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by Allan F. Cook, II, *and* Fred A. Franklin

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Astrophysical Observatory,
Smithsonian Institution.

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Optical Properties of Saturn's Rings: I. Transmission

By Allan F. Cook, II,[1] and Fred A. Franklin[2]

In the present paper, the first in a projected series on the rings of Saturn, we propose to discuss observations bearing upon the transmission properties of the three rings of Saturn. Aspects of this problem have been treated by Bobrov (1952) in one of a series of six interesting papers (Bobrov, 1940, 1951, 1952, 1954, 1956a, 1956b) on the ring system. Forthcoming papers will consider other physical aspects of the rings and their interaction with the interplanetary medium.

Rough estimates of the optical thickness of Saturn's rings may be obtained from material available from occultations of stars by the rings. The most important of these occurred on March 14, 1920, when observers in the Union of South Africa (Reid et al., 1920) and in India (Bhaskaran, 1920) followed the passage of BD+11° 2269 through rings A and B. If we assume that the diffraction disk of the star could be seen if its brightness were 1 percent that of the surrounding ring, we can show that the optical thickness, τ, of ring B must be 0.59. Such a low value of the contrast is valid only because the star was visibly very orange, having a spectral class of K3.

Ainslie (1917a, 1917b) and Knight (in Ainslie and Knight, 1917) observed the occultation of the star 212 B. Geminorum by ring A on February 9, 1917. Since both observers followed the star (whose visual magnitude was 6.68 and spectral class M1) through ring A, its transparency is amply demonstrated. Any attempt to estimate the optical thickness of ring A by an argument similar to that used for the 1920 occultation leads to a lower limit greater than that for ring B. One final occultation was reported (Leonard, 1940) of BD+6° 259 on November 24, 1939. This star was not visible through the ring system, probably because of its faintness and solar spectral class, and the generally poor seeing and transparency. Bobrov (1952) has used this observation to determine a lower limiting value of 0.83 for the optical thickness of ring B.

In his 1952 paper Bobrov claims that, on a reproduction in L'Astronomie of a photograph taken by Camichel (1946), the ball of Saturn is clearly discernible as a bright area through ring B. Bobrov's argument is greatly weakened by the presence of other bright spots on the ring. It seems exceedingly unlikely that with his value of 0.7 for the average optical thickness of ring B, corresponding to a contrast of 3 percent, the ball of the planet should be at all visible in such a reproduction. From a detailed study of the original photographs, Camichel (private communication) is of the same opinion. We object not to Bobrov's value of τ, but to this particular means of inferring it.

[1] Harvard College Observatory.
[2] Harvard College Observatory, and Smithsonian Astrophysical Observatory.

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The eclipse of Iapetus by the shadow of the rings, as observed by Barnard (1890) on November 1–2, 1889, provides probably the best transmission data concerning the optical thickness of both rings B and C. Only for the latter, however, do we have measures of the optical thickness as a function of distance from the planet, because oncoming daylight prevented Barnard from following the satellite more than just into the penumbra of ring B. Buchholz (1895) discussed Barnard's observations, using several simplifying approximations; he assumed the sun to be a uniformly illuminated disk, and Iapetus to be a point reflector. In the present report we have removed these two assumptions by considering a limb-darkened sun and by trying to represent the distribution of light on the finite surface of Iapetus. Our first problem then is to discuss the light curve of Iapetus and to find a way of representing it. Photometric measurements of Iapetus have been carried out by Pickering (1879) in 1878–79, Wendell (1909) from 1896 to 1900, Guthnick (1910) from 1905 to 1908, Graff (1939) in 1922, and Widorn (1952) in 1949, who summarized the data. In general, the light variation ranges from about magnitude 10.0 to magnitude 12.0 while the period of the light curve is virtually the same as that of revolution about Saturn, with maximum brightness occurring at western elongation.

From this fact one may infer that Iapetus always presents the same face toward Saturn; hence, the inclination of the axis of rotation of Iapetus to its orbit is very probably near 90°, and the normal to the plane of the orbit is inclined to the plane of the sky by 18° or less. This implies that mean intensities of meridianal zones may be determined at least approximately.

As a first attempt we compared the following model with the above observations. Let $A(\lambda)$ be the average brightness as a function of the longitude λ, A_1 and A_2 be average relative brightnesses based upon a two-hemisphere model for the bright and dark hemispheres respectively, and A_3 be the brightness of a periodic term in cos 2λ; then:

$$-\pi/2 \leq \lambda \leq \pi/2 \qquad A(\lambda) = A_1 + A_3 \cos 2\lambda$$
$$\pi/2 \leq \lambda \leq 3\pi/2 \qquad A(\lambda) = A_2 + A_3 \cos 2\lambda.$$

Then for the range $0 \leq \lambda_0 \leq \pi$ we have essentially a Fourier series representation of the average brightness:

$$\bar{\bar{A}}(\lambda_0) = \frac{1}{2}\left[\int_{\lambda_0 - \pi/2}^{\pi/2} (A_1 + A_3 \cos 2\lambda) \cos (\lambda - \lambda_0) d\lambda + \int_{\pi/2}^{\lambda_0 + \pi/2} (A_2 + A_3 \cos 2\lambda) \cos (\lambda - \lambda_0) d\lambda\right].$$

This leads to the result:

$$\bar{\bar{A}}(\lambda_0) = 1/2(A_1 + A_2) + 1/2(A_1 - A_2) \cos \lambda_0 + 1/3 A_3 \cos 2\lambda_0,$$

which gives a negative brightness for a small region just on the dark side of the boundary between the bright and dark hemispheres.

In model II we tried:

$$0 \leq |\lambda| \leq \pi/4 \qquad \bar{A}(\lambda) = A_1 + A_3$$
$$\pi/4 \leq |\lambda| \leq \pi/2 \qquad \bar{A}(\lambda) = A_1 - A_3$$
$$\pi/2 \leq |\lambda| \leq 3\pi/4 \qquad \bar{A}(\lambda) = A_2 - A_3$$
$$3\pi/4 \leq |\lambda| \leq \pi \qquad \bar{A}(\lambda) = A_2 + A_3.$$

We can now carry out an integration similar to the one above and then determine the best set of values for A_1, A_2 and A_3 from the observed light curves. Finally, one must ask how well these three can represent the entire light curve. Such a model gives quite satisfactory agreement with the observed results of Guthnick (1910) and Wendell (1909), but requires an extremely small brightness for two octants. More explicitly, for Guthnick's data we have: $A_1 = 0.765$, $A_2 = 0.125$ and $A_3 = 0.110$, while for Wendell's results: $A_1 = 0.780$, $A_2 = 0.115$ and $A_3 = 0.105$. For each case the derived curve deviates from the observed one by less than ± 0.06 magnitudes.

A model such that

$$0 \leq |\lambda| \leq \alpha \qquad \bar{A}(\lambda) = A_1$$
$$\alpha \leq |\lambda| \leq \pi \qquad \bar{A}(\lambda) = A_2$$

where from Wendell's data A_1 was determined to be 0.855, A_2 to be 0.145 and α to be 76°, fits about as well. Finally a more elaborate model was tried on the data of both Guthnick and Wendell:

$$0 \leq |\lambda| \leq \lambda_1 \qquad \bar{A}(\lambda) = A_1$$
$$\lambda_1 \leq |\lambda| \leq \lambda_2 \qquad \bar{A}(\lambda) = A_2$$
$$\lambda_2 \leq |\lambda| \leq \pi \qquad \bar{A}(\lambda) = A_3$$

with the added restriction that $\lambda_1+\lambda_2=\pi$. For Wendell's curve: $A_1=0.679$, $A_2=0.205$, $A_3=0.116$ and $\lambda_1=58°$, while for Guthnick's data: $A_1=0.691$, $A_2=0.212$, $A_3=0.097$ and $\lambda_1=53°$. These results fit the respective observed curves to within ± 0.03 magnitudes.

