

Few of the larger mystical numbers have survived the shock of occidental contact; but they abound in the Orient. The coincidental-augmental 61 prevails in Tibet, where Sven Hedin found a lama, 1 out of 61 of co-ordinate rank, who professed survival for sixty-one millenniums, through a succession of exoteric deaths and esoteric reincarnations at uniform periods of sixty-one years;[1] and this odd value is explained by the designation of the sixty-first figure in the Mongolian hexagram—"The Right Way" or "In the Middle"[2]—which at

[1] Through Asia, by Sven Hedin, 1899, vol. II, p. 1132.
[2] Chinese Philosophy, p. 12.

the same time connects the Book of Changes with the nearly world-wide Cult of the Quarters and its mystical Middle. The numbers 63 and 65 are also mystical in Chinese philosophy, though their potency would seem to be dwarfed by the mechanical-arithmetical structure of the octonal square to which they have been adjusted evidently during recent centuries. Among the Hindu more or less mystical numbers abound, and many of these are found on analysis to correspond with conventional almacabalic augmentals and coincidentals; while the Buddhistic ritnals and series of aphorisms often run in measures of fives, with an initial or final supernumerary—the feature being apparently fixed by a mnemonic finger-count superposed on the almacabalic system, much as the octonal count is superposed on the mystical figures in the Chinese hexagram.

Suggestive vestiges of the mystical number-groups persist widely in the form of irrational and functionless supernumeraries, such as the thirteenth loaf in the baker's dozen, the twenty-first skerret in the coster's score, the thousand-and-first night of Arabian tale, and the conventional overplus in the legal "year and a day." It is possible that the supernumerary habit was crystallized in some cases by simple object-counting so conducted as to include an additional object as a tally; but there are many indications that the habit originally sprang from almacabalic augmentation, in which the sum is always one more than the multiple of the even-number basis. Moreover, the supernumerary habit is especially characteristic of countries and culture-stages in which mystical number-jumbles are rife.

Certain of the graphic vestiges of the quaternary-quinary system are of special significance; for just as the hexagrams of the senary-septenary system bridged the way from mystical almacabala to rational geometry, so the mechanical development of symbols exoterically quatern but esoterically quinary carried intelligence across the chasm dividing the morass of almacabala from the algorithmic forelands rising into the firm ground of arithmetic. True, the passage was made easier by the coincidental structure of the hand, that natural abacus which undoubtedly served to fix the quinary system in all minds trained up to the contemplation of fives; yet the way was apparently so long from the habitual perception of lowly twos and fours to the ready grasp and combination of fives that mechanical structure was even more efficient than organic structure in guiding progress. The graphic number symbols of the Mexican codices illustrated and discussed by Dr Thomas and others epitomized the growth of a vigesimal system crystallized by the coincidence of manual and pedal structures, while both the terms and the gestures of the Zuñi finger-count analyzed by Cushing point the way in which binary prepossessions passed into quinary practices despite the obstruction of the senary

concept.[1] The most conspicuous and persistent graphic vestiges are those of the barbaric Roman notation, which barred arithmetical progress for ages, and even to-day saps vitality by its crude extravagance in form and function. In certain aspects this notation may be considered binary, or rather dichotomous, and a reciprocal of the bifurcate classification of Aristotle with the Tree of Porphyry,[2] although, as has been well shown by Cushing, the integers of the system stand for fingers and represent in their combinations the ordinary finger-counts employed throughout the lower medial strata of cultural development. In reality the system is neither perfectly binary nor fully quinary, and still less is it susceptible (by reason of the indefiniteness[3] as well as the inelasticity of the notation) of development into a complete decimal system; yet its survival as a mere enumerative system opens a vista through the millenniums to a thought-plane in which men managed to exist without arithmetic, without number systems save of the crudest, without numerical bases of ratiocination, without traceable germs of ideas now fundamental in daily thought. The Chinese number symbols also show traces of genesis and development from the lowly plane of finger-counting; but to the Aryan mind the most striking vestiges of essentially prescriptorial thought relating to numbers are those conserved in the Roman notation.

The various vestiges, verbal, proverbial, and graphic (vestiges far too many for full enumeration), at once illumine prerational numeration and seem to establish that course of development of number-concepts suggested by the customs of people still living in the lower culture-stages. Conversely, the definition of almacabala serves to explain certain curious vestiges of primitive thought prevailing even today and in the highest culture; and the vestiges and developmental outlines combine to form a useful means of tracing the general course of intellectual progress from the obscure beginnings in lower savagery toward the present culmination in modern enlightenment.

[1] Manual Concepts, American Anthropologist, vol. v, 1892, pp. 289–317. It is to be observed that throughout this luminous discussion, than in which his genius never shone more brightly, Cushing confined himself to the middle strata of development in which numerical concepts are quinary, and in which counting is habitually manual, and made no reference to the lower strata of numerical conceptuality represesented by peoples less advanced than the Zuñi.

[2] The Foundation of Science, The Forum, vol. XXVII, 1899, p. 177.

[3] Thus a prodigal publisher may burden his title-page with the cabala MDCCCCI; if a shade less prodigal of ink, he may substitute the sign MDCDI; or if still more economical of ink and no less inconsiderate of the convenience of readers, he may recast the formula as MCMI.