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The Upper Atmosphere and Satellite Drag

By R. J. STIRTON

The atmospheric drag force on artificial satellites is of interest both for orbit computations and for computing the density of the upper atmosphere from the observations of satellite motion. Density determination by this means has been suggested by Davis, Whipple, and Zirker (1956), Spitzer (1956), and Jones and Bartman (1956). Harris and Jastrow (1958) and Sterne, Folkart, and Schilling (1958) have given values of the density obtained from orbit data of Satellites 1957 a1, 1957 a2, and 1957 b.

These determinations depend on a knowledge of the drag coefficient $C_D$, which is defined by the usual equation for the drag force $F$,

$$F = \frac{1}{2} \rho C_D v^2 A,$$

where $\rho$ is the atmospheric density, $v$ is the satellite velocity, and $A$ is a characteristic area. For spherical satellites the value $C_D = 2$ (based on cross-section area) has been used (Davis et al., 1956; Spitzer, 1956). For other shapes Sterne and Schilling (1958) suggest that $C_D = 2$ is a good approximation if one fourth of the total area of the body is used for $A$. Harris and Jastrow (1958) used $C_D = 2.3$ in their determination of air density.

These values of $C_D$ are based on the following assumptions:

(a) The mean free path of the molecules is much greater than the satellite dimensions ("free-molecule flow").
(b) The satellite velocity $v_o$ is much greater than the mean random velocity $\bar{v}$ of the gas molecules.
(c) The mechanisms that reflect the impinging molecules range from specular reflection to inelastic impact and diffuse evaporation at an effective surface temperature $T_e$.

In general, at a given density, the drag coefficient for free-molecule flow depends on $\bar{v}$ (or the absolute kinetic temperature $T_e$ and the molecular weight $M$ of the gas).

The drag coefficients of a sphere for specular reflection $C_{D_1}$, inelastic impact $C_{D_2}$, and diffuse evaporation $C_{D_3}$ are derived in pages 12-13 and are graphed in figure 1 as functions of the parameter $a$:

$$a = \frac{v_o}{\sqrt{\frac{2\nu}{\overline{C}}}} = \frac{\sqrt{m}}{2kT}$$

where $m$ is the molecular mass and $k$ is the Boltzmann constant. It is noteworthy that for a sphere $C_{D_2} = C_{D_3}$, a condition that is unique to this shape. The derivation is based on the Maxwell-Boltzmann distribution of velocities of the gas molecules, but it does not impose condition (b) above. The curve of $C_{D_3}$ in figure 1 is for complete temperature accommodation, and therefore represents the upper bound of the drag coefficient for diffuse evaporation (see eq. 18, p. 13).

As a specific example, consider a spherical satellite with velocity $v_o = 7.5 \times 10^4$ cm/sec. The drag coefficients are shown in figure 2 as functions of the temperature and for various values of the molecular weight. Even for very high gas temperatures, and with the low densities in the upper atmosphere, the surface of the

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Figure 1.—The drag coefficients for a sphere.
satellite certainly does not completely accommodate, and the drag coefficient for evaporation is shown for \( T' = 100^\circ K \) to \( 1,000^\circ K \) only.

Clearly the drag coefficient \( C_D = C_f \) does not depart seriously from the limiting value of 2 unless the molecular weight is low and the temperature is high. One is fully justified in using the limiting values of \( C_D \) for the density determinations cited above. However, it is of interest to consider possible situations in which the drag coefficient \( C_D \) could depart from the limiting value of \( C_D = 2 \) for the sphere.

