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SOME SUNSPOT AND FLARE STATISTICS

by Barbara Bell *and* Harold Glazer

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Some Sunspot and Flare Statistics

By **Barbara Bell**[1] and **Harold Glazer**[2]

Hale and Nicholson (1938) have studied the distribution of the 1915–1924 sunspots among the magnetic classes. Giovanelli (1939) has investigated the relation of flare frequency to sunspot area and to magnetic class, using 1935–1937 data. For our recent study (Bell and Glazer, 1958) of the relations between some properties of sunspots and geomagnetic conditions, we accumulated on punch cards a substantial quantity of sunspot and flare data for the years 1937–1953, which can be used to bring up to date and supplement some aspects of these earlier studies.

The present investigation employs information on 5,940 sunspots, observed from 1937 through 1953 at the Mount Wilson Observatory and reported in the Publications of the Astronomical Society of the Pacific, and 8,403 flares reported during the same period in the I. A. U. Quarterly Bulletin of Solar Activity. For each sunspot group listed by Mount Wilson in the P. A. S. P., the following quantities relevant to the present discussion were punched: magnetic classification, maximum magnetic field strength, a duration factor, and the number of flares observed in the region. We also used the area data on spot groups larger than 500 millionths of the solar disc, published by the Greenwich Royal Observatory (1955).

The Mount Wilson classification of sunspots according to their magnetic properties provides the basic data for our investigation. This system, developed by Hale and Nicholson (1938) from observation of hundreds of spot fields and the distribution of their associated calcium plages, recognizes unipolar (α), bipolar (β), and complex (γ) groups, with the following distinguishing properties:

Unipolar groups are single spots or groups of spots having the same magnetic polarity. They are subdivided according to the position of the group in its calcium plage into the following classes:

α: Spots having a fairly symmetrical distribution of calcium plages preceding and following the group.

αp: Spots situated in the preceding part of an elongated plage.

αf: Spots situated in the following part of an elongated plage.

Bipolar groups in their simplest form consist of two spots of opposite polarity. Often, however, the bipolar group is a stream of spots, those in the preceding and following parts of the group being of opposite polarity. Bipolar groups are subdivided into the following classes:

β: Groups whose preceding and following members, whether single or multiple, are approximately equal in area.

βp: Groups whose preceding member is the principal component.

βf: Groups whose following member is the principal component.

Semicomplex ($\beta\gamma$) groups have bipolar characteristics, but no clearly marked dividing line between the spots of opposite polarity. This class includes groups whose preceding or following members are accompanied by small spots of the opposite polarity.

Complex (γ) groups are those which have the polarities so irregularly distributed that they cannot be classified as bipolar; sharply bounded regions of opposite polarity sometimes exist within the same penumbra.

The Mount Wilson classification places the symbols *l* or *d* before the magnetic class to indicate whether the spot rotated around the east limb or was born on the disc; similarly, an *l* or *d* after the magnetic class indicates whether the spot rotated around the west limb or died on the disc. From these data, plus the published dates when the spot group was first and last observed, we have constructed a "duration factor" which

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divides sunspots into the following ten categories:

ll: Groups that cross the disc, from east limb to west limb.

ld_e, ld_c, ld_w: Groups that appear around the east limb and die, respectively, on the disc east, within one day either side of central meridian passage (CMP), or west.

d_el, d_cl, d_wl: Groups that are born on the disc east, within one day either side of CMP, or west, respectively, and disappear around the west limb.

d_ed_e, d_wd_w, d_ed_w: Groups that are born and die on disc east, on disc west, and those that are born east and die west.

Each spot group was placed in one of these ten categories, usually without uncertainty. But in some cases, where cloudy weather prevented a definite classification, we made assignments more or less arbitrarily. We have no reason to believe that these occasional arbitrary assignments have introduced any serious error in any of the results.

Distribution of spot groups among magnetic classes

Table 1 shows the frequency distribution of the 5,940 sunspot groups among the various magnetic classes. The spots are subdivided by date, according to phase of the sunspot cycle. While the exact dates of division are of course arbitrary, the intervals do correspond roughly to the maximum, declining (decl.), and rising phases of the solar activity cycle.

Table 1 gives the percent of spot groups in each magnetic class for the five periods. The percents of the classified spot groups are taken with reference to the total number of classified groups, the quantity given in the next to last column; but the frequency of unclassified spots is taken relative to the total number of all groups, given in the last column. The rows labeled "North" and "South" give the average distribution of spot groups in the respective solar hemispheres, while the row labeled "mean" gives the over-all average results for the 17 years and 5,940 spot groups. In the final row of the table we reproduce the frequencies obtained by Hale and Nicholson (1938), to facilitate comparison.

The most striking difference between the last two rows of table 1 is the great increase in the frequency of unclassified spot groups in the later period. In the earlier period the Mount Wilson observers attempted to obtain daily polarity measures for every group, whereas in more recent years they have attempted to obtain polarities only once or twice during the life of a group. This change in policy (see Publ. Astron. Soc. Pacific, vol. 48. p. 124, 1936) appears to explain fully the increase in unclassified groups which occur primarily among the small and short-lived spots.

We see also for the more recent period a decline in the relative frequency of α and βp groups with a corresponding increase in the frequency of αp and β. If we consider merely the unipolar and bipolar categories, without regard to the p or f subdivisions, we find agreement within one percent with the distribution obtained by Hale and Nicholson. Nicholson states (private communication) that the differences can probably be adequately explained by changes in the methods of classification.

When the first classifications were made and the laws of polarity distribution were not well established, a group would be classified as α or as αp solely by its position in the surrounding calcium plage. Now, however, if a spot is known to be the surviving leader-spot of a bipolar group, it will be classified as αp even though its position in the plage on a majority of days might be closer to that of a simple α. Since many unipolar spots were formerly bipolar, this change in procedure clearly tends to increase the percentage of αp and diminish that of α. The absence of any increase in the αf frequency results from the fact that the follower spot usually disappears some time before the leader spot.

In the early classifications of β and βp, a very slightly greater field strength in the leader would yield a βp. More recently, however, a group is likely to be called β unless the difference between the spots is greater than the probable error of the field strength or area. Thus β's have become more common at the expense of βp, and αp at the expense of α, although the total percentages of unipolar and bipolar groups remain unchanged.

Table 2 shows the percentage of sunspot groups in each magnetic class when the spots are subdivided according to their maximum magnetic field intensity, H. The bottom row repeats the mean frequencies to facilitate comparisons. In subsequent discussion we shall sometimes refer to these as the "expected" frequencies. Table 2 shows that sunspots of strong and of weak magnetic field intensities differ in their distribution among the magnetic classes. Among the spots of large field strength, the classes αp, βp, $\beta\gamma$, and γ appear in excess of the expected frequency, while among the groups of small field strength the α, αf, β, and βf are more numerous. The increase in percent of βp and corresponding decrease in β is especially noteworthy as the field strength increases. The results in table 2 suggest that groups in which the follower spot is larger than or equal to the leader are much less likely to attain a large field strength and size than those in which the leader spot is the larger. One would infer that a more stable configuration exists when the leader spot is the larger, or is the only one present.

Table 3 shows the percentage distribution among magnetic classes for the spots of at least 2,000 gauss magnetic field intensity, subdivided according to mean area of the spot group. It has previously been found that the central magnetic field intensity increases systematically with the area of the individual spot up to an area about 300 millionths and remains substantially constant for larger spots (Nicholson, 1933; Houtgast and van Sluiters, 1948). Group areas are not closely related to the magnetic field strength of any member spot. Previously published figures on field intensities and areas of large ($A \geq 500$) spot groups (Bell and Glazer, 1958, tables 1 and 2) indicate that any relation between the two quantities is slight, particularly for the β and α spots, and the scatter is large. In general, however, one can certainly say that the spots of $H \leq 900$ gauss are small, those of 1,000 to 1,900 gauss are small to medium, and those of $H \geq 2{,}000$ gauss are medium to large in area.

Table 3 shows a dearth of unipolar spots of large area, as would be expected when group areas are considered. Although there is a dearth of β and βf spots in all categories where $H \geq 2{,}000$ gauss, this deficiency is no more conspicuous for groups of very large area than for moderately large groups. If we consider only the combined bipolar population with $H \geq 2{,}000$ gauss, the proportion of $\beta + \beta f$ groups is greater for areas ≥ 750 than for areas < 750 millionths.

The most noteworthy feature shown in table 3, however, is the marked increase in relative frequency of magnetically complex (γ) and semicomplex ($\beta\gamma$) sunspots with increase in area of the group; this suggests that magnetic complexity may be a form of instability tending to occur when a spot group becomes unusually large. Such an instability may be qualitatively explained by a recent theory of active regions (Krook and Menzel, 1957; Krook and Wild, 1957) which indicates that the inhibition of convection within a spot by its magnetic field leads to a compensating enhanced convection in the facular regions surrounding the spots. The complex fields seen in so many of the largest spot groups may arise from the conflict, over an unusually large area, between the forces of convection and of magnetic inhibition. Menzel (private communication) has suggested that the forces of convection may churn up the magnetic lines of force and associated electric currents, redistributing them in the observed complex and irregular patterns.

Table 4 presents a further subdivision, with the duration factor added. This table gives the ratio of frequencies, observed divided by expected, where the "expected" is the mean distribution, given in tables 1 and 2, and the "observed" is the frequency percentage distribution for the specific magnetic field strength and duration class. For convenience in discussion we may speak of the duration groups as containing mature (*ll*), dying (*ld*), young (*dl*), and short-lived (*dd*) sunspots.

Inspection of table 4 reveals that mature sunspots show a marked preference for the classes αp, βp, $\beta\gamma$, and γ, a preference which is more marked for the larger mature spot groups but which is still evident in those of middle size. Very few small spots survive from limb to limb. Dying spots, when large, appear in excess in the unipolar categories αp and α and in α, αp, αf, when small. The excess of unipoles and dearth of bipoles is greatest for the groups that die east of CMP (ld_e) and least for those dying west of CMP; i. e., greatest in those groups where the spots are nearest to extinction. Young spots, on the other hand, tend to be bipolar and to concentrate in βp, βf, $\beta\gamma$ if large, and in simple

β if small. Short-lived spots tend to be small and to favor α, αf, and β in excess of the expected frequencies.

These quantitative results are in accord with the more general observations of Hale and Nicholson (1938), who pointed out a conspicuous tendency for spots to begin life as bipolar groups, with the follower spot tending to break up first and leave the group as unipolar for its last several days. They also emphasized the predominance of the bipolar condition, and the tendency of unipolar groups to hover on the edge of bipolarity, as evidenced by the preponderance of αp (bipolar plages) and the occasional detection of invisible magnetic spots in groups that never attained a visibly bipolar condition.

Among the spot groups of area $\geq$ 500 millionths, we find 78 percent *ll* ($\Sigma\beta$, $\beta\gamma$, and all α, γ), 20 percent *dl* ($\Sigma\beta$, $\beta\gamma$), and only 2 percent *ld* ($\Sigma\beta$, $\beta\gamma$). The frequencies of *dl* and *ld* appear compatible with the Greenwich (1925) finding that, in the case of large long-lived spot groups, the size of individual spots tends to increase rapidly at the start, reach a maximum around the ninth day, and then slowly decline.

East-west asymmetry

The classification according to duration can be used to investigate some aspects of the question of an apparent east-west asymmetry in sunspot distribution on the solar disc. An east-west asymmetry was found originally by Maunder (1907) from Greenwich sunspot data comprising 15,721 observations of 2,870 spot groups for the years 1889–1901. She found that (1) the eastern solar hemisphere contained about 51 percent of the total sunspot area and 52½ percent of the total number of spot groups; (2) of the total spots observed, 33 percent appeared by rotation at the east limb while only 27 percent disappeared by rotation over the west limb, or 6 percent more spots dissolved on the visible disc than were born there; and (3) about twice as many groups were seen to develop east than west of the central meridian. Partial confirmation of these results was found by Pocock (1918) for the years 1902–1917, and by Rodés (1922) for 1910–1920 sunspots. The third aspect of the asymmetry has been confirmed by Waldmeier (1939) for 1925–1929, and by Mme. d'Azambuja (1955) for 1919–1947 spots. Mme. d'Azambuja finds not only that about twice as many centers of activity appear to arise east than west of the central meridian, but also that the eastern-born centers are more durable. (For other references see Kiepenheuer, 1953; Waldmeier, 1955, p. 160; Link, 1954.)

Table 5 gives the percent of spot groups in each of the ten duration classes and three field-intensity groups, with the entire table, rather than each row, totaling 100 percent.

In table 6 we have combined certain columns of table 5 to better illuminate some aspects of east-west asymmetry. The last two columns, of spot groups visible east and west, give a western excess of about 2 percent (E 75 and W 77 percent). The first two columns reveal that in the 1937–1953 period 2 percent more spot groups by rotation departed at the west limb (WL, 36 percent) than entered at the east limb (EL, 34 percent), and 2 percent more spot groups thus arose than dissolved on the visible disc. These results indicate that parts 1 and 2 of the eastern excess found by Maunder do not appear in the Mount Wilson data for 1937–1953.

On the other hand, from the central columns of table 6, we see that 2.1 times more sunspots appear to be born east than west of the central meridian, while 2.3 times more spots appear to die west than east of CM, in good agreement with the third aspect of east-west asymmetry found by Maunder, as well as the recent results of Mme. d'Azambuja. For this eastern excess in spot births, Schuster (1911) has given what appears to be the most probable explanation, that "the predominance of the eastern half mainly points to an increased facility of detecting a sunspot when it is near the center of the solar disc." From a discussion of a generalized visibility function he concluded that "this excess, therefore, only proves an increased condition of visibility near the center, but, in itself, no want of symmetry."

The probable correctness of Schuster's explanation is indicated, we believe, by the individual columns of table 5. These columns are separated by vertical lines into geometrically similar groups. The first three columns have no bearing on the east-west asymmetry in spot birthplaces. The next two, comprising short-lived spots, give an excess of less than one percent in favor of east (E 13.5 and W 12.8

percent). This small eastern excess does not appear significant, particularly in view of the greater degree of uncertainty in the classification of short-lived (*dd*) groups. (In the case of short-lived groups we have no d_c category for spots born or dying on the day of CMP, and such spots have been arbitrarily assigned to d_e or d_w; spots seen only on the day of CMP were classified as $d_e d_w$.)

The key to the asymmetry would appear to lie in the last five duration classes. If, in fact, independently of whether they are detected, equal numbers of spots are born east and west of central meridian, then the numbers $N(d_e d_w + d_e l)$ should equal $N(d_w l)$. Actually, among the detected spot groups, we see that $N(d_w l) < N(d_e l)$, without even considering the large $d_e d_w$ group. However, since all spots in the $d_e d_w$ and the $d_e l$ groups must cross the central meridian, one may assume that only an insignificant percent would escape detection. Spots of the $d_w l$ group never cross the central meridian and many may escape observation. If we assume an equality of births east and west, 214 spots should exist undetected in the $d_w l$ group for every 67 that are seen. Presumably most of these would fall in the group with low field strength. Applying similar reasoning to the asymmetry of deathplace, we find that 239 spots should exist undetected in the $l d_e$ group for every 47 that are seen. A greater percent of $l d_e$ spots than of $d_w l$ spots appears to escape detection.

The principal asymmetries manifested in table 5 can be explained, at least qualitatively, by two facts: sunspots are much more easily detected near the center of the solar disc; and spot groups tend to grow rapidly in their early stages and to die out more slowly. Any quantitative discussion of the observed asymmetries and of the adequacy of the suggested interpretations, as well as any analysis of the visibility function, would require a finer subdivision of spot lifetimes and birthplaces than is tabulated on our IBM cards.

Flares and magnetic class of spot groups

Tables 7 and 8 present the distribution of the 8,403 solar flares observed in the years 1937–1953 according to the magnetic class and field intensity of the sunspot groups with which the flares are associated. Table 7 gives total numbers of flares, and 8 gives the percent in each category of sunspots. Tables 9 and 10 show the average number of flares observed per sunspot group of the various types. Tables 7 to 10 show that for a given magnetic class the frequency of flare production increases markedly with increasing field strength of the associated spot group. This result is not unexpected, since Giovanelli (1939) previously found a relation between frequency of flares and area of the spot group, and, as we discussed earlier, there is at least a general relation between area of a spot group and the maximum magnetic intensity of its largest component spot.

Tables 7 to 10 also show that the frequency of flare production depends on the magnetic class of the associated sunspot. Bipolar spot groups produce more flares than unipolar groups of comparable field intensity, while magnetically complex groups produce the largest average number of observed flares.* This result is in accord with that of Giovanelli (1939) although no detailed comparison is possible because he studied only those spot groups that produced at least one observed flare, and counted each group anew each day it appeared on the disc. However, the results in table 8 suggest a stronger dependence of flare productivity on sunspot magnetic class than Giovanelli found. This difference may be partially explained by area considerations (to be discussed in the next section) and partially by Giovanelli's omission of groups possessing no observed flares. Flareless spots are relatively rare among complex groups, and most numerous among unipolar groups. If, therefore, we considered only those groups with one or more observed flares, the average number of flares per spot group would be increased sub-

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*NOTE ADDED IN PROOF: At the Moscow (1958) meeting of the International Astronomical Union, A. B. Severny reported that, according to observations with the solar magnetograph at the Crimean Astrophysical Observatory, flares originate at "neutral points of sunspot magnetic fields, when the gradient of field strength is sufficiently high in the vicinity of such points."

The greater flare productivity of γ and $\beta\gamma$ spots can be readily understood, at least qualitatively, in the light of Severny's observations, because the required neutral points should be most numerous in magnetically complex spot groups where the positive and negative polarities are more or less intermingled.

At the same meeting Hoyle and Gold presented a theoretical paper outlining a method of storage and sudden release of flare energy in the chromosphere. Also according to their theory, neutral lines should be the seat of flares; and the energy storage would require uncommon field configurations, which should occur more often in complicated spot regions than in simple ones.

stantially for unipolar spots, moderately for bipolar, and scarcely at all for complex groups; the dependence on magnetic class would thus be minimized.

This point may be further illuminated by table 11, which shows the percent of spot groups in each magnetic class that produced no observed flares, and by table 12, which shows the percent of groups that produced ten or more observed flares. A large $\beta\gamma$ or γ group ($H \geq$ 2,000 gauss) appears almost certain to produce a few flares and likely to produce a large number; 60 percent of these complex spots produced ten or more observed flares, in contrast to 17 percent of the bipolar and a mere 3 percent of the large unipolar spot groups. An essentially similar result was obtained by Richardson (1951). Newton (1943) has pointed out that the majority of very large (3+) flares are also associated with magnetically complex spot groups.

In this connection it is interesting that Behr and Siedentopf (1952) found that, in the years around the 1947 sunspot maximum, the ratio between the number of flares and the sunspot relative number was some 25 percent less than around the 1937 maximum. The data in table 1 show that complex spot groups were relatively less frequent around the 1947 maximum than around the previous maximum, and this deficiency in 1947 (or excess in 1937) may be related to the findings of Behr and Siedentopf.

The duration classes have been used to investigate in a preliminary way whether a significant difference in flare productivity exists between recently born, mature, and dying spot groups. Table 13 shows the average number of observed flares per spot group for the three basic magnetic types, various duration classes, and the two categories of larger field intensity. Since the data on our IBM cards provide no means of obtaining exact mean lifetimes for each group, it seemed advisable to present the table in this form rather than to attempt an approximate and possibly misleading tabulation of flares per spot group per day. Thus, in table 13 one may meaningfully compare the group ld_w with dl but neither of these with any other duration group unless one keeps in mind the difference in number of observing days for the group.

It has commonly been said that young spot groups produce more flares than do dying groups. When all classes are taken together, the data in table 13 support this opinion: *dl* groups produce more flares than corresponding *ld* groups. However, if we compare spot groups of a given magnetic class, we see that this apparent advantage for young spot groups all but disappears. The data in table 13 seem to indicate that magnetic class rather than phase in the life cycle of a spot is the more important factor in the rate of flare production. The greater flare productivity of young spot groups may well be connected with the tendency of young groups to be bipolar and dying spots to be unipolar, rather than with the phase of the life cycle in itself.

Flare frequency in groups of large area

While Giovanelli found a general over-all relation between field strength of a spot and its flare productivity, he also found the latter property to be more closely related to area of the spot group. For spot groups in a particular area interval, 300 to 400 millionths, he found very little relation between maximum field intensity and flare productivity.

We have investigated the relation between area and flare productivity in spot groups of mean area greater than 500 millionths. For the years 1937–1953, Greenwich (1955) lists 271 spot groups of this size, of which 248 had a maximum field strength greater than 2,000 gauss, while 23 fell in the range 1,000 to 1,900 gauss. In table 3 we gave the distribution of the spot groups of large area among the various magnetic types. Table 14 lists the average number of flares observed in spot groups of maximum field strength greater than 2,000 gauss, subdivided according to the area of the group. In each magnetic class flare frequency increases in a conspicuous and systematic way with increase in area of the group. Within a given area interval complex spot groups produce more flares than do bipolar groups, but, because of the tendency of complex groups to be large in area, the contrast between magnetic types is less striking than when area is not considered, and the relation is closer to that found by Giovanelli.

Table 15 presents the average number of flares observed per unit (10^{-5} of solar disc) of spot group area (or, $10\overline{F}/\overline{A}$) based on the 271 spot groups with mean area greater than 500 millionths.

Figure 1 shows the mean number of observed flares plotted against mean spot-group area and various intervals of area. Each magnetic class is plotted separately. The lines were determined from the plotted points by the method of least squares to fit the relation $\overline{F}_t = C + B\overline{A}_t$. The top section of table 16 gives the parameters of these least squares solutions, where each mean point was weighted according to the number of spot groups contributing to it. The table also gives s_F, the standard error[3] of $\overline{F}_t$ predicted from the equation, and r^2 where r is the coefficient of correlation. As we previously noted, the complex spot groups produce the most flares per unit area and the unipolar spots the fewest. For the large spots in our sample this difference between magnetic classes appears in the intercepts rather than in the slopes of the lines, which do not show any systematic variation with magnetic class.

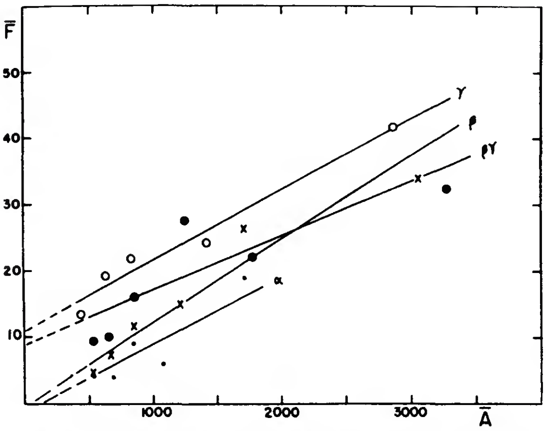

Figure 1.—Relation between the average number of observed flares (F̄) and the average area of the sunspot group (Ā), for sunspot groups larger than 500 millionths of the solar disc. The spot groups are divided into four magnetic classes, which are distinguished by the following symbols: dots (·), α spots; crosses (X), β spots; filled circles (●), βγ spots; and open circles (o), γ spots.

The intercepts for the β and the α spots pass satisfactorily close to the origin, so that these relations may be assumed to apply also to spots of smaller area. For the βγ and the γ spots, however, the intercepts are large. The slope and the intercept for the βγ spots are strongly influenced by the five spots of area ≥ 2000 millionths. If these are omitted, we obtain $B = 1.55 \times 10^{-2}$ and $C = 2.20$; but the improvement in s_F and r^2 is negligible. If the βγ and γ lines were forced to pass through the origin, the correlations would be lower but the slopes would then show a systematic increase with magnetic complexity, of the sort found by Giovanelli (1939). For example, if the lines were drawn arbitrarily through the origin and the mean, we should obtain $B(\gamma) = 1.90 \times 10^{-2}$, $B(\beta\gamma) = 1.55 \times 10^{-2}$, $B(\beta) = 1.20 \times 10^{-2}$, and $B(\alpha) = 0.8 \times 10^{-2}$. If we ignore the two points corresponding to the largest areas and pass the lines through the origin, we obtain $B(\gamma) \approx 2.7 \times 10^{-2}$ and $B(\beta\gamma) \approx 1.8 \times 10^{-2}$. This situation clearly suggests that the relation between flare productivity and area of the spot group is not linear over the entire range of areas, particularly for the γ and the βγ spots. Even from figure 1 there is an indication that spots of intermediate area, around 1,000 millionths, may be more efficient flare producers per unit area than are the very largest spot groups.

