

## Limits on inference of Mars small-scale topography from MOLA data

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[1] The surface roughness of Mars at scales from  $\sim 1$ –200 m is of interest for lander hazard and radar-sounder clutter analysis, but MOLA data are the only current source of global topographic information. We use synthetic fractal profiles to determine the uncertainties associated with deriving self-affine statistics from detrended segments of MOLA tracks. The mean Hurst exponent derived from such segments is an underestimate of the true value,  $H$ , for  $H > 0.5$ , and an overestimate for  $H < 0.5$ . Values of  $H$  from independent samples along a profile are distributed about this biased mean value. The magnitudes of the bias and variance in  $H$  increase for shorter segments. Terrestrial topography data show that extrapolation of roughness statistics to smaller scales based on self-affine relationships is uncertain. Roughness at the 1–15 m scales may be poorly correlated with topography at  $>100$ -m scales. We suggest that any extrapolation be regarded as a lower bound on the true terrain statistics. **INDEX TERMS:** 5464 Planetology: Solid Surface Planets: Remote sensing; 5480 Planetology: Solid Surface Planets: Volcanism (8450); 6225 Planetology: Solar System Objects: Mars. **Citation:** Campbell, B. A., R. R. Ghent, and M. Shepard, Limits on inference of Mars small-scale topography from MOLA data, *Geophys. Res. Lett.*, 30(3), 1115, doi:10.1029/2002GL016550, 2003.

### 1. Introduction

[2] The topography of natural surfaces is often described by statistical measures, such as the root-mean-square (rms) height or slope. These statistical descriptors are of interest for understanding the physical processes that form and modify a landscape, and in developing remote sensing models that relate scattered or emitted energy to surface properties. Recent work has shown that many natural surfaces are well described by self-affine, or fractal, descriptions. A self-affine surface has a power-law relationship between the horizontal scale of interest (profile length or step size) and the vertical roughness statistic (rms height or slope) [Turcotte, 1992]. The power-law dependence of rms height or slope is represented by the Hurst exponent,  $H$ , which takes on values between 0 and 1. Any given Hurst exponent represents roughness over some range of horizontal scales, and field topography data demonstrate that different values of  $H$  may occur for different ranges of scale [Shepard *et al.*, 2001].

[3] The Mars Orbiter Laser Altimeter (MOLA) collected topographic data with a vertical resolution of the order of 1 m, and a typical sampling interval of  $\sim 300$  m [Smith *et al.*, 1999]. These data have been used to characterize roughness at a range of spatial scales [Kreslavsky and Head, 2000], and to estimate Hurst exponents from profiles representing segments of MOLA orbit tracks [e.g., Garneau and Plaut, 2000; Aharonson *et al.*, 2001; Orosei *et al.*, 2001]. A typical profile length used in such studies is 100 MOLA points ( $\sim 30$  km), which permits analysis of topography on horizontal scales from  $\sim 300$  m to  $\sim 3$  km, based on the sampling criteria of Shepard *et al.* [2001]. For 30-km profile segments, the martian surface is characterized by relatively high  $H$  values, with a mean of  $\sim 0.7$  [Orosei *et al.*, 2001].

[4] Topographic statistics at scales smaller than the MOLA sampling interval are of interest for landing site planning and modeling of potential clutter effects in sounder data (e.g., for the MARSIS and SHARAD instruments) [Plaut *et al.*, 2001]. For landing site evaluation, the topography at scales on the order of 1 m is required. Radar sounders (with free-space wavelengths,  $\lambda$ , of  $\sim 15$  m for SHARAD and 60–167 m for MARSIS) measure clutter from roughness at horizontal scales on the order of  $\lambda$  or greater [e.g., Peeples *et al.*, 1978]. In theory, the roughness of a self-affine surface at one scale together with the Hurst exponent may be used to extrapolate estimates of roughness at other scales. Pending the acquisition of high-resolution stereo photographs by future missions, extrapolation from the MOLA data may provide the only mechanism for roughness estimation over large regions of Mars.

[5] The need for information on small-scale topography based on MOLA data raises two major questions that are addressed here. First, are Hurst exponents derived by fits to segments of MOLA data biased by the small sample size, and what are the statistical uncertainties in  $H$ ? We address this issue through analysis of synthetic fractal profiles. Second, is it reasonable to extrapolate roughness statistics at 1–170 m scales based on fits to data over the 300–3000 m scale range? We address this question through examples of terrestrial surface roughness over a similar scale range.

