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A CRITERION FOR THE MODE OF ABLATION OF STONE METEORS

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A Criterion for the Mode of Ablation of Stone Meteors

By Allan F. Cook[1]

We shall present evidence that Meteor 19816 of the program of the Harvard Meteor Expedition (Cook, Jacchia, and McCrosky, 1963) passed from ablation by melting and spraying to ablation by vaporization, and back to ablation by melting during its luminous flight through the atmosphere. We shall then propose a criterion for the mode of ablation which, in the case of ablation by vaporization, becomes the ratios of the heat transports parallel to and normal to the surface of the liquid layer just beneath that surface. A critical value of this criterion will then be established from this meteor.

Drag coefficient

We shall assume that an oblate spheroid with minor axis oriented in the direction of flight of the meteor adequately represents its shape. From the shape factor found by Cook et al. (1963) in their equation (93), we have for the ratio of the minor axis to the major axis the value 0.812 (only the first figure has any significance).

Let ϕ denote the azimuthal angle about the minor axis, and θ the polar angle from that axis. Let θ' denote the polar angle of the normal to the surface. Then we have for the drag on the Newtonian approximation:

$$\Gamma=\frac{\int_0^{\pi/2}\int_0^{2\pi}\cos^2\theta'\cos\theta\sin\theta d\phi d\theta}{\int_0^{\pi/2}\int_0^{2\pi}\cos\theta\sin\theta d\phi d\theta},$$

$$\tan\theta'=\alpha\tan\theta,\ \alpha=\frac{c}{a},\ b=a, \qquad (1)$$

where a, b, c are the principal axes of the ellipsoid forming the surface of the meteoroid. Integration over ϕ is immediate in both numerator and denominator and over θ, as well as in the denominator. The result is

$$\Gamma=2\int_0^{\pi/2}\frac{\cos^3\theta\sin\theta}{\alpha^2+(1-\alpha^2)\cos^2\theta}d\theta$$

$$=2\int_0^1\frac{u^3du}{\alpha^2+(1-\alpha^2)u^2}, \qquad u\equiv\cos\theta, \qquad (2)$$

where the second relation of equation (1) was used. A further change of variable is convenient:

$$\Psi\equiv\text{arc tan}\left[\left(\frac{1-\alpha^2}{\alpha^2}\right)^{1/2}u\right],$$

$$\Gamma=\frac{2\alpha^2}{(1-\alpha^2)^2}\int_0^{\text{arc tan}\left[\left(\frac{1-\alpha^2}{\alpha^2}\right)^{1/2}\right]}\tan^3\Psi d\Psi,$$

$$\Gamma=\frac{1}{1-\alpha^2}+\frac{\alpha^2}{(1-\alpha^2)^2}\ln(\alpha^2). \qquad (3)$$

In our case, $\Gamma=0.568$. In free molecular flow, $\Gamma=1$. In terms of the shear-transfer coefficient Λ' we have, instead of equation (60) (Cook et al.), the relation

$$\Gamma\simeq0.568+0.432\Lambda'. \qquad (4)$$

The shear-transfer coefficient

We replace equation (35) of Cook et al. by

$$\Lambda'\simeq(0.84a/l)^{1/2}, \qquad (5)$$

where a is the semi-axis major of the ellipsoid, and l is a mean free path in air. In terms of the presentation area $\mathscr{A}$ we have

$$a=(\mathscr{A}/\pi)^{1/2}. \qquad (6)$$

The heat-transfer coefficient

We replace equation (2) of Cook et al. by

$$\Lambda\simeq(1.6a/l)^{1/2}. \qquad (7)$$

The heat of ablation

Cook et al. give sufficient data for two stone meteors, 1242 and 19816 of the Harvard pro-

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gram, to allow the computation of Λ', Λ, and Γ by equations (5), (6), (7), and (4) above, whence we can compute the heat of ablation ζ from

$$\zeta=\frac{\Lambda}{2\Gamma\sigma}, \tag{8}$$

taking σ from tables 3 and 4 of Cook et al. The resulting values of ζ are given in table 1.