Considerable theoretical and experimental work has been done on the properties of the upper atmosphere.\(^2\) Four recent papers are particularly pertinent to the present discussion. Miller (1957) has given the temperature and molecular weight of the atmosphere to 600 km. Nicolet (1958) has indicated that the densities derived from observations of satellites (Harris and Jastrow, 1958; Sterne and Schilling, 1958; Sterne, Folkart, and Schilling, 1958) require both higher temperatures and higher densities for the upper atmosphere than was previously expected. See for example Nicolet's (1954) atmosphere. Chapman (1957) has treated the effect of the solar corona on the upper atmosphere. From these papers it is interesting to attempt to construct a crude atmosphere extending above 500 or 600 km. Temperature and molecular weight are of particular interest since they can have important effects on the drag coefficient.

Chapman (1957) suggested that the temperature of the coronal gas in the region of the earth's orbit may be on the order of \( 220,000^\circ K \). He later revised the estimate to about \( 100,000^\circ K \) (Chapman, in press), and also suggested that the earth's atmosphere would merge with the coronal gas at a distance of about 50 earth radii. Using these data let us extrapolate Nicolet's (1954) atmosphere to \( 3 \times 10^8 \) km (50 earth radii), as shown in figure 3. Nicolet (1958) has recently indicated that the temperature may be on the order of \( 3,000^\circ K \) at 500 km. An extrapolation from this temperature point is also shown.

Miller (1957) has given the molecular weight of the atmosphere to 600 km. We might suggest, as a first guess, that the molecular weight would continue to fall until the nitrogen and oxygen were completely dissociated and ionized. Also the very high regions of the upper atmosphere might be expected to be composed largely of hydrogen in an ionized state. At about \( 30,000^\circ K \) it should be nearly fully ionized and therefore the molecular weight should have a limiting value of \( M = \frac{1}{2} \) when the temperature reaches \( 30,000^\circ K \). From these ideas and the temperature curve of figure 3, Miller's molecular weight is extrapolated as shown in figure 4.

Now let us consider a family of circular satellite orbits with various radii, and estimate the
drag coefficient as a function of distance above the surface. If we use the estimated molecular weight and temperature curves of figures 3 and 4, and an $r^{-1}$ dependence of the satellite velocity on the radius of the orbit, we can compute $C_D^{0}$, shown in figure 5 as a function of the altitude. Under these circumstances $C_D^{0}$ departs from the limiting value of 2 around $10^4$ km.

If, as has been proposed by Jones and Bartman (1956) and the Technical Panel (1958), large balloon-like satellites are used to measure atmospheric density at very high altitudes, precise results require that some consideration be given to the departure of the drag coefficient from the limiting value corresponding to $v_0 \ll C$.

Even with the value of $C_D$ based on the discussion given here, a number of effects should be considered before the density is inferred from the observed motions of satellites in the very high atmosphere. The gas is probably not in equilibrium in the thermodynamic sense. Solar activity will affect the amount and depth of penetration of coronal gas in the earth's atmosphere. The free path is comparable to the size of the earth, and the particles of the gas can be in orbital motion around the earth rather than in the random motion postulated for the Maxwell-Boltzmann distribution. Ionization complicates this problem further, and the region of the so-called ring current (see Technical Panel, 1958) will certainly not have random velocity distribution.

The satellite will carry some electrostatic charge, which may modify the drag. Perhaps a satellite with an electrometer would yield some interesting results in this connection. Eddy currents induced in the conducting parts of the satellite will result in an energy loss, which will be a draglike effect.

The molecular-flow drag on a sphere

The problem of the resistance to the motion of a sphere moving through a fluid has been treated extensively in the literature. Newton considered the fluid as being composed of mass points at rest. G. C. Stokes later treated the problem and obtained the well-known Stokes law. Others, notably Millikan (1923) and Epstein (1924), refined the theory to apply to sphere velocities much smaller than the mean thermal velocity of the gas molecules. More general treatments have been carried out by others. For example, Heineman (1948) considers the sphere and a number of other shapes for gas densities so low that the mean free path is much greater than the body size. No restriction was imposed on the sphere velocity.