### 2. Estimating Hurst Exponents From Profile Segments

[6] The most common method for estimating the Hurst exponent is the variogram, a plot of Allan variance versus horizontal step size. The Allan variance corresponds to the

mean-square height difference between points separated by a distance  $\Delta x$ :

$$v^2(\Delta x) = \langle [z(x) - z(x + \Delta x)]^2 \rangle \quad (1)$$

where  $\langle \rangle$  denotes an ensemble average. For a fractal surface, the Allan variance and step size are related by a scaling coefficient,  $C$ , and the Hurst exponent:

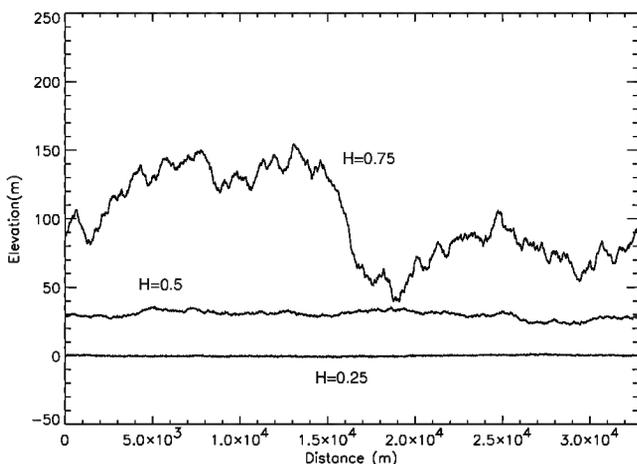
$$v^2(\Delta x) = C^2(\Delta x/\Delta x_0)^{2H} \quad (2)$$

where  $\Delta x_0$  is a reference scale typically taken to be 1 m. The values of  $C$  and  $H$  are obtained by a linear fit to the log-log variogram plot. In general, topographic data are detrended to reduce the effects of roughness on scales much larger than the profile length. The rms slope,  $s$ , at a given horizontal increment is:

$$s = C(\Delta x/\Delta x_0)^{H-1} \quad (3)$$

[7] To simulate variograms from segments of MOLA orbit tracks, we created synthetic fractal surface profiles of varying Hurst exponent, with lengths of 32,000 points (Figure 1). The profiles are generated using the power-spectrum filtering method of *Turcotte* [1992]. As with any statistical distribution, a segment of such a profile provides only a limited sample of the self-affine behavior, and the Hurst exponents for independent samples along the profile are distributed about some mean. While we found empirically that a Gaussian function adequately represents the distribution of  $H$  values for a collection of segments, the mean value of  $H$  may be offset from that of the full profile.

[8] To demonstrate these behaviors, each synthetic profile was divided into non-overlapping segments of  $N$  points, and we determined the best-fit value of  $H$  for each detrended segment over variogram step sizes,  $\Delta x$ , of  $1-N/10$  points (the upper limit suggested by *Shepard et*



**Figure 1.** Synthetic topographic profiles for Hurst exponent values of 0.25, 0.5, and 0.75. All profiles have identical rms slope at the unit scale. Profiles are offset for clarity. Note that higher values of  $H$  lead to more rapid increases in roughness with horizontal scale.

**Table 1.** Results of Fitting Hurst Exponents to Detrended Profile Segments<sup>a</sup>

H	N	Mean H	$\sigma_H$
0.25	50	0.32	0.10
0.24	100	0.30	0.07
0.26	1000	0.25	0.04
0.50	50	0.52	0.10
0.50	100	0.51	0.08
0.50	1000	0.48	0.05
0.75	50	0.68	0.10
0.76	100	0.68	0.09
0.74	1000	0.70	0.06

<sup>a</sup>Value of  $H$  for long profile (32,000 points), number of points,  $N$ , in each profile segment, and resulting mean and standard deviation of Hurst exponent distribution. Variogram fits performed over length scales of  $1-N/10$  points.