We accordingly also plotted the mean numbers of observed flares against the logarithms of the mean areas, with the result shown in figure 2. The middle section of table 16 gives the least squares parameters for the relation $\overline{F}_t = C + B \log \overline{A}_t$. If we compare the corresponding values of r^2, we see that the linear relation gives the better fit for the α and the γ spots, while the log relation fits the β spots slightly better and the βγ spots markedly better. Except for the βγ's the differences are slight, however, and the linear and the log relations fit the data about equally well. From an inspection of the intercepts, C, we infer that the log relations would not apply to small spots. If we take account both of the above results for large spot groups and of the results of Giovanelli, it appears very probable that there is a gradual transition from a linear relation for small spots to a logarithmic relation for very large spots. The spot size around which the transition occurs appears to be smaller for complex spots (500–1000 millionths) than for bipolar spots (≥ 2000 millionths).

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[3] These standard errors are underestimates because they do not take account of the uncertainty in B and C; furthermore, they can be only approximate because F does not actually have a normal distribution about the line.

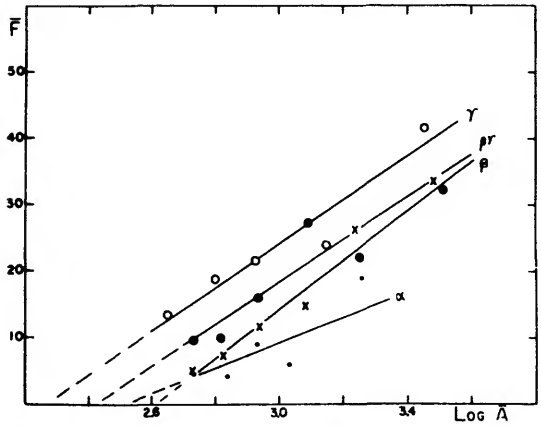

Figure 2.—Relation between the average number of observed flares ($\bar{F}$) and the logarithm of the average spot-group area (log $\bar{A}$), for spot groups larger than 500 millionths of the solar disc. The spot groups are divided into four magnetic classes, which are distinguished by the following symbols: dots (·), α spots; crosses (X), β spots; filled circles (●), βγ spots; and open circles (o), γ spots.

On the average there is a good relation between the mean area and the flare productivity of a spot group. But for individual spot groups in a given area range and a single magnetic class, there is a wide variation in the number of observed flares, as shown, for example, by the βγ spots plotted in figure 3. The least squares parameters for the linear relation, $F_t = C + BA_t$, for individual spot groups is given in the bottom section of table 16. While the slopes and intercepts are close to those shown in the top section of the table, r^2 has decreased and s_F has increased to a point where the relations can hardly be considered good.

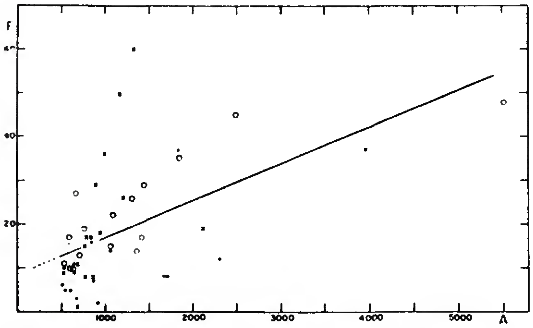

Figure 3.—Relation between the number of observed flares (F) and mean area (A) of individual βγ sunspot groups larger than 500 millionths of the solar disc. The symbols distinguish the season of the year in which the spot group occurred: dots (·), November through February; crosses (x), May through August; and circles (o), March, April, September, and October.

Since most flare observations have been made at observatories located in the Northern Hemisphere, there will tend to be more observing hours in summer than in winter; thus a seasonal factor probably contributes to the scatter in the number of observed flares per unit spot area. To estimate the importance of this factor, we divided each class of spots into three groups according to the date of central meridian passage. These three groups are distinguished by separate symbols in figure 3. Table 17 gives the average flare productivity, $10\bar{F}/\bar{A}$, for the three seasonal groupings of each magnetic type of sunspots. The influence of seasonal observational selection in observed flares per unit area, $10\bar{F}/\bar{A}$, seems apparent in the case of the βγ and the γ spot groups, and negligible in the β samples, which, it should be noted, are the largest and thus presumably the most reliable.

A further test for a seasonal factor in the frequency of flares observed in β spots is given in table 16. In the last two rows of each section, the β spots were divided into two seasonal groups, β(April–September) and β(October–March), which gave values of $10\bar{F}/\bar{A}$ of 0.129 and 0.105 respectively. This division by season produces no improvement in the parameters s_F and r^2, and the scatter appears to be greater in summer, when the observations are more complete. The summer and winter slopes are in better agreement for the log than for the linear relation.

While the scatter can certainly be attributed in part to variation in completeness of the observations, it also seems clear that the relation between area and flare productivity of a sunspot group is only a general tendency, subject to wide and real variation for individual spot groups.

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TABLE 1.—*Percent of sunspot groups in each magnetic class*

Interval	Phase of spot cycle	Magnetic class								Unclassified	Total groups classified	Total of all groups
		α	αp	αf	β	βp	βf	$\beta\gamma$	γ			
Jan. 1937–Nov. 1940	max.	9.9	22.0	2.9	26.8	25.8	8.3	3.4	0.7	23.0	1434	1863
Dec. 1940–May 1944	decl.	5.6	24.2	4.9	23.3	26.2	10.5	3.8	1.3	22.2	446	573
June 1944–Nov. 1946	rise	11.5	19.7	4.8	29.0	23.7	8.6	2.5	0.3	22.0	524	672
Dec. 1946–May 1950	max.	7.5	27.5	3.9	30.8	21.2	6.1	2.5	0.5	20.4	1626	2043
June 1950–Dec. 1953	decl.	13.3	28.0	4.4	20.8	21.6	9.7	0.9	1.1	19.2	638	789
1937–1953 (north)	all	9.4	25.2	4.4	27.1	21.8	8.3	3.0	0.8		2350	
1937–1953 (south)	all	9.2	24.3	3.3	27.5	25.2	7.6	2.3	0.6		2318	
1937–1953 (mean)	all	9.3	24.7	3.9	27.3	23.5	8.0	2.7	0.7	21.4	4668	5940
*1915–24	all	14.0	20.4	4.0	20.9	29.2	7.9	3.0	0.8	6.9	2025	2174

*Frequencies obtained by Hale and Nicholson (1938)

TABLE 2.—*Percent of spot groups in each magnetic class, subdivided by maximum magnetic field intensity*

Field strength (gauss)	Magnetic class								Unclassified	Total groups classified	Total of all groups
	α	αp	αf	β	βp	βf	$\beta\gamma$	γ			
≤900	13. 2	22. 9	7. 4	35. 1	12. 9	7. 9	0. 6	0. 1	35. 5	2107	3351
1000–1900	5. 7	22. 5	1. 2	27. 7	29. 9	9. 8	2. 7	0. 5	1. 5	1528	1551
2000–2900	6. 5	32. 9	0. 7	12. 4	32. 9	6. 1	6. 6	2. 0	0. 7	709	714
≥3000	7. 1	28. 7	0. 3	7. 1	41. 4	4. 3	7. 7	3. 4		324	324
Total	9. 3	24. 7	3. 9	27. 3	23. 5	8. 0	2. 7	0. 7	21. 4	4668	5940

TABLE 3.—*Percent of spot groups in each magnetic class, with field strength ≥2000 gauss, and subdivided by area of spot group*

Area (10^{-6} of solar disk)	Magnetic class								Combined classes			Total No.
	α	αp	αf	β	βp	βf	$\beta\gamma$	γ	$\Sigma\alpha$	$\Sigma\beta$	$\gamma+\beta\gamma$	
≥2000	—	—	—	13. 3	6. 7	6. 7	33. 3	40. 0	—	26. 7	73. 3	15
≥1500	—	3. 4	—	13. 8	13. 8	3. 4	34. 6	31. 0	3. 4	31. 0	65. 6	29
≥1000	—	5. 1	—	13. 6	23. 7	6. 8	35. 6	15. 2	5. 1	44. 1	50. 8	59
750–1000	—	5. 5	—	16. 7	38. 9	7. 4	22. 2	9. 3	5. 5	63. 0	31. 5	54
500–750	1. 5	14. 1	0. 7	8. 9	53. 3	5. 9	11. 9	3. 7	16. 3	68. 1	15. 6	135
<500	8. 6	38. 3	0. 6	10. 4	33. 2	5. 2	2. 9	0. 8	47. 5	48. 8	3. 7	787
Mean	6. 8	31. 4	0. 6	10. 7	35. 6	5. 5	7. 0	2. 4	38. 8	51. 8	9. 4	1035

TABLE 4.—*Ratios of observed to expected frequencies of sunspots, subdivided by magnetic class, magnetic field strength, and duration factor*

Field strength (gauss)	Duration class	Magnetic class								Unclassified	Total No. classified
		α	αp	αf	β	βp	βf	$\beta\gamma$	γ		
≥ 2000		.71	1.28	.15	.39	1.51	.69	2.60	3.43	—	1035
	ll	.86	1.51	.18	.22	1.42	.44	2.82	5.00	—	687
	ld	1.27	2.36	0	.16	.73	.81	.78	0	—	93
	dl	.10	.15	0	.91	2.22	1.29	2.74	.70	—	213
	dd	.52	.77	.61	1.14	1.11	1.49	1.78	0	—	42
1000–1900		.61	.91	.31	1.05	1.27	1.22	1.00	.72	—	1530
	ll	1.02	1.26	.33	.65	1.23	.66	1.81	1.40	—	304
	$ld_{e,c}$	1.68	1.79	.41	.42	.87	.71	.30	0	—	122
	ld_w	.98	1.75	.43	.51	.91	.83	1.17	1.00	—	286
	d_el	0	.15	0	1.42	1.79	1.21	1.70	1.30	—	216
	$d_{c,w}l$	.18	.25	.30	1.62	1.33	1.67	.80	0	—	231
	d_id_i	.68	.58	.40	1.45	.81	2.38	0	0	—	63
	d_ed_w	.17	.53	.40	1.26	1.49	1.75	.10	0	—	308
<1000		1.42	.92	1.90	1.20	.55	.99	.22	.14	—	2103
	ll	.70	.60	0	1.80	.60	1.70	0	0	—	14
	ld_e	1.89	2.08	2.25	.70	.10	0	.30	0	2.35	125
	ld_c	1.05	1.70	1.40	.79	.42	1.14	.80	0	.60	143
	ld_w	1.67	1.06	1.20	1.31	.51	.60	0	1.70	.16	84
	d_el	.20	.26	0	2.42	.81	.80	0	0	.19	47
	d_cl	.10	.36	.60	1.96	.81	.95	.40	0	.43	89
	d_wl	.74	.48	1.00	1.69	1.06	.70	.20	0	2.17	160
	d_ed_e	2.14	.98	2.82	1.09	.34	.87	.10	0	2.24	401
	d_wd_w	1.88	.84	2.20	1.29	.43	.95	.10	0	2.20	382
	d_ed_w	1.10	.79	1.82	1.34	.68	1.24	.20	0	1.23	679

TABLE 5.—*Percent of sunspots in various duration and field intensity classes*

Field strength (gauss)	Duration classes (see p. 26)										Total No. sunspots
	ll	ld_c	d_cl	d_ed_e	d_wd_w	d_ed_w	ld_e	d_wl	ld_w	d_el	
≤ 900	0.2	2.8	1.6	13.0	12.1	15.1	4.2	5.0	1.5	0.8	3347
1000–1900	5.1	1.6	2.6	0.4	0.6	5.3	0.5	1.5	4.8	3.7	1553
≥ 2000	11.6	0.1	0.8	<0.1	<0.1	0.6	<0.1	0.2	1.4	2.7	1040
Total	16.9	4.5	5.0	13.5	12.8	20.9	4.7	6.7	7.7	7.2	5940

TABLE 6.—*Percent sunspots in various categories, illustrating east-west asymmetry*

Field strength (gauss)	Spots rotating over		Spots first seen		Spots last seen		Spots visible	
	EL	WL	East	West	East	West	East	West
≤ 900	8.7	7.6	28.9	17.1	17.2	28.7	37.6	36.3
1000–1900	12.0	12.9	9.4	2.1	0.9	10.7	21.4	23.6
≥ 2000	13.2	15.3	3.3	0.2	0.1	2.0	16.4	17.3
Total	33.8	35.8	41.6	19.5	18.2	41.4	75.4	77.2

TABLE 7.—*Total number of flares observed in 1937–1953 in various magnetic classes*

Field strength (gauss)	Magnetic class								Unclassified	Combined classes			Total
	α	αp	αf	β	βp	βf	$\beta\gamma$	γ		$\Sigma\alpha$	$\Sigma\beta$	$\gamma+\beta\gamma$	
≤ 900	34	90	33	234	116	60	16	0	77	157	410	16	660
1000–1900	80	273	8	762	844	228	257	61	14	361	1834	318	2527
≥2000	87	577	17	630	1881	321	1127	568	8	681	2832	1695	5216
Total	201	940	58	1626	2841	609	1400	629	99	1199	5076	2029	8403

TABLE 8.—*Percent of flares occurring in spot groups of various magnetic classes*

Field strength (gauss)	Magnetic class								Unclassified	Combined classes			All classes
	α	αp	αf	β	βp	βf	$\beta\gamma$	γ		$\Sigma\alpha$	$\Sigma\beta$	$\gamma+\beta\gamma$	
≤ 900	0. 4	1. 1	0. 4	2. 8	1. 4	0. 7	0. 2	0	0. 9	1. 9	4. 9	0. 2	7. 9
1000–1900	1. 0	3. 2	0. 1	9. 1	10. 1	2. 7	3. 1	0. 7	0. 2	4. 3	21. 8	3. 8	30. 1
≥2000	1. 0	6. 9	0. 2	7. 5	22. 4	3. 8	13. 4	6. 8	0. 1	8. 1	33. 7	20. 2	62. 0
Total	2. 4	11. 2	0. 7	19. 4	33. 9	7. 2	16. 7	7. 5	1. 2	14. 3	60. 4	24. 2	100. 0

TABLE 9.—*Average number of flares observed per spot group in each magnetic class*

Field strength (gauss)	Magnetic class								Unclassified	Combined classes			All classes
	α	αp	αf	β	βp	βf	$\beta\gamma$	γ		$\Sigma\alpha$	$\Sigma\beta$	$\gamma+\beta\gamma$	
≤900	. 12	. 19	. 21	. 32	. 43	. 36	1. 33	0	. 06	. 17	. 35	1. 23	. 20
1000–1900	. 92	. 80	. 40	1. 79	1. 85	1. 53	6. 37	8. 7	. 6	. 80	1. 79	6. 62	1. 63
≥2000	1. 24	1. 77	2. 83	5. 67	5. 11	5. 63	15. 65	22. 70	1. 6	1. 70	5. 28	17. 48	5. 02
Mean	. 46	. 81	. 32	1. 28	2. 60	1. 64	11. 20	19. 1	. 08	. 68	1. 85	12. 82	1. 42

TABLE 10.—*Average number of flares observed in limb-to-limb (ll) spot groups of each magnetic class*

Field strength (gauss)	Magnetic class								Unclassified	Combined classes			All classes
	α	αp	αf	β	βp	βf	$\beta\gamma$	γ		$\Sigma\alpha$	$\Sigma\beta$	$\gamma+\beta\gamma$	
1000–1900	1. 94	1. 03	. 50	3. 13	2. 86	4. 25	5. 53	13. 7	----	1. 23	3. 09	6. 90	2. 53
≥2000	1. 34	2. 04	3. 40	8. 95	5. 80	8. 63	16. 80	23. 60	1. 0	1. 95	6. 46	19. 00	5. 70

TABLE 11.—*Percent of spot groups in each magnetic class producing no observed flares*

Field strength (gauss)	Magnetic class								Unclassified	Combined classes		
	α	αp	αf	β	βp	βf	$\beta\gamma$	γ		$\Sigma\alpha$	$\Sigma\beta$	$\gamma+\beta\gamma$
≤ 900	93	92	94	85	79	80	58	0	97	93	83	54
1000–1900	72	70	70	49	41	52	15	0	83	71	46	12
≥ 2000	60	56	50	22	22	19	4	4	40	56	22	4

TABLE 12.—*Percent of spot groups in each magnetic class producing ten or more observed flares*

Field strength (gauss)	Magnetic class								Unclassified	Combined classes		
	α	αp	αf	β	βp	βf	$\beta\gamma$	γ		$\Sigma\alpha$	$\Sigma\beta$	$\gamma+\beta\gamma$
≤ 900	0	0	1	1	0	0	8	0	1	1	1	8
1000–1900	2	1	0	4	3	1	27	43	0	1	3	29
≥ 2000	0	4	17	19	16	21	57	68	0	3	17	60

TABLE 13.—*Average number of observed flares in spot groups of different magnetic classes*

Field strength (gauss)	Duration class	Combined magnetic classes			All classes	Total flares	Total spots
		$\Sigma\alpha$	$\Sigma\beta$	$\gamma+\beta\gamma$			
≥ 2000	ll	1.95	6.46	19.00	5.70	3964	688
	ld_w	.82	3.35	12.0	1.88	158	84
	d_el	.72	4.87	14.1	5.61	897	160
	ld_c	.14	—	—	.14	1	7
	d_cl	0	3.37	1.0	3.07	135	44
	d_ed_w	1.3	.83	1.5	1.0	36	36
1000–1900	ll	1.23	3.09	6.90	2.53	770	304
	ld_w	.99	2.24	7.54	1.78	508	286
	d_el	.38	2.35	6.42	2.51	541	216
	ld_c	.21	.84	1.0	.46	44	96
	d_cl	.58	1.78	10.5	1.80	275	153
	ld_e	.29	1.22	—	.62	16	26
	d_wl	.33	1.45	1.33	1.33	104	78
	d_ed_w	.38	.80	8.0	.75	231	308

TABLE 14.—*Average number of flares observed per large spot group with magnetic strength ≥2000 gauss*

Area (10^{-4} of solar disk)	Magnetic class								Combined classes			All classes
	α	αp	αf	β	βp	βf	$\beta\gamma$	γ	$\Sigma\alpha$	$\Sigma\beta$	$\gamma+\beta\gamma$	
≥ 1000	—	10.3	—	21.1	20.2	16.2	28.7	35.8	10.3	19.9	30.8	25.0
750–1000	—	9.0	—	11.3	12.0	11.5	15.3	21.6	9.0	11.8	17.2	13.3
500–750	4.5	4.05	4.0	5.78	6.80	5.87	12.0	21.2	4.10	6.53	13.6	7.22
<500	1.15	1.47	2.6	3.62	3.28	3.98	6.95	5.34	1.42	3.42	6.63	2.59
Mean	1.24	1.77	2.83	5.67	5.11	5.63	15.65	22.70	1.70	5.28	17.48	5.02

TABLE 17.—*Seasonal effects on the frequency of observed flares*

Magnetic class	Area	Season	No.	Mean area ($\bar{A}$)	No. flares ($\bar{F}$)	$10\bar{F}/\bar{A}$
$l\gamma l$	≥400	Nov.–Feb.	5	1280	14.8	.116
		Equinox	10	1420	28.8	.202
		May–Aug.	9	1180	26.0	.220
$l\beta\gamma l$	≥1000	Nov.–Feb.	5	1730	15.8	.092
		Equinox	8	2070	29.5	.143
		May–Aug.	5	1960	38.4	.196
$l\beta\gamma l$	500–999	Nov.–Feb.	10	680	7.4	.108
		Equinox	7	650	15.3	.236
		May–Aug.	11	800	15.4	.192
$l\alpha l$	≥500	Nov.–Feb.	3	670	4.3	.064
		Equinox	10	800	5.6	.070
		May–Aug.	16	600	5.6	.093
$l\beta l$	≥800	Nov.–Feb.	11	1660	18.5	.112
		Equinox	14	1180	16.8	.142
		May–Aug.	16	1150	16.5	.143
$l\beta l$	500–799	Nov.–Feb.	21	610	6.1	.100
		Equinox	20	690	6.8	.109
		May–Aug.	33	630	6.4	.102

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Abstract

This paper presents some statistics on 5,940 sunspot groups observed at the Mount Wilson Observatory during the years 1937 through 1953. The principal topics studied are: the frequency of spot groups occurring in the various Mount Wilson magnetic classes, and the dependence of this frequency on the size and age of the spot groups; some aspects of the apparent east-west asymmetry in spot distribution on the solar disc; and the distribution of 8,403 observed solar flares among spot groups of various magnetic classes, sizes, and ages.

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TABLE 15.—*Flares observed per unit (10^{-6} of solar disk) of spot group area*

Area (10^{-6} of solar disk)	Magnetic class								Combined classes			All classes
	α	αp	αf	β	βp	βf	$\beta\gamma$	γ	$\Sigma\alpha$	$\Sigma\beta$	$\gamma+\beta\gamma$	
$\geq$1500	—	.11	—	.111	.171	.075	.111	.145	.11	.125	.126	.125
1000–1500	—	.06	—	.160	.132	.074	.230	.24	.06	.127	.232	.165
750–1000	—	.106	—	.118	.139	.115	.178	.256	.106	.129	.199	.150
600–750	.06	.056	—	.078	.109	.141	.180	.373	.058	.104	.222	.129
500–600	—	.084	.07	.123	.107	.073	.177	.12	.083	.106	.162	.106
Mean	.06	.083	.07	.113	.124	.090	.167	.184	.081	.117	.173	.135

TABLE 16.—*Least squares parameters relating flare production and mean area for large spot groups* (see p. 31)

Equation	Mag. class	No.	Mean No. flares ($\overline{F}$)	Mean area ($\overline{A}$)	Least squares parameters		Standard error (s_F)	Coef. of correlation (r^2)
					(C)	(B)		
$\overline{F}_i=C+B\overline{A}_i$								
	lγl	5	24.88	1302	10.95	1.07×10^{-2}	1.60	.977
	lβγl	6	18.55	1196	8.72	0.82×10^{-2}	4.23	.667
	lβl	6	10.27	868	−0.73	1.27×10^{-2}	1.61	.940
	lαl	5	5.48	677	−1.49	1.03×10^{-2}	1.46	.769
	β(summer)	6	10.38	806	−4.13	1.80×10^{-2}	1.57	.927
	β(winter)	6	10.11	961	−1.44	1.20×10^{-2}	0.98	.986
$\overline{F}_i=C+B\log\overline{A}_i$								
	lγl	5	24.88	3.0103	−76.94	33.82	2.35	.951
	lβγl	6	18.55	3.0081	−77.42	31.90	3.41	.827
	lβl	6	10.27	2.8958	−97.27	37.13	1.62	.968
	lαl	5	5.48	2.8104	−49.53	19.57	1.59	.658
	β(summer)	6	10.38	2.8812	−101.56	38.85	2.14	.873
	β(winter)	6	10.11	2.9175	−97.91	37.02	2.15	.933
$F_i=C+BA_i$								
	lγl	24	24.88	1302	11.72	1.01×10^{-2}	12.4	.421
	lβγl	47	18.55	1196	8.81	0.83×10^{-2}	11.5	.310
	lβl	115	10.27	868	0.49	1.13×10^{-2}	7.85	.378
	lαl	29	5.48	677	−1.47	1.03×10^{-2}	3.86	.326
	β(summer)	69	10.38	806	−2.57	1.61×10^{-2}	8.28	.300
	β(winter)	46	10.11	961	+0.36	1.02×10^{-2}	6.84	.554