In the present paper, the drag on a sphere is calculated for free-molecule flow. The method follows the general approach of Epstein (1924) but removes the restriction of low velocity of the sphere. In particular, the drag is found for molecular impact, specular reflection, and diffuse reflection (or impact and evaporation) without complete temperature accommodation. These cases seem pertinent to the artificial earth satellite and other space vehicles. The intermediate results,
equations (4) and (8), are equivalent to those obtained by Heineman (1948).

The gas is assumed to be in thermal equilibrium, which the sphere's motion does not disturb. The Maxwell-Boltzmann distribution of molecular velocities then may be used. Consider the sphere at rest and that the gas flows over it with velocity \( v_0 \). Then, the distribution of velocities relative to the sphere is that given in equation (1):

\[
f_{v_x,v_y,v_z} = N \beta^3 \frac{\beta^2}{\pi^2} \exp \left[ -\beta^2 \left( (v_x + \lambda v_0)^2 + \right. \right.
\left. (v_y + \mu v_0)^2 + (v_z + \nu v_0)^2 \right] \tag{1}
\]

where \( N \) is the number density; \( \lambda, \mu, \) and \( \nu \) are the direction cosines of \( v_0 \); \( v_x, v_y, \) and \( v_z \) are the components of the molecular velocity \( v \); and also where \( \beta^2 = m/2kT \); \( m \) is the molecular mass, \( k \) is the Boltzmann constant, and \( T \) is the absolute temperature.

The number of molecules with velocity \( v \) striking a surface element \( ds \) per second is \( \mathbf{v} \cdot ds \), and the momentum transferred per second to \( ds \) is then \( m\mathbf{v} \cdot ds \). Further, by symmetry arguments, the components of momentum balance except along \( v_0 \), so that the desired momentum component is \( m(\lambda v_x + \mu v_y + \nu v_z)v_0ds \) if the surface element \( ds \) is along \( y \). The momentum transferred per second to \( ds \) is given by the equation

\[
\dot{P}_vds = m \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{v_x,v_y,v_z}(v_x+\mu v_0+\nu v_0) v_0 v_x d\xi d\eta d\phi ds. \tag{2}
\]

By introducing the variables, \( \xi = v_x + \lambda v_0, \eta = v_y + \mu v_0, \) and \( \zeta = v_z + \nu v_0 \), equation (2) may be reduced to

\[
\dot{P}_vds = \frac{Nm}{4\pi^2 \beta^2} \int_{\xi=0}^{\xi=1} \left\{ \pi^{1/2 + 2} \int_{0}^{\infty} e^{-x^2} dx \right\} + \frac{2ae^{-x^2}}{a^3} ds. \tag{3}
\]

where \( a = v_0 \beta \).

We now integrate equation (3) over the entire surface of the sphere of radius \( r \). The element of area is \( ds = r^2 \sin \theta d\theta d\phi \), where \( \theta \) is the colatitude and \( \phi \) is the longitude of \( v_0 \). It may be seen that \( \mu = \cos \theta \), and \( \dot{P} \) becomes

\[
\dot{P}_v = \frac{\pi^{1/2} Nm}{2\pi^2 \beta^2} \left[ \int_{0}^{\pi/2} \cos \theta \sin \theta \left(1 + 2a^2\right) \left\{ \pi^{1/2 + 2} \int_{0}^{\infty} e^{-x^2} dx \right\} + \frac{2ae^{-x^2}}{a^3} \right] d\theta. \tag{4}
\]

By some integration by parts the total drag force due to molecular impact becomes

\[
\dot{P}_v = \frac{\pi^{1/2} Nm a^2}{2\pi^2 \beta^2} \left[ \frac{4a^4 + 4a^2 - 1}{a^4} \int_{0}^{\pi} e^{-x^2} dx + \frac{1 + 2a^2}{a^3} e^{-a^2} \right]. \tag{5}
\]

