Scale-Dependent Surface Roughness Behavior and Its Impact on Empirical Models for Radar Backscatter

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Abstract—One goal of radar remote sensing is the extraction of terrain statistics and surface dielectric properties from backscatter data for some range of wavelengths, incidence angles, and polarizations. This paper addresses empirical approaches used to estimate terrain properties from radar data over a wider range of roughness than permitted by analytical models. Many empirical models assume, at least implicitly, that roughness parameters like rms height or correlation length are independent of the horizontal length scale over which they are measured, in contrast to recent surveys of natural terrain, which show that self-affine, or power-law scaling, between horizontal scale and roughness statistics is very common. The rms slope at the horizontal scale of the illuminating wavelength \( \lambda \) is directly related to the variogram or structure function of a self-affine surface, can be readily obtained from field-measured topography, and, when used in an empirical model, avoids the need for arbitrary wavelength-dependent terms. To facilitate comparison with earlier approaches, an expression that links the rms height at some profile length with the rms (Allan) deviation at an equivalent horizontal sampling interval is obtained from numerical simulations. An empirical model for polarimetric scattering as a function of \( s(\lambda) \) at \( 35^\circ-60^\circ \) incidence from smooth to rugged lava surfaces is derived and compared with earlier models for backscatter from modestly rough soil surfaces. The asymptotic behavior of polarization ratios for the lava flows occurs as a first-order process, likely through single-scattering-like facets. Hagfors [1] postulated that the radar wavelength \( \lambda \) acts as a low-pass filter on the target terrain, such that the derived rms slope represents roughness on scales large compared to the wavelength, but the dominant horizontal scale is not determined. Efforts to invert the physical and geometric optics models, small-perturbation model (SPM), and integral equation model (IEM) lead to similar issues of appropriate terrain description and solution uniqueness (e.g., [2]-[8]).

I. INTRODUCTION

RADAR backscatter measurements have been long studied as a technique for estimating the physical properties of natural terrain. The interpretation of such observations, as with many other remote sensing methods, cannot be reduced to a unique inverse analytical model because descriptions of the potential variations in surface roughness and near-surface dielectric properties exceed the available free parameters of any plausible measurement. In consequence, quantitative radar remote sensing of sites for which no ground truth is available (such as an isolated terrestrial location or a planetary surface) is founded on one of the following three approaches.

1) Direct inversion of radar measurements to geophysical parameters based on a theoretical model for the relationship of the scattering process to surface topography and dielectric properties. A crucial aspect of such an inversion is whether the target surface roughness conforms to the mathematical restrictions invoked to obtain a solution for the backscattered echo and polarization state, and whether the chosen statistical parameters have a unique meaning (i.e., could they be measured in the field?). Perhaps the most familiar example of such a model in planetary studies is that of Hagfors [1], which describes the variation of near-nadir backscattered power with incidence angle in terms of surface reflectivity and a roughness term related to the statistical distribution of mirrorlike facets. Hagfors [1] postulated that the radar wavelength \( \lambda \) acts as a low-pass filter on the target terrain, such that the derived rms slope represents roughness on scales large compared to the wavelength, but the dominant horizontal scale is not determined. Efforts to invert the physical and geometric optics models, small-perturbation model (SPM), and integral equation model (IEM) lead to similar issues of appropriate terrain description and solution uniqueness (e.g., [2]-[8]).

2) Development of “semiempirical” models that seek a reduced number of free parameters for the inversion of analytic expressions. The basic approach is to calculate the backscatter coefficient, using an analytical model such as the IEM [9], for a large number of surface roughness and dielectric scenarios, then either identify parameters that play a second-order role in modulating the echoes or group some of these parameters into composite descriptors. This methodology has been explored in applications of the IEM, which requires up to three terrain parameters (rms height, correlation length, and the shape of the correlation function), to soil moisture and roughness studies. Several studies show that a composite roughness parameter can be related to the measured HH- or VV-polarized backscatter coefficients, but typically limit the IEM to single-scattering terms and modest roughness [10]. and may introduce additional parameters to describe the correlation function shape [11]. Combining the rms height and correlation function descriptors without an explicit dependence on horizontal scale [12], however, obscures the physical significance with respect to measurable terrain statistics.
3) Development of empirical models that relate radar echoes to measured terrain statistics and dielectric properties, guided to varying degree by assumptions about the functional form of the relationship based on simple physical scattering models. Empirical models with little constraint from theoretical studies may still yield significant insights into the general mechanisms of scattering and point the way toward more robust analytical approaches. This is particularly true when studying scattering linked with roughness outside the validity range of existing theory. One challenge to empirical studies is to obtain a diverse data set that encompasses the full range of interest for every parameter or a data set that spans a wide range for one parameter with little change in others.

With all of these methods, it is important to define the statistical descriptions of surface topography that are tractable for mathematical models of scattering, realistic in their representation of variations in roughness with horizontal scale, and "compact" in the sense of having only as many descriptive parameters as are required by the observed behaviors. Section II reviews recent studies on the statistical properties of natural surfaces, discusses the need for consistent approaches to measuring and characterizing topography, and advocates the use of a wavelength-scale rms slope parameter in studies relating surface roughness to backscatter. Section III reviews empirical modeling efforts for radar backscatter based on topographic and dielectric data, attempts to compare these within a common framework for the roughness description, and presents new models for roughness effects on polarimetric backscatter derived for a suite of lava flow surfaces with little dielectric contrast. Section IV summarizes the implications of these studies for the importance of particular scattering mechanisms in different polarization states, the degree to which physical parameters may be extracted from radar backscatter data, and implications for future studies of the scattering process.

II. STATISTICAL DESCRIPTIONS OF SURFACE ROUGHNESS

We define roughness as the statistical behavior of surface height $z$ here over a profile $z(x)$ but, most generally, over a surface $z(x, y)$. Two main approaches exist for describing terrain statistics, which differ in their treatment of roughness as a phenomenon that can have a strong dependence on the horizontal scale of measurement.

A. Autocorrelation Power Spectrum Description

Many radar scattering models, including the SPM and IEM, describe roughness through the autocorrelation function $\rho(\Delta x)$, where $\Delta x$ is the offset or lag between shifted versions of profile or surface data, and the correlation length $l$ is defined as the lag at which the normalized autocorrelation function $\rho(\Delta x)/\sigma^2$ falls to $1/e$ or about 0.37. The normalizing parameter $\sigma^2$ is implicitly the height variance over the profile length $L$ used in the analysis. Since the power spectral density (PSD) $W(f)$ of surface height is related to the Fourier transform of the autocorrelation function, the choice of $\rho(\Delta x)$ also determines the shape of the PSD

$$W(f) = \sigma^2 F[\rho_0(\Delta x)]$$

where $f$ is the spatial frequency in m$^{-1}$, and $F[\rho_0(\Delta x)]$ denotes the Fourier transform of the normalized autocorrelation function. Measured surface profiles often have an approximately power-law form PSD

$$W(f) = c \left(\frac{f}{f_0}\right)^{-\alpha}$$

where $c$ is the spectrum offset (with units of m$^3$, or topographic "power" in m$^2$, per unit spatial frequency), $\alpha$ is the dimensionless slope, and $f$ is normalized to a reference frequency of $f_0 = 1$ m$^{-1}$. For a 2-D $(x, y)$ representation, $c$ has units of m$^4$. The use of fixed values for $\sigma$ and $l$ to describe a surface implies that the roughness does not change with $L$, at least over the horizontal scales of interest (i.e., the range of radar wavelengths used in remote sensing). As discussed in Section II-B, this type of stationary behavior appears to be, at best, an end-member property among natural surfaces.

B. Self-Affine Description

Several authors note that natural surfaces can exhibit a strong dependence of roughness parameters on either the length of the profile $L$ or the sampling interval between profile points $\Delta x$ [13]–[26]. These observations suggest that a self-affine, or fractal, description of topography is appropriate, with significant consequences for exploring the link between roughness and radar backscatter. The topography of a self-affine surface is a noise process, in that there are no preferred spatial frequencies or periodic components, and the distribution of any sampled set of heights is Gaussian. The rms height of this noise signal scales as a power-law with the profile length but has no dependence upon the spatial sampling interval

$$\sigma(L) = \sigma(L_0) \left[\frac{L}{L_0}\right]^H$$

where $L_0$ is a reference length scale often taken to be 1 m for convenience. The Hurst exponent $H$ takes on values between zero and one, and is related to both the fractal dimension [13] and the PSD slope of the profile [14]

$$H = (\alpha - 1)/2.$$  \hspace{1cm} (4)

The rms slope $s$ of the surface height distribution is defined as the ratio of the rms, or Allan deviation $\nu(\Delta x)$, to the spatial sampling interval along the profile

$$s(\Delta x) = \frac{\nu(\Delta x)}{\Delta x} = \sqrt{\frac{\langle[z(x) - z(x + \Delta x)]^2\rangle}{\Delta x}} = s(\Delta x_0) \left(\frac{\Delta x}{\Delta x_0}\right)^{H-1}$$
where $\Delta x_0$ is a reference sampling interval, again typically taken to be 1 m. The rms height at some length scale is independent of the horizontal sampling interval; thus, even two points separated by $L$ provide one estimate of $\sigma(L)$. This has a high degree of uncertainty so finer sampling is required, and robust estimates of $\sigma$ typically come from $L > 10\Delta x$. The rms slope, however, is well characterized at the sampling interval for a reasonable number of profile points [26], which allows robust characterization of roughness even with what might be considered coarse sampling (e.g., 25-cm intervals as representative of roughness related to L-band backscatter). Such sampling does not reveal the properties of the roughness at finer scales, except by extrapolation based on the $H$ value derived for larger scales. As shown in Section III, many aspects of radar scattering appear to be well characterized by $s(\Delta x = \lambda)$, without further assumptions about the finer scale nature of the surface.

Self-affine characteristics can lead to strong variations in the roughness of a surface as a function of the probing radar wavelength, which raises concerns about linking theoretical models that use the autocorrelation power spectrum description with field topography measurements. The correlation length increases with $L$ [13], [14], [27], as does the rms height, but not in a linear sense that permits $\sigma(L)/l(L)$ to be scale invariant for all $H$. Unless $H = 0$, the values of $\sigma$ or $l$ based on profiles of different lengths will yield different predictions of backscatter when used with theoretical models based on the autocorrelation-PSD terminology. For example, difficulties in obtaining good model inversion results (particularly with the IEM) using Gaussian or exponential correlation functions have led to the development of hybrid functions that use at least one additional parameter to adjust the shape of $p(\Delta x)$ to the range exhibited by natural terrain [30], [31]. These modified forms may be interpreted as approximations to the behavior of self-affine surfaces with varying values of $H$.

The measurement of topography, typically in the form of profiles of finite length and discrete sampling, apply a spatial filter to the surface power spectrum. The behavior of the rms height as a function of profile length $L$ and the power spectrum descriptors $c$ (offset with units of m$^5$) and $\alpha$ (dimensionless spectrum slope) is given in [32]. This relationship is written here to preserve the unit dimensions noted and to include an explicit upper spatial-frequency limit

$$
\sigma(L, \alpha > 1) = \left[ \frac{1}{L} \int_{1/L}^{1/\Delta x} c \left( \frac{1}{f_0} \right)^{-\alpha} df \right]^{1/2} = \left[ \frac{c f_0^\alpha}{\alpha - 1} (L^{\alpha - 1} - \Delta x^{\alpha - 1}) \right]^{1/2}
$$

where $f_0$ is a reference spatial frequency taken for convenience to be 1 m$^{-1}$, and $\Delta x$ is the sample spacing along the profile. The value of the upper spatial frequency considered $1/\Delta x$ is only important as $\alpha$ approaches unity ($H = 0$), provided $L \gg \Delta x$.

In most cases, this expression can be simplified (dropping $f_0$) to a form consistent with (3)

$$
\sigma(L, \alpha > 1) = L^H \left[ \frac{c}{2H} \right]^{1/2}.
$$

There is some ambiguity in characterizing a surface with constant rms height ($H = 0$) over a range of profile length. Such behavior is commonly observed as a roll-off in the variogram of natural terrain [26], separating the spatial-wavelength interval between small-scale roughness induced by geologic (or agricultural) processes from larger scale structure associated with various landforms. In this case, the low spatial-frequency cutoff in (6) is the roll-off scale $L_r$, and the integration to define the constant rms height is carried out over finer length scales (i.e., where $H \neq 0$).

Extensive surveys of natural terrain [26] show the following:

1. $H$ tends to have a relatively narrow range (e.g., 0.3 to 0.7) about a value of 0.5, termed as “Brownian”; 2) there may be different values of $H$ for different ranges of horizontal scale (“multifractal” behavior); and 3) there is often a plateau, or roll-off, in the value of $\sigma(L)$ at a profile length anywhere from tens of centimeters to several meters. Some authors suggest hybrid surface descriptions incorporating large-scale tilts and smaller scale self-affine roughness [21] or a combination of scale-independent small-scale roughness and larger scale self-affine behavior [22], [24], but these introduce unnecessary complexity relative to a multifractal approach (for example, with $H$ approaching zero for a horizontal scale range over which is observed a nearly constant rms height).

The third observation is relevant to the applicability of field-measured topography as input to a theoretical model. If all profiles collected have a length greater than the roll-off scale of a particular terrain, there is no bias introduced by having profiles of varying length scale. The problem arises in not knowing from relatively short profiles (e.g., the 1-m length used by many investigators) whether this roll-off has been reached, but for relatively smooth surfaces this is probably a reasonable assumption. Possible errors in estimating self-affine descriptive parameters from topographic data sets limited in number and/or sampling length are further addressed in [19], [24], [26], and [33].

C. Wavelength-Scaled Roughness Parameters

The derivations of theoretical models for radar backscatter from natural surfaces often refer to the roughness parameters as having an implicit relationship to the probing wavelength. In the case of the SPM, the first-order backscatter coefficients are directly related to the PSD of topography at spatial wavelengths appropriate to yield a resonance phenomenon similar to Bragg scattering [34], [35]. Other authors discuss the concept of the radar wavelength as a low-pass filter for the topography, with spatial wavelengths at some scale finer than $\lambda$ contributing negligibly to the echo [1]. Given these results, Campbell and Shepard [17] proposed that scattering from self-affine terrain be described as a function of the wavelength-scaled rms height, $\gamma$, or rms slope $s(\lambda)$. 

$$
\gamma = \frac{2\pi}{\lambda} s(\lambda)
$$

$$
\gamma = \left[ \frac{2\pi}{\lambda} s(\lambda) \right]^{1/2}
$$

As shown, $\gamma$ is well characterized at the sampling interval for a reasonable number of profile points [26], which allows robust characterization of roughness even with what might be considered coarse sampling (e.g., 25-cm intervals as representative of roughness related to L-band backscatter). Such sampling does not reveal the properties of the roughness at finer scales, except by extrapolation based on the $H$ value derived for larger scales. As shown in Section III, many aspects of radar scattering appear to be well characterized by $s(\Delta x = \lambda)$, without further assumptions about the finer scale nature of the surface.
If we define the reference length as 1 m, and the rms height at this scale to be $\sigma_1$, then

$$\gamma = \frac{\sigma(\lambda = 1)}{\lambda} = \frac{\sigma_1}{\lambda} \left[ \frac{\lambda}{1} \right]^H = \sigma_1 \lambda^{H-1}. \quad (8)$$

The wavelength-scale rms slope, relative to that for a sampling interval of 1 m, $s_1$, is

$$s(\lambda) = s_1 \lambda^{H-1}. \quad (9)$$

The rms slope is suggested as the most robust parameter for modeling efforts, due to its ease of field measurement (profiles with sampling intervals of $\Delta x = \lambda$) and lower degree of sensitivity, relative to estimates of $\sigma$, to modest residual tilts along a profile segment [26]. To facilitate comparison with earlier models, we derived an empirical fit, for numerous simulated surface profiles with a range of $H$ and vertical scaling values, between the rms height and rms (Allan) deviation at some scale of measurement

$$\frac{\sigma(L)}{\nu(\Delta x = L)} = \exp(-2H) \frac{\sigma_1 \lambda^{H-1}}{\sqrt{2}}. \quad (10)$$

This expression matches the predicted behavior of a stationary surface when $H = 0$ [13].

III. EMPIRICAL STUDIES OF RADAR BACKSCATTER FROM ROUGH SURFACES

The empirical approach outlined in Section I is based on fitting functions, usually guided by expected forms from simple physical models, between measured surface properties and radar backscatter observations. The utility of a model must be judged on the scope of the reference data set (does it encompass the full range of possible conditions?) and the asymptotic limits of the chosen function (are the predicted values at very low and very high roughness physically reasonable?). Several authors have derived empirical fits between radar backscatter and surface roughness, and these models differ in their functional form, range of applicability, and asymptotic behavior.

A. Empirical Models Using the RMS Height and Correlation Length Parameters

Oh et al. [36] studied four bare soil fields, averaging the rms height for at least ten 1-m profiles in each field, and measured the dielectric properties at the surface and at a depth of 4 cm. The field-measured values of rms height at this 1-m horizontal scale varied from 0.3 to 3 cm. A truck-mounted radar system with wavelengths of 2.7, 6.3, and 24 cm was used to collect fully polarimetric backscatter data for incidence angles of 10° to 70°. The extrapolation of roughness to other horizontal scales was accommodated through the use of a $k\sigma = (2\sigma_0/\lambda)$ parameter. Oh et al. [37] revise this model, using a larger number of field sites, with roughness determined by averaging values from 1- and 3.5-m profiles. We cannot assess the bias introduced by averaging characteristics for different sample lengths; a Brownian ($H = 0.5$) surface would exhibit about a twofold increase in rms height over this change of $L$, but it is possible that the maximum relief is attained at some $L_r < 1$ m. The derived fits are

$$\sigma_{HH}^0 = 0.11m_\nu^{0.7}(\cos \phi)^{2.2} \left[ 1 - \exp \left( -0.32(k\sigma_1)^{1.8} \right) \right] \quad (11)$$

where $m_\nu$ is the soil moisture, $\phi$ is the radar incidence angle, and

$$\sigma_{VV}^0 = 0.10 \left[ \frac{\sigma_1 + \sin(1.3\phi)}{\lambda} \right]^{1.2} \left[ 1 - \exp \left( -0.9(k\sigma_1)^{0.8} \right) \right] \quad (12)$$

$$\sigma_{HH}^0 = 1 - \left( \frac{\phi}{90^\circ} \right)^{0.35m_\nu^{0.65}} \exp \left( -0.4(k\sigma_1)^{1.4} \right). \quad (13)$$

Oh [38] subsequently drops the dependence of the model on the correlation length

$$\sigma_{HH}^0 = 0.095 \left[ 1 + \sin(1.5\phi) \right]^{1.4} \left[ 1 - \exp \left( -1.3(k\sigma_1)^{0.9} \right) \right]. \quad (14)$$

As expected, the copolarized ratio (13) approaches unity at high values of roughness parameter. The cross-polarized ratio (14) approaches a value of about 0.1 for very rough surfaces when $\phi = 45^\circ$, which is lower than typical of rugged terrain [39]. This upper limit constrains the use of the model, since higher values of $\sigma_{HH}^0/\sigma_{VV}^0$ yield no solution for the roughness.

Dubois et al. [40] derived empirical functions for $\sigma_{HH}^0$ and $\sigma_{VV}^0$ from soil surfaces, using the roughness data of [36] and a study of eight additional sites. The derived fits [with corrections for typographical errors provided by J. van Zyl (personal communication, 2007) for $\phi = 30^\circ - 65^\circ$ are

$$\sigma_{HH}^0 = 10^{-2.75} \frac{\cos 1.5\phi}{\sin 5\phi} 10^{0.028\varepsilon'} \tan\phi \left( \frac{2\pi}{\lambda} \frac{\sigma_1 \sin \phi}{\lambda} \right)^{1.4} \lambda^{0.7} \quad (15)$$

$$\sigma_{VV}^0 = 10^{-2.35} \frac{\cos 3\phi}{\sin^5 \phi} 10^{0.046\varepsilon'} \tan\phi \left( \frac{2\pi}{\lambda} \frac{\sigma_1 \sin \phi}{\lambda} \right)^{1.1} \lambda^{0.7} \quad (16)$$

where $\varepsilon'$ is the real dielectric constant of the surface, and $\lambda$ is in centimeters. The variation of backscatter with wavelength has power-law forms of $\lambda^{-0.7}$ for HH and $\lambda^{-0.4}$ for VV polarization, which may be interpreted as a means to extrapolate $\sigma_1$ to the range of horizontal scales (radar wavelengths of 2.7 to 24 cm) used in the study. There is no roughness-induced variation in the dependence of $\sigma^0$ on the incidence angle, where other observations suggest that the angular scattering function has a shallower slope with $\phi$ for rougher terrain (e.g., [41]). The polarimetric behavior of this model may be illustrated by setting $\phi = 45^\circ$ and $\varepsilon' = 6$

$$\frac{\sigma_{HH}^0}{\sigma_{VV}^0} = 1.61 \left( \frac{\sigma_1}{\lambda} \right)^{0.3}. \quad (17)$$

For an L-band ($\lambda = 24$ cm) observation, a change of rms height from 0.3 to 3.0 cm yields an increase of $\sigma_{HH}^0/\sigma_{VV}^0$ from
TABLE I

<table>
<thead>
<tr>
<th>Site</th>
<th>Description</th>
<th>(H)</th>
<th>(s(25\text{ cm}))</th>
<th>(\sigma_y(m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ponded pahoehoe</td>
<td>0.62</td>
<td>0.073</td>
<td>0.014</td>
</tr>
<tr>
<td>2</td>
<td>Ponded flows with tumuli</td>
<td>0.68</td>
<td>0.172</td>
<td>0.036</td>
</tr>
<tr>
<td>3</td>
<td>Platy pahoehoe</td>
<td>0.51</td>
<td>0.436</td>
<td>0.085</td>
</tr>
<tr>
<td>4</td>
<td>Billowy pahoehoe toes</td>
<td>0.48</td>
<td>0.324</td>
<td>0.062</td>
</tr>
<tr>
<td>5</td>
<td>A' a</td>
<td>0.26</td>
<td>0.705</td>
<td>0.120</td>
</tr>
<tr>
<td>6</td>
<td>Ropy pahoehoe</td>
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<td>0.222</td>
<td>0.041</td>
</tr>
<tr>
<td>7</td>
<td>A' a</td>
<td>0.29</td>
<td>0.586</td>
<td>0.105</td>
</tr>
<tr>
<td>8</td>
<td>Pahoehoe sheet flows</td>
<td>0.49</td>
<td>0.225</td>
<td>0.046</td>
</tr>
<tr>
<td>9</td>
<td>Pahoehoe sheet flows</td>
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<td>0.147</td>
<td>0.032</td>
</tr>
<tr>
<td>10</td>
<td>Ponded pahoehoe</td>
<td>0.63</td>
<td>0.076</td>
<td>0.017</td>
</tr>
</tbody>
</table>

Fig. 1. AIRSAR radar image of Kilauea Volcano and lava flows in the adjacent Ka'u Desert. Radar wavelength: 24 cm; VV polarization. Image width: About 15 km. Radar flight line is along the top of the image. Labeled sites correspond to the location of topographic profiles shown in Fig. 2. Each site was imaged at three different incidence angles from about 35° to 60° in three parallel offset flight lines.

![Image](image.png)

Fig. 2. Detrended (tilt removed) topographic profiles, at 25-cm sample spacing, for ten Kilauea lava flows. Profiles are offset vertically for clarity but have identical vertical scaling.

![Image](image.png)

Fig. 3. Plot of HH- versus VV-polarized backscatter coefficients (in decibels) at 24-cm wavelength for the ten lava flow sites, at three values of the incidence angle for each. The line has a slope of \(\sigma_{HH}/\sigma_{VV} = 1.0\).

0.43 to 0.86. This is in reasonable agreement with the range observed for Hawaii lava flows when \(\gamma\) is less than 0.05 [42]. Dubois et al. [40], however, point out that the model predicts unrealistic values of \(\sigma_{HH}/\sigma_{VV} > 1\) as roughness increases. A comparison of Dubois model predictions using Kilauea topography data with Airborne Synthetic Aperture Radar (AIRSAR) L-band data near 45° [41] suggests that the model overestimates the echoes by 5-6 dB on very smooth terrain and by 0.5 to 3 dB near the applicability limit of \(\sigma_1 > 5\) cm (see also [43]). These mismatches for dry rocky terrain may reflect the fact that the Dubois model was developed for moist soils with relatively high dielectric constants.

B. Empirical Models Based on Scale-Dependent Roughness Parameters

Lava flows in Hawaii offer an interesting comparative data set for the development of empirical models. Their dielectric constants span a narrower range than moist fields, and they exhibit a wide range of surface texture at the centimeter to meter scale. This allows a focus on the effect of roughness on backscatter as a function of horizontal scale length and radar incidence angle. Using AIRSAR data (5.7-, 24-, and 68-cm wavelengths, Fig. 1) and topographic profiles (Fig. 2; Table I) for lava flows on Kilauea Volcano, Campbell and Shepard [17] derived a fit between \(\sigma_{HH}^0\) and the wavelength-scale rms slope using a functional dependence on roughness similar to (11). The use of the \(s(\lambda)\) parameter offered a means to directly interpret echoes at a range of radar wavelengths as samples of the variogram, or structure function, at different horizontal scales, and avoided the ad hoc power-law scaling with \(\lambda\) included in many other models.

A complete set of empirical backscatter models is developed here from the Kilauea topography data and the AIRSAR 24-cm radar echoes. Each of the ten field sites was observed at three incidence angles via the image data from three parallel offset flight lines. We begin with a relationship between the surface roughness, incidence angle, and the HH- or VV-polarized echoes, which are similar for all but very smooth terrain [41] (Fig. 3)

\[
\sigma_{HH,\text{VV}}^0 = 0.16 \left[ 1 - \exp \left\{ -70.372s(\lambda)^2 \exp(-0.0644\phi) \right\} \right]
\]  

(18)

The cross-polarized linear echo is represented by

\[
\frac{\sigma_{HH}^0}{\sigma_{VV}^0} = \frac{1}{3} \left( \frac{\phi}{90} \right) (1 - \exp[-4.5s(\lambda)])
\]  

(19)
where $\phi$ is in degrees. This expression has an asymptotic limit of $1/3$ for very rough surfaces at high incidence angle and approaches zero at nadir ($\phi = 0$). The ratio of linear and circular “depolarized” components has a rather narrow range, represented by

$$\frac{\sigma_{0LL}^0}{\sigma_{0L}^0} = 0.3 + 0.2 \left(1 - \exp[-4.5s(\lambda)]\right)$$

and an asymptotic limit of 0.5 for rough terrain [39]. The ratio between circular and linear “polarized” components has little dependence on roughness or incidence angle (Fig. 4)

$$\frac{\sigma_{0LR}^0}{\sigma_{0VV}^0} = 0.9.$$  \hspace{1cm} (21)

Thus, we can readily derive the circular polarization ratio (CPR) as a function of roughness and incidence angle (Fig. 5). These expressions provide a good fit to the 24-cm backscatter coefficients and CPR over the large range of roughness represented by the Kilauea lava flow sites (Fig. 6). Based on the results of

![Image](image-url)
extending the use of empirical models to rocky terrain on the derived values of the rms height. Campbell [46] used the three computer simulations of these rocks. The radar backscatter curve in geologic settings where the dominant cause of roughness is individual rocks at or near the surface. For example, Deroin et al. [45] measured the size distribution and maximum height of rocks within areas 0.5 m on a side in the western Sahara and inferred the rms height of the topography from computer simulations of these rocks. The radar backscatter values for these sites were compiled from C-band European Remote Sensing 1 data, and they found good agreement between backscatter and maximum rock height using the function $h_{\text{max}} = 23\exp(0.2a^2)$. A similar functional dependence was found between the backscatter coefficient and the simulation-derived values of the rms height. Campbell [46] used the three wavelengths provided by the AIRSAR system (5.7, 24, and 68 cm) and showed that the rms height of a rock-strewn surface in Hawaii modulates the backscatter in a similar manner to "continuous" rough terrain [17]. This result is important in extending the use of empirical models to rocky terrain on the Moon and Mars.

IV. SYNTHESIS AND CONCLUSION

1) At the broadest level, it appears that radar backscatter has a simple functional relationship with roughness descriptions anchored at the horizontal scale of the probing wavelength. This does not imply a single dominant physical mechanism or scale length in surface scattering, but rather, that the roughness giving rise to each mechanism on various terrains has some common relationship with that near the wavelength scale. Practical experience shows that the wavelength-scale rms slope is the most robust parameter that can be obtained from field-measured profiles, given the need to sample only at a horizontal spacing near $\lambda$ and the modest sensitivity of $s$ to background tilts. The adoption of a self-affine description of the terrain is not a prerequisite to the application of such a parameter but provides a useful means of predicting the scaling of $\sigma^0$ with $\lambda$.

2) The cross-polarized linear $\sigma^0_{\text{HH}}$ and $\sigma^0_{\text{HV}}$ and same-sense circular $\sigma^0_{\text{LL}}$ and $\sigma^0_{\text{RR}}$ backscatter coefficients (often termed "depolarized" in planetary radar literature) have asymptotic behaviors with roughness that mimic those of randomly oriented dipolelike scatterers ($\sigma^0_{\text{HH}}/\sigma^0_{\text{LL}} = 0.5, \sigma^0_{\text{HH}}/\sigma^0_{\text{VV}} = 1/3$) [39, 47]. Such scattering features might correspond to rock edges, ground surface cracks, or other topographic discontinuities. The HV-polarized returns emerge as a second-order component in most theoretical treatments of scattering and are sometimes interpreted as solely arising from multiple scattering (e.g., [9]). The relative strength (up to 15%-20% of the like-polarized echo) and the relationship between the linear- and circular-polarized components argue that these returns instead occur primarily via single scattering from surface topographic discontinuities [48]. Such a mechanism was described at least as early as [49]: "... returns at oblique angles of incidence arise through single scattering from discrete objects. These discrete scatterers may, as a first approximation, be thought of as linear dipoles of more or less random orientation." The dipolelike elements are complemented by mirrorlike parts of the surface, which produce strong like-polarized linear (with $\sigma^0_{\text{HH}} = \sigma^0_{\text{VV}}$) or opposite-sense circular reflections e.g., [50]. The inability of current theoretical treatments, even those incorporating self-affine scaling properties [51, 52], to address topographic discontinuities poses a challenge to adequately capturing the first-order scattering components, for a wide range of surface roughness, across all polarization states.

3) For some geologic environments, such as dry lava flows on Venus, the empirical functions discussed in Section III provide a robust tool for estimating surface roughness parameters. In other settings, there may be substantial additional effects due to near-surface dielectric changes (e.g., soil moisture on Earth) and/or the importance of subsurface returns arising from interfaces or suspended scatterers (e.g., rocks in the lunar regolith or cracks in the icy shells of outer-planet satellites). The estimation of the dielectric constant is possible only in very constrained situations. For example, the $\sigma^0_{\text{HH}}/\sigma^0_{\text{VV}}$ ratio depends only upon the radar incidence angle and $\alpha^2$ for the first-order SPM, but the smooth surfaces amenable to such analysis may also permit strong modulation of the echo polarization via the Fresnel transmission coefficients of the surface and the strength of the volume-scattered component.
(e.g., [42] and [53]). Polarimetric analysis methods can suggest the occurrence of subsurface scattering but do not provide strong constraints on $e'$ (e.g., [54]). There is also evidence that multiple scattering is a significant component of echoes in settings where smooth-sided rocks or other objects are closely spaced at the surface or suspended in a low-loss matrix [55] or where a very low-loss medium (particularly ice) with included voids permits coherent superposition of signals traveling along time-reversed paths back to the observer (e.g., [56] and [57]). New studies are needed to understand single and multiple scatterings among arbitrary-shaped rocks [58] on a surface or suspended in a low-loss medium, in order to capture the range of properties exhibited by the lunar or martian regolith. It may be more plausible to model the dielectric constant when $e'$ is large enough, or surface scattering so predominates, that we can ignore penetration effects. In this case, it appears that the $\sigma_{HV}/\sigma_{VV}$ ratio (14 or 19) can be used to isolate the effects of roughness and to thus solve for the reflectivity-induced changes in radar brightness. In planetary remote sensing, however, the surface dielectric constant may be too low (e.g., 2.7 for the lunar regolith and 2.5–3.2 for outer-planet ices) to ignore subsurface scattering except at very small $\lambda$.

4) There is considerable opportunity for further work. There are still a small number of terrain profile databases, and these vary in their spatial sampling and included range of roughness and dielectric properties. Given the increasing capability of computational methods, it may be most effective to construct large numbers of self-affine synthetic surfaces, with resolution adequate to approximate topographic discontinuities, and calculate their backscatter as a function of polarization, $s(\lambda)$, dielectric constant, and incidence angle. This may yield more refined empirical representations and reveal the feasible degree of discrimination between roughness and $e'$ changes from multipolarization data.

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REFERENCES