No reasonable model can represent the light curves of either Graff or Pickering. The former is considerably asymmetric while in the latter case the satellite was compared directly with an image of Saturn introduced by low power optics. Furthermore, the considerable color difference between Saturn and Iapetus should have made Pickering's observations subject to the Purkinje effect.

A general superposition of all curves except Pickering's shows agreement to within ± 0.10 magnitudes. In view of this fact and the relatively acceptable agreement given by all models, it seems most reasonable to adopt the simplest, i. e., a two-hemisphere model which agrees to within ± 0.10 magnitudes for the results of Guthnick and Wendell when the values $A_1=0.879$ and $A_2=0.121$ are employed.

This assumption is supported by the work of Widorn (1952) who showed that his observations may be represented extremely accurately on the basis of a two-hemisphere model. With this problem settled we can proceed to consider the eclipse of Iapetus.

Adopting the orbital elements of Iapetus given by G. Struve (1933), we can determine the position of the satellite during the eclipse. We can now rectify the observed light curve of Barnard in order to obtain a continuous curve of the optical thickness of ring C. We shall first assume that the light intensity, $I(r,s)$, transmitted to the satellite can be represented by a quadratic, whence,

$$I(r,s)=I(r)+b\cos\theta\frac{\Delta r}{R}s+c\cos^2\theta\left(\frac{\Delta r}{R}\right)^2 s^2, \quad (1)$$

where r is the radial distance in the ring plane of the point through which passes the straight line joining the centers of the sun and Iapetus, s is measured in the direction of motion of the satellite, and θ is the angle between a line perpendicular to the s-direction and the direction defined by the isolines of light intensity transmitted by the rings, assumed to be straight lines over the satellite disk. This assumption introduces an error of less than 1 percent. R is the radius of the solar disk of confusion, which at the time of the eclipse was 0.″2775. The quantity Δr is the change in r as one moves a distance R along the direction perpendicular to the isolines. The displacement in this latter direction we shall denote by H, so that $s=H\sec\theta$.

Using the familiar form of the equation of an ellipse, we have

$$\frac{\Delta r}{R}=\frac{[(1-e^2)^2x^2+y^2]^{1/2}}{[(1-e^2)\{(1-e^2)x^2+y^2\}]^{1/2}}, \quad (2)$$

where e is the eccentricity of the ellipses formed by projection of circles in the ring plane on the plane of the sky as seen from the sun, and x, y are the rectangular coordinates of Iapetus relative to Saturn and the principal axes of the projection of the rings on the plane of the sky. The coefficients b and c are given by the expressions:

$$b=\frac{I(r_{max}-I(r_{min})}{2R}; \quad c=\frac{I(r_{max})+I(r_{min})-2I(r_0)}{2R^2}, \quad (3)$$

where $I(r_{max})$ and $I(r_{min})$ are the intensities at a distance of one radius of the solar disk of confusion within and without r in the direction perpendicular to the isolines.

We shall first introduce the effect of a limb-darkened sun on equation (1) by the following integration:

$$I'(r)=1/F\int_{-R}^{+R}dH\int_{-\sqrt{R^2-H^2}}^{+\sqrt{R^2-H^2}}L(S,H)I(r,s)dS \quad (4)$$

where F, a normalizing divisor, equals $4\pi R^2/5$; $L(S,H)$, the solar limb darkening, is $\frac{2}{5}+\frac{3}{5}R\times\sqrt{R^2-H^2-S^2}$, and S is a coordinate lying in the direction of the isolines. This integration yields:

$$I'(r)=I(r)+\tfrac{2}{10}\,c(\Delta r)^2. \quad (5)$$

In order to consider the effect of the disk of confusion of the satellite itself, we must carry out another integration over the satellite disk:

$$I''(r)=1/F'\Bigg\{A_1\Bigg[\int_{-R'}^{-R'\sin\theta}dH\int_{-\sqrt{R'^2-H^2}}^{+\sqrt{R'^2-H^2}}I'(r)dS+\int_{-R'\sin\theta}^{R'\sin\theta}dH\int_{-\sqrt{R'^2-H^2}}^{H\cot\theta}I'(r)dS\Bigg]+A_2\Bigg[\int_{-R'\sin\theta}^{R'\sin\theta}dH\int_{H\cot\theta}^{\sqrt{R'^2-H^2}}I'(r)dS+\int_{R'\sin\theta}^{R'}dH\int_{-\sqrt{R'^2-H^2}}^{\sqrt{R'^2-H^2}}I'(r)dS\Bigg]\Bigg\}. \quad (6)$$

In equation (6), $F'=(A_1+A_2)\ (\pi R'^2/2)$, R' is the angular radius of Iapetus which was taken as 0″.090. A_1 and A_2 are the brightnesses of the trailing and the leading halves of the satellite respectively and are quoted above. Since maximum and minimum light of Iapetus occur at western and eastern elongations respectively, it seems reasonable to assume that during the eclipse the bright and dark regions possessed the same area.

This expression reduces upon integration to:

$$I''(r)=I(r)+\frac{4b\Delta r}{3}\frac{A_1-A_2}{A_1+A_2}\frac{R'}{R}\cos\theta[\sin^2\theta-\cos^2\theta]+c\left[9/40(\Delta r)^2+\frac{(\Delta r)^2}{4}\left(\frac{R'}{R}\right)^2\right]. \quad (7)$$

Barnard's measures, which in our notation are denoted by $I''(r)$, are eye estimates relative to the two satellites Tethys and Enceladus, whose difference in brightness he takes equal to 1.0 magnitudes. More recent photometry (Guthnick, 1914) suggests that this difference is more nearly 1.1 magnitudes. Barnard's curve was therefore corrected for this difference.

In applying equation (7) we can use values of $I''(r)$ to approximate the b's and c's and hence obtain a first approximation to $I(r)$. If necessary one can repeat the process until no change in $I(r)$ occurs. For the crape ring, since the corrections to $I(r)$ are small, one iteration is sufficient. Final values of $I(r)$ are shown in figure 1.

As mentioned previously, Barnard carried his observations just into the shadow of ring B before on-coming dawn made further estimates impossible.

We can reduce his few measures in a fashion similar to the one employed for ring C. We consider the effect of the sun by the integration:

$$I'(r)=1/F\int_{-R}^{\beta}dH\int_{-\sqrt{R^2-H^2}}^{+\sqrt{R^2-H^2}}L(S,H)I(r,s)dS+\frac{\alpha}{F}\int_{\beta}^{R}dH\int_{-\sqrt{R^2-H^2}}^{+\sqrt{R^2-H^2}}L(S,H)I(r,s)dS, \quad (8)$$

where β is the perpendicular distance from the center of the solar disk of confusion to the as-

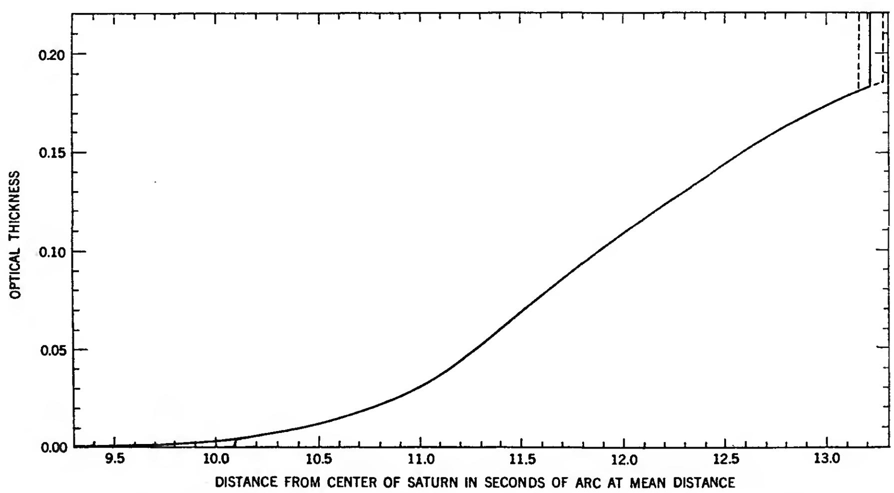

FIGURE 1.—The optical thickness of ring C plotted against distance from the planet's center.

sumed sharp straight-line boundary of ring B, and α is the transmission of this ring. Upon integration, equation (8) yields:

$$I'(r)=\frac{1}{8\pi}[4I(r)+c(\Delta r)^2]\times$$
$$[\gamma\sqrt{1-\gamma^2}+\sin^{-1}\gamma+\pi/2]$$
$$-\frac{\Delta r}{12}(1-\gamma^2)^{3/2}[4b+3c\gamma\Delta r]$$
$$+\frac{3}{8}\Big[(\gamma+1)I(r)+(\gamma^2-1)\frac{b\Delta r}{2}+(\gamma^3+1)\frac{c(\Delta r)^2}{3}$$
$$-(\gamma^3+1)\frac{I(r)}{3}-(\gamma^4-1)\frac{b\Delta r}{4}-(\gamma^5+1)\frac{c(\Delta r)^2}{5}\Big]$$
$$+\frac{\alpha}{8\pi}[4\pi-3\pi\gamma+\pi\gamma^3-4\gamma\sqrt{1-\gamma^2}-4\sin^{-1}\gamma]. \quad (9)$$

The quantity γ in this last equation equals β/R. It is impossible to integrate equation (9) over the satellite disk, so the following procedure will be adopted. We can obtain values of b and c by extrapolating the observed curve for ring C; that is, by ignoring the effect of ring B. Equation (9) then yields a set of values for $I'(r)$. We shall now assume that the effect of passing over the ring B boundary can be treated by applying an equation similar to (7), viz.:

$$I''(r)=I'(r)$$
$$+\frac{4b'\Delta r}{3\pi}\frac{R'}{R}\frac{A_1-A_2}{A_1+A_2}\cos[\sin^2\theta-\cos^2\theta]$$
$$+\frac{c'(\Delta r)^2}{4}\left(\frac{R'}{R}\right)^2. \quad (10)$$

The quantities b' and c' are found from the computed curve for $I'(r)$ given by equation (9) in accordance with the relation

$$I'(r,s)=I'(r)+b'\cos\theta\frac{r}{R'}s+c'\cos^2\theta\left(\frac{r}{R}\right)^2 s^2. \quad (11)$$

The use of equations (9), (10), and (11) permits us to calculate a family of curves of $I''(r)$ for differing values of the ring B transmission, α, and inner boundary, ρ. This latter quantity enters into the solution through its dependence upon γ, $\gamma=\frac{\rho-r}{\Delta r}$. The best value of ρ, 13.″22 ± 0.″06 at the mean distance of Saturn, is somewhat higher than those commonly found in the literature which are usually the result of night-time micrometer measures. However, Lowell and E. C. Slipher (see Lowell, 1915) obtained 13.″21, a value corrected for irradiation by means of daylight measures. We suspect that values much lower than this are incorrect due to the presence of irradiation.

The second quantity obtainable is a probable lower limit to the transmission of ring B. The best value of the transmission, α, as viewed during the eclipse is 0.05. As the elevation of the sun above the ring plane at the same time was −11° 11′, this value of the transmission corresponds to an optical thickness of 0.58. It may be seen that the extreme limits on the transmission are $0\leq\alpha\leq0.10$ whence those on the optical thickness are $\infty\geq\tau\geq0.45$. For further details the reader is referred to figure 2.