The momentum transfer for a molecule striking \( ds \) and reflecting specularly is \( 2mv \) along \( ds \). Again by symmetry arguments, the component along \( v_0, 2mv_0 \mu, \) is all that is required. The momentum transferred to \( ds \), by use of distribution (1), is

\[
\dot{P}_v = 2m \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{v_x,v_y,v_z} v_0 v_x d\xi d\eta d\phi ds. \tag{6}
\]

By changing to the variables \( \xi, \eta, \) and \( \zeta \) as above, equation (6) becomes

\[
\dot{P}_v = \frac{Nm}{4\pi^2 \beta^2} \left[ \mu \pi^{1/2 + 2} \int_{0}^{\infty} e^{-x^2} dx + 2ae^{-x^2} \right] ds. \tag{7}
\]

Integration over the sphere gives the total drag force \( \dot{P}_v \) for specular reflection:

\[
\dot{P}_v = \frac{Nm \pi^{1/2} \left[ \pi^{\cos \theta \sin \theta} + \right. \right.
\left. 2a^2 e^{-a^2} \cos \theta \sin \theta + 2a^2 e^{-a^2} \cos^2 \theta \sin \theta + \right. \right.
\left. (2 \cos \theta \sin \theta + 4a^2 \cos^2 \theta \sin \theta) \right] \int_{0}^{\pi} e^{-x^2} dx \right\} d\theta; \tag{8}
\]

and again, by some integration by parts, equation (8) reduces to

\[
\dot{P}_v = \frac{\pi^{1/2} N m a^2}{2\pi^2 \beta^2} \left[ \frac{4a^4 + 4a^2 - 1}{a^4} \int_{0}^{\pi} e^{-x^2} dx + \frac{1 + 2a^2}{a^3} e^{-a^2} \right], \tag{9}
\]

which is identical with equation (5).
The third case discussed here is the drag due to molecular impact plus that due to evaporation of the molecules. Assume that the molecules that impact upon the surface transfer all their momentum and then evaporate with a Maxwell distribution of velocities corresponding to a surface temperature $T'$, which is constant over the sphere. This may be regarded as an assumption of perfect thermal conductivity of the sphere. Current artificial earth satellites (before the orbits decay into dense atmosphere) are generally at temperatures on the order of a few hundred degrees Kelvin rather than possibly the thousands of degrees Kelvin corresponding to the kinetic temperature of the gas at very high altitudes. Complete thermal accommodation does not occur with these artificial satellites, and the energy balance is maintained largely by radiation processes.

The distribution of velocities of the evaporated molecules relative to the sphere is given by the equation

$$f_{v_0} = B \exp \left[ -\beta^2 (v_a^2 + v_0^2 + v_b^2) \right]$$ (10)

where $\beta^2 = 2kT'/m$, and the normalizing factor $B$ is determined by equating the number of molecules striking and evaporating from $ds$.

The number of molecules striking $ds$ per second with velocity $v$ is, as before, $v \cdot ds$, and with distribution (1) the total number striking $ds$ per second is

$$n ds = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{v_0} v_0 dv_0 dv_0 dv_0 ds.$$ (11)

After the introduction of the variables $\xi$, $\eta$, and $\zeta$ and the parameter $a = v_0/\beta$, equation (11) reduces to the following:

$$n ds = \frac{N}{2\beta^4} \left\{ \pi^2 a + e^{-a^2} + 2\pi a \int_{-\infty}^{\infty} e^{-a^2} dx \right\} ds.$$ (12)

The number of molecules evaporating, $n'$, from $ds$ is

$$n' ds = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{v_0} v_0 dv_0 dv_0 dv_0 ds = \frac{\pi}{2\beta^4} B ds,$$ (13)

or

$$B = \frac{N\beta^4}{\beta^4} \left\{ \pi^2 a + e^{-a^2} + 2\pi a \int_{-\infty}^{\infty} e^{-a^2} dx \right\}.$$ (14)

The net momentum carried away by a molecule evaporating from $ds$ is $mv$, since by symmetry the other components balance. The total momentum carried away per second from $ds$ is then

$$\dot{P} ds = m \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{v_0} v_0 dv_0 dv_0 dv_0 ds = \frac{B m \pi^2 \rho}{4\beta^4} ds.$$ (15)

Only the component of equation (15) along $v_0$ is required. Then with the value of $B$ from equation (14) the drag force $F_{D0}$ due to evaporation is

$$F_{D0} = \frac{\pi^2 N \rho}{2\beta^4} \int_{0}^{\pi} \left\{ \pi^2 \cos^2 \theta \sin \theta + 2a \cos^2 \theta \sin \theta \int_{0}^{\infty} e^{-a^2} dx \right\} d\theta = \frac{N \rho \pi^2 \rho}{6} \frac{\pi}{\beta} N \pi^2 v_0 C',$$ (16)

where $C'$ is the average velocity of the evaporated molecules.

The drag coefficients corresponding to either equation (5) or (9) and to equation (16) are readily obtained from the definition of $C_D$ in the equation for the drag force for the sphere,

$$F = \frac{1}{2} \rho C_D \pi^2.$$

The factor $Nm$ is just the mass density, $\rho$, and the coefficients for the various mechanisms become the following:

(a) Molecular impact or specular reflection:

$$C_D^{(o)} = C_D^{(o)} = \frac{1}{\pi^4} \left[ \frac{4a^4 + 4a^2 - 1}{a^4} \int_{0}^{\infty} e^{-a^2} dx \right]$$ (17)

(b) Evaporation:

$$C_D^{(e)} = \frac{\pi C'}{3} \frac{2a^2}{3a^2}.$$ (18)

(c) Molecular impact and evaporation (diffuse reflection):

$$C_D^{(o)} = C_D^{(o)} + C_D^{(e)}.$$ (19)
Two limiting cases are of interest. First, the case studied by Epstein (1924) assumed that \( v_o \ll C \), where \( C = 2/(\pi \beta) \) is the average speed of the gas molecules. It is readily seen that either equation (5) or (9) approaches Epstein's formula for the drag force due to molecular impact or specular reflection, namely, \( 4/3 \pi \sigma v_0 C \), and that equation (16) is the same as his formula for evaporation drag for the perfect thermal conductor. Second, Newtonian flow assumes that \( v_o >> C \). Equation (17) has the limiting value of \( C_D = 2 \).

Epstein (1924) considered in some detail several mechanisms of diffuse reflection or evaporation, some of which, he and Millikan (1923) pointed out, violate the second law of thermodynamics. All of the allowable mechanisms of diffuse reflection lead to formulas similar to equation (16), except that the factor of \( \pi/6 \) may be replaced by factors ranging from \( 3\pi/16 \) for the perfect thermal nonconductor, to \( 16\pi/27 \) for "conservation of velocity" (see Epstein, 1924). The latter value seems physically unlikely, and for the present purpose equation (16) will be used.

Loeb (1934) discussed specular reflection and indicated that it can occur at small angles on polished surfaces. Some specular reflection may occur with highly polished metallic satellites but it seems reasonable that most of the re-emission of molecules occurs by an evaporation mechanism.

Molecular impact without some form of re-emission may seem unlikely, at first thought. However, if the satellite consists of a thin-walled plastic balloon, as has been proposed for measurements of atmospheric density (Jones and Bartman, 1956; Technical Panel, 1958), a portion of the more energetic molecules may in fact penetrate the wall.

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Nicolet, M.
UPPER ATMOSPHERE AND SATELLITE DRAG

ROCKET PANEL


SPITZER, L., Jr.


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TECHNICAL PANEL


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Abstract

The drag on a sphere moving in the regions of the upper atmosphere is investigated. Calculations show that the drag coefficient departs from the classical value of $C_D=2$ for spheres in circular orbits of altitude greater than $10^6$ km. The drag coefficient may well become much greater than 2 at sufficiently high altitudes.