The mean value of ζ for Meteor 1242 is 2.6×10^{10} ergs gm^{-1}. The last epoch of Meteor 19816 clearly exhibits the same value. The anticipated value of this quantity (Öpik, 1958) is 2.0×10^{10} ergs gm^{-1} for ablation by melting. It is clear that melting and spraying was the mode of ablation for Meteor 1242. It is equally clear that over most of the trajectory of Meteor 19816 this was not the case. Transition from ablation by vaporization to ablation by melting and spraying was in progress at t=2.824 sec, and melting and spraying occurred at t=3.124 sec. At all earlier epochs the table suggests that ablation occurred by vaporization.

Since we are now faced with ablation by vaporization, we must consider the effects of the departing meteoric vapors on Λ', Γ, and Λ.

Shielding by own vapors

We shall make the approximate assumption that the shielding effects of the departing vapors can be estimated by increasing the air density in the ratio

$$\left(1-\frac{dm/dt}{\mathscr{A}\rho_a V}\right)^{-1/2},$$

where dm/dt is the rate of change of mass of the meteoroid and $\mathscr{A}\rho_a V$ is the rate of interception of air mass by the cross section of the meteoroid. We have the ablation equation

$$\frac{dm}{dt}=-\frac{\Lambda}{2\zeta}\mathscr{A}\rho_a V^3, \tag{9}$$

$$\Lambda=\Lambda_a\left(1-\frac{dm/dt}{\mathscr{A}\rho_a V}\right)^{-1/2},$$

$$\Lambda_a\equiv(1.6l/a)^{1/2}.$$

From equation (9) it follows that

$$\frac{\Lambda_a V^2}{2\zeta}\left(\frac{\Lambda}{\Lambda_a}\right)^3+\left(\frac{\Lambda}{\Lambda_a}\right)^2-1=0. \tag{10}$$

A table of $[\Lambda_a V^2/(2\zeta)]$ can be prepared with argument Λ/Λ_a. Inverse interpolation in terms of the known quantity $\Lambda_a V^2/(2\zeta)$, where $\zeta=8\times10^{10}$ ergs gm^{-1} for vaporization (Öpik, 1958), yields Λ/Λ_a. We assume that the same conversion factor applies to the shear-transfer coefficient Λ', whence

$$\Lambda'=\frac{\Lambda}{\Lambda_a}\Lambda'_a, \quad \Lambda'_a\equiv(0.84l/a)^{1/2}. \tag{11}$$

Table 1.—*Values of various quantities for Meteors 1242 and 19816 on the assumption that ablation is by melting and spraying*

Meteor No.	Epoch, t (sec)	Mass, m (gm)	Semi-axis major, a (cm)	Λ'	Λ	Γ	ζ (ergs gm^{-1})
1242	1.0526	276.0	3.398	0.0881	0.1215	0.606	2.8×10^{10}
	1.6842	262.0	3.250	0.0683	0.0943	0.598	3.0×10^{10}
	2.3158	245.0	3.190	0.0534	0.0736	0.591	2.7×10^{10}
	2.9474	219.8	3.060	0.0448	0.0576	0.586	2.5×10^{10}
	3.5790	193.1	2.934	0.0330	0.0448	0.582	2.4×10^{10}
	4.2105	168.2	2.802	0.025	0.0354	0.579	2.0×10^{10}
19816	0.764	10.65	1.116	0.537	0.742	0.800	0.77×10^{11}
	1.135	9.81	1.083	0.331	0.457	0.711	1.11×10^{11}
	1.860	7.16	0.978	0.181	0.250	0.646	1.07×10^{11}
	2.038	3.12	0.742	0.172	0.237	0.642	1.08×10^{11}
	2.425	2.19	0.660	0.141	0.195	0.629	1.29×10^{11}
	2.623	1.66	0.602	0.128	0.176	0.623	0.94×10^{11}
	2.824	0.92	0.494	0.126	0.174	0.622	0.54×10^{11}
	3.124	0.26	0.324	0.131	0.180	0.624	0.24×10^{11}

Finally, we must remember that vaporization will take place at the side of the meteoroid, not in front (Cook et al., eq. 58), if we are only slightly beyond the regime of ablation by melting and spraying. In particular, for a spherical meteoroid melt is neither accumulating nor being removed by the flow field near $\theta=55°$, but near $\theta=0°$ it is being very efficiently removed by the flow field. We estimate that the heat and shear-transfer coefficients take their unshielded values, Λ_a, Λ'_a, over half the presentation area $\mathscr{A}$, and their shielded values over the other half. The deduced values of ζ for Meteor 19816, together with the shielded and adopted values of the coefficients, are given in table 2. The first and last values indicate a mixed mode of ablation. The mean of the other five is 8.7 $\times 10^{10}$ ergs gm^{-1}, which is in satisfactory agreement with Öpik's estimate.

Theory of ablation of meteoric stone

We shall apply the theory of ablation of glassy materials developed by Bethe and Adams (1959). It can be fitted into the structure of the theory of the ablation of iron given by Cook et al. The first change is that the law for the dynamic viscosity given by their equation (21) is replaced by

$$\mu=6\times10^{-4}\times10^{8680/T}. \tag{12}$$

The exponential form is that introduced by Bethe and Adams, while the constants have been determined from the values of μ quoted by Öpik (1958) to give a rough fit.

The approximate form of the equation of heat conduction given by Cook et al. in their equation (24) may be integrated to yield

$$\frac{\partial T}{\partial y}=\frac{\partial T}{\partial y}\bigg|_{y=0}\exp\left[\frac{1}{k}\int_0^y v\,dy'\right]. \tag{13}$$

Here T is the temperature, y is measured normal to the surface (positive outward), k is the thermometric conductivity, v is the y-component of the velocity, and the surface lies at $y=0$. We assume that the interior temperature of the meteoroid is negligible. Following Bethe and Adams, we assume that the molten layer is so thin that we can use $v\simeq v_w$, the limit on v as $y\rightarrow-\infty$, in equation (13). Then we have the temperature distribution

$$\delta_T\equiv k/v_w,\quad T(y)=T_0\exp(y/\delta_T). \tag{14}$$

We next introduce an approximate form for the dynamic viscosity μ, suggested by Bethe and Adams (1959):

$$\lim_{T\rightarrow T_0}\mu=\mu_0\left(\frac{T_0}{T}\right)^n,\quad n=\frac{20{,}000}{T_0}, \tag{15}$$

where T_0 is the surface temperature, μ_0 the surface viscosity, and n is determined by taking the indicated limit of equation (12). The distribution of viscosity is then found by substitution of equation (14) to be

$$\delta=\delta_T/n,\quad \mu(y)=\mu_0\exp(-y/\delta). \tag{16}$$

We substitute this distribution into equation (23) of Cook et al. for the component of the flow velocity parallel to the surface to find

$$u(y)=\delta\,\frac{T_0}{\mu_0}\exp(y/\delta)-\frac{\delta^2}{\mu_0}\frac{\partial p}{\partial x}\left(1-\frac{y}{\delta}\right)\exp(y/\delta), \tag{17}$$

where the boundary condition $\lim_{y\rightarrow-\infty} u=0$ has been imposed. Here τ_0 is the shearing stress upon the surface, and $\partial p/\partial x$ is the pressure gradient parallel to the surface. We next

TABLE 2.—*Values of various quantities for Meteor 19816 on the assumption that ablation is by vaporization*

Epoch, t (sec)	Shielded Λ'	Shielded Λ	Adopted Λ'	Adopted Λ	Adopted Γ	ζ (ergs gm^{-1})
0.764	0.190	0.263	0.364	0.502	0.725	5.8×10^{10}
1.135	0.136	0.187	0.233	0.322	0.669	8.3×10^{10}
1.860	0.090	0.124	0.136	0.187	0.626	8.3×10^{10}
2.038	0.088	0.121	0.130	0.179	0.624	8.6×10^{10}
2.425	0.079	0.109	0.110	0.152	0.616	10.3×10^{10}
2.623	0.077	0.104	0.102	0.141	0.612	7.8×10^{10}
2.824	0.080	0.111	0.103	0.142	0.612	4.5×10^{10}

substitute (17) into equation (18) of Cook et al. (the equation of continuity) to find

$$v(y)=v_0+\frac{1}{r_2\sin\theta}\frac{\partial}{\partial x}\left\{r_2\sin\theta\,\frac{\delta^2}{\mu_0}\left[\tau_0\langle 1-\exp(y/\delta)\rangle -\delta\frac{\partial p}{\partial x}\left\langle 2\langle 1-\exp(y/\delta)\rangle+\frac{y}{\delta}\exp(y/\delta)\right\rangle\right]\right\}, \quad (18)$$

$$v_w=v_0+\frac{1}{r_2\sin\theta}\frac{\partial}{\partial x}\left\{r_2\sin\theta\,\frac{\delta^2}{\mu_0}\left[\tau_0-2\delta\frac{\partial p}{\partial x}\right]\right\}. \quad (19)$$

Here r_2 is the radius of curvature in the plane containing the normal to the surface, and perpendicular to the plane containing the axis of symmetry of the meteoroid and the normal to the surface, the axis of symmetry being taken along the direction of flight.

Comparison with equation (33) of Cook et al. for iron shows that δ here plays a role similar to that of $-y_m$ in their analysis for iron. In the present case their equation (53) becomes

$$\lim_{\theta\to 0}(v_w-v_0)=2\left(\frac{\delta}{r}\right)^2\frac{r\rho_aV^2}{\mu_0}\times\left\{\Lambda'+2[2-\Gamma A(\rho_m/m)^{1/3}r]\frac{\delta}{r}\right\}. \quad (20)$$

Here r is the frontal radius, and v_0 is the velocity of passage of stone from the liquid to the vaporized state across the surface. We again make the assumption that $v_0=0$ and use

$$v_w=\frac{\Lambda}{2}\frac{\rho_aV^3}{\zeta\rho_m}. \quad (21)$$

The procedure is the same as for iron, and yields the result that near $\theta=0$ the material flows away as melt and does not vaporize.

We now assume for mathematical convenience that our meteoroid is spherical, and consider the state of affairs near $\theta=54.7°$. We recall that equation (13) is integrated from equation (24) of Cook et al. We further note that their equation is based on the assumption that

$$u\left|\frac{\partial T}{\partial x}\right|\ll v\frac{\partial T}{\partial y}. \quad (22)$$

In that case, even as for iron, we must have ablation by vaporization in this region, i.e., $v_0=v_w$. We neglect the heat radiated away from the surface and employ

$$v_w=v_0=\frac{\Lambda}{2}\frac{\rho_aV^3}{\zeta\rho_m}\cos\theta,\qquad \zeta=c_pT_0+\zeta_v, \quad (23)$$

and

$$\frac{\Lambda}{2}\rho_aV^3\cos\theta=\left(1.109\cdot 10^{14}\sqrt{T_0}+\frac{7.39\cdot 10^{17}}{\sqrt{T_0}}\right)10^{-\frac{13{,}500}{T_0}}, \quad (24)$$

where the right-hand side of equation (24) is derived from the vaporization law given by Öpik (1958). Here c_p is the specific heat, and ζ_v the heat of vaporization.

Criterion for mode of ablation

If ablation is to occur by melting and spraying, the surface near $\theta=54.7°$ must be cooled by advection of cooler melt from smaller x, i.e., the assumption in equation (22) must break down. In that case, let us introduce the criterion

$$\mathscr{C}\equiv\frac{u_0(\partial T_0/\partial x)}{V_0(\partial T/\partial y)|_{y=0}}. \quad (25)$$

If the criterion is very small, ablation is certainly by vaporization. If it is very large, it is certainly caused by melting. We desire the critical value near which transition between the two modes of ablation occurs. We also adopt the unshielded values of Λ and Λ' in making the computation.

We have from equation (17), at $y=0$,

$$u_0=\frac{\delta}{\mu_0}\left(\tau_0-\delta\frac{\partial p}{\partial x}\right). \quad (26)$$

Substitution of equations (34) and (44) of Cook et al., together with the assumption of spherical shape and substitution of equation (12) above with $T=T_0$, yields

$$u_0=\frac{\Lambda'(c_pT_0+\zeta_v)\rho_mk}{6\Lambda V}T_0\times 10^{-\frac{8680}{T_0}}\sin\theta\times\left[1+10^{-4}\frac{(c_pT_0+\zeta_v)\rho_mk}{\Lambda\Lambda'\rho_aV^3r}\times T_0\sec^2\theta\left(2\cos\theta-\frac{3}{4}\Gamma\right)\right]. \quad (27)$$

Differentiation of equation (24) with respect to x yields

$$\frac{\partial T_0}{\partial x}=-\frac{\Lambda\rho_a V^3}{2r}\sin\theta \times\frac{T_0^2\times10^{E/T_0}}{\frac{1}{2}T_0\left(C_1\sqrt{T_0}-\frac{C_2}{\sqrt{T_0}}\right)+2.3026E\left(C_1\sqrt{T_0}+\frac{C_2}{\sqrt{T_0}}\right)}, \quad (28)$$

where

$$\frac{\partial\theta}{\partial x}=\frac{1}{r},\quad C_1=1.109\times10^{14},\quad C_2=7.39\times10^{17}. \quad (29)$$

Equation (23) gives v_0, and equation (24) can be solved for T_0. Finally, equation (14) yields

$$\left.\frac{\partial T}{\partial y}\right|_{y=0}=\frac{T_0}{\delta_T},\qquad \delta_T=k/v_w, \quad (30)$$

where v_w is given by equation (21).

Combination of all the above results yields

$$\mathscr{C}=\frac{2\Lambda'}{3\Lambda^2}\frac{k^2}{r\rho_a V^4}\times\frac{[\rho_m(c_pT_0+\zeta_v)]^3T_0^2}{T_0(C_1\sqrt{T_0}-C_2/\sqrt{T_0})-4.605E(C_1\sqrt{T_0}+C_2/\sqrt{T_0})}\times10^{\frac{E-8680}{T_0}}\times\tan^2\theta\times\left[1+10^{-4}\frac{k\rho_m(c_pT_0+\zeta_v)}{\Lambda\Lambda'\rho_aV^3r}T_0\sec^2\theta\left(2\cos\theta-\frac{3}{4}\Gamma\right)\right]. \quad (31)$$

The final term within the brackets is negligible, whence we have

$$\mathscr{C}\simeq\frac{2\Lambda'}{3\Lambda^2}\frac{k^2}{r\rho_a V^4}\times\frac{[\rho_m(c_pT_0+\zeta_v)]^3T_0^2}{T_0(C_1\sqrt{T_0}-C_2/\sqrt{T_0})-4.605E(C_1\sqrt{T_0}+C_2/\sqrt{T_0})}\times10^{\frac{E-8680}{T_0}}\tan^2\theta. \quad (32)$$

The results are given in table 3. The relative run of the temperature is, of course, much more accurate than the absolute value. It is evident that transition between melting and vaporization occurs near $\mathscr{C}=1.2$. If we had used the shielded values of Λ' and Λ, we would have found a somewhat larger value of $\mathscr{C}$; it would also entail extra work in some applications. The unshielded forms were therefore preferred. It is also interesting that $\mathscr{C}=0.8$ already implies vaporization alone. The upper limit on the transition zone must remain more vague for the present.

TABLE 3.—*Temperature at $\theta=54.^\circ7$, and criterion $\mathscr{C}$ for mode of ablation, assumed to be by vaporization, for Meteor 19816 for spherical meteoroid*

Epoch t (sec)	Surface temperature T_0	Criterion $\mathscr{C}$
0. 764	2364° K	1. 3
1. 135	2458	0. 8
1. 860	2584	0. 5
2. 038	2636	0. 5
2. 425	2658	0. 7
2. 623	2640	0. 7
2. 824	2612	1. 1
3. 124	2498	4. 1

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Abstract

Evidence is presented that Meteor 19816 of the program of the Harvard Meteor Expedition passed from ablation by melting and spraying to ablation by vaporization, and back to ablation by melting during its luminous flight through the atmosphere. A criterion is proposed for the mode of ablation. If ablation by vaporization is assumed, it is possible to compute the ratio of the heat transports parallel and normal to the surface of the liquid layer just beneath that surface. A critical value of this ratio of approximately one is found from the meteor.

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Die mikroskopische Beschaffenheit

der

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