As a final result, the measures of the optical thickness of the crape ring can be used to determine an expression proportional to the pressure of the gas, probably originating from the solar corpuscular stream, in this ring. The argument runs as follows: Let the rate of introduction of particles of radius l across the inner edge of ring B be $W(l)$ per unit length of the circumference of the ring, and let the number of particles in a column one square centimeter in cross section in the ring be $n(r,l)$ where r is the distance from Saturn. Let a steady state of inward flow of particles due to loss of angular momentum to the gas be assumed. Then the rate of loss of angular momentum by the particles gives us the equation:

$$\frac{4\pi}{3}\rho_p l^3 n(r,l)\frac{d}{dt}(rv)=-\pi k_1(r)\,n(r,l)\,l^2, \quad (12)$$

where v is the velocity of the ring particles and ρ_p their density. The quantity $k_1(r)$ is a factor measuring the effective drag on a particle's cross section, πl^2. Using the fact that the period of revolution equals $k_2r^{3/2}$, we may write equation (12) as:

$$\frac{4\pi^2}{3}\frac{\rho_p l^3\,n(r,l)}{k_2 r^{1/2}}\frac{dr}{dt}=-\pi k_1(r)\,n(r,l)\,l^2. \quad (13)$$

In the steady state we must have:

$$2\pi\rho W(l)=-n(r,l)2\pi r\frac{dr}{dt} \quad (14)$$

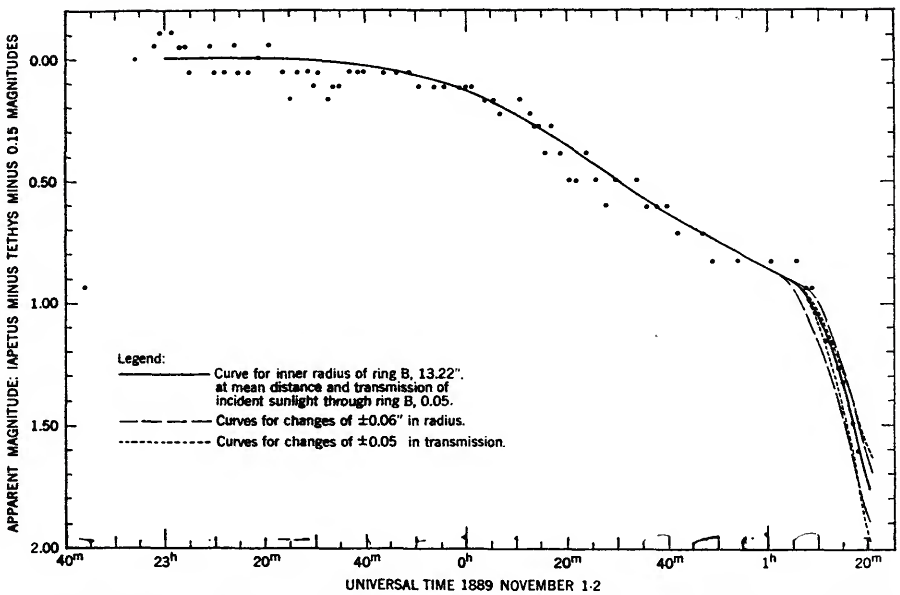

FIGURE 2.—The filled circles are Barnard's magnitude estimates which have been used to deduce limiting and most probable values for the transmission of ring B and the location of its inner boundary.

where ρ is, as before, the inner radius of ring B. Rearranging this last equation and combining it with equation (13), we obtain:

$$\pi n(r,l)l^2=\frac{4\pi^2}{3}\frac{\rho_p\rho l^3W(l)}{k_2k_1(r)r^{3/2}}. \tag{15}$$

By definition the optical thickness is

$$\tau(r)=\pi\int_0^\infty n(r,l)l^2dl=\frac{4\pi^2}{3}\frac{\rho_p\rho}{k_2k_1(r)r^{3/2}}\int_0^\infty l^3W(l)dl. \tag{16}$$

Letting k_3 be the unknown constant,

$$k_3=\frac{4\pi^2}{3}\frac{\rho_p\rho}{k_2}\int_0^\infty l^3W(l)dl, \tag{17}$$

we have

$$k_1(r)=\frac{k_3}{r^{3/2}\tau(r)}. \tag{18}$$

Note that $k_1(r)$ increases sharply inward. If Saturn possesses a magnetic field which is operative at a distance equal to that of the crape ring, then this field would divert ionized particles from the corpuscular stream away from the ring plane. If on the other hand Saturn possesses no effective magnetic field at such distances, then for strong gravitational focussing (i. e., very slow particles) $k_1(r)$ should be proportional to r^{-2}. The increase will be less steep for faster particles. The fact that the increase is much steeper than this seems to imply that the solar corpuscular stream cannot be directly responsible for the shape of $k_1(r)$. We are therefore led to postulate the existence of a cloud of neutral gas, the shape of whose pressure curve we are in a position to determine.

The factor $k_1(r)$ has the form:

$$k_1(r)=k_4\rho_g(v_c-v) \tag{19}$$

where ρ_g is the gas density, v_c the circular velocity and k_4 a constant of proportionality. If P is the gas pressure, then in equilibrium:

$$\frac{dP}{dr}=-a\rho_g, \tag{20}$$

where

$$a=\frac{v_c^2-v^2}{r}. \tag{21}$$

If we combine this with equation (19) we obtain:

$$\frac{dP}{dr}=-\frac{k_1(r)}{k_4}\frac{(v_c+v)}{r}\cong\frac{k_1(r)}{k_4}\frac{2v_c}{r}. \tag{22}$$

We must now introduce equation (18) into this last result in order to obtain the equation:

$$\frac{k_4}{k_3}\frac{dP}{dr}\cong-\frac{4\pi}{k_2 r^3 \tau(r)},\text{ or}$$

$$P-P(\rho)=k_5\int_r^{\rho}\frac{dr'}{r'^3\tau(r')}. \tag{23}$$

Equation (23) was integrated graphically from the inner boundary of ring B ($r=\rho=13''.22$), where the integrand is practically zero, to $r=10''.3$, beyond which point the crape ring becomes invisible. The result of this integration, that is, the shape of the gas pressure curve as a function of the distance from Saturn, is exhibited in figure 3.

We are pleased to thank Dr. Fred L. Whipple for directing our attention to this problem and for his continued aid and encouragement. Miss Frances W. Wright also deserves our gratitude for aid in the translation of Russian articles.

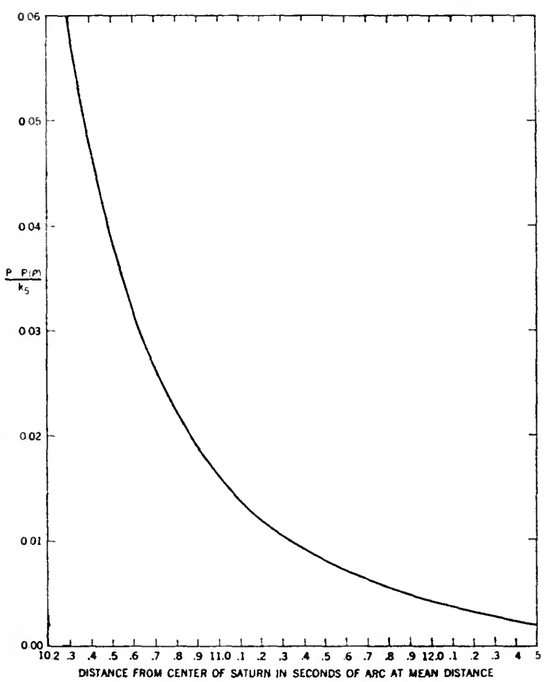

FIGURE 3.—The variation of $\frac{P-P(\text{P})}{k_5}$ (see equation (23)) with distance from the planet's center.

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Abstract

All existing observational data relating to the transmission properties of Saturn's rings are discussed. From such data the optical thickness of the crape ring is determined as a function of its distance from the planet. Limiting values for ring B and a new value for the inner radius of ring B are established. The shape of the pressure curve of a gas cloud evidently associated with ring C can be computed from its optical thickness.

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