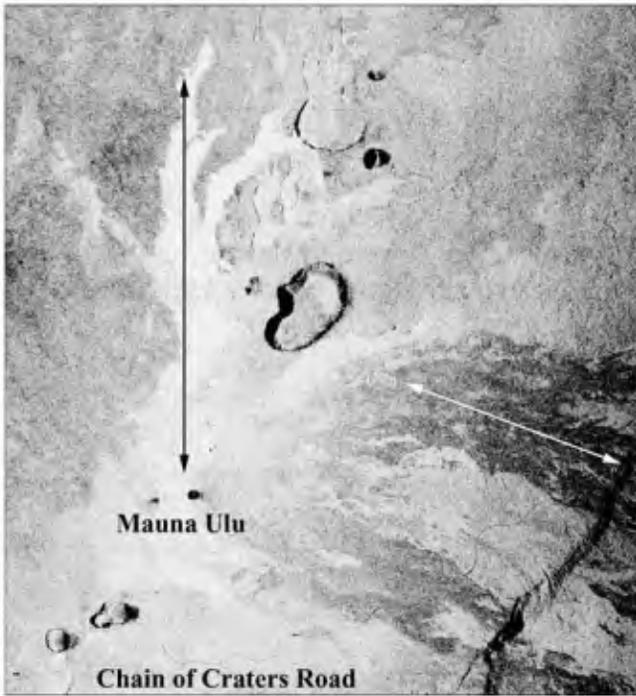
*al.* [2001]). Table 1 shows the mean and standard deviation of  $H$  as a function of the true mean value (which can be slightly different from the ideal value due to the finite length of the full profile) and the segment length,  $N$ .

[9] These results show that the mean value of  $H$ , based on finite profile segments, is overestimated when  $H < 0.5$  and underestimated when  $H > 0.5$ ; the magnitude of the offset increases as the segment length decreases. The individual values of  $H$  derived from the segments are also distributed more widely about this biased mean as the segment length declines. For high- $H$  surfaces, the detrended profile segments effectively high-pass filter the true topography. For low- $H$  surfaces, the bias does not appear to be linked to the detrending process, and may reflect under-sampling of the long-wavelength structure.

### 3. Extrapolating Surface Roughness Information

[10] Given the rms slope of a surface at some reference scale, and the value of  $H$ , we may in principle use (3) to estimate the rms slope at other scales. This approach requires, however, that the scaling relationship of (2) apply over the desired range of  $\Delta x$ . *Shepard et al.* [2001] show that self-affine scaling is often linked to particular surface formation mechanisms which operate over a finite range of scales. For example, the cm-scale roughness of lava flows may be determined by the weathering of glassy rinds, while the meter-scale roughness is dictated by the fluid dynamic features of flow emplacement (billows, ridges, channels). Likewise, the topography of boulder fields exhibits self-affine scaling up to the size of the largest rocks, but above this scale reflects the statistically dissimilar undulations of the underlying terrain [*Campbell, 2001*]. Any inference of Mars small-scale topography from MOLA data assumes similarity of surface formation processes between the 300-m step size and the desired scale. The validity of this assumption may vary considerably over the surface of Mars.

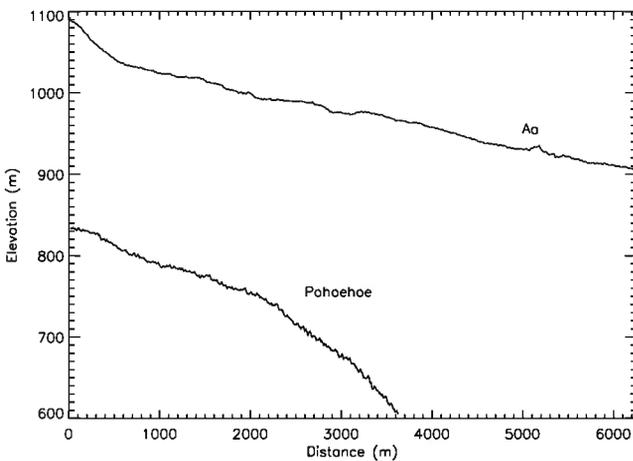
[11] As examples, we present variogram plots for lava flows in Hawaii that have been profiled in the field, and are covered at larger scales by TOPSAR interferometric radar data [*Zebker et al., 1992*]. The TOPSAR data used here have 5 m horizontal resolution and a 0.07-m vertical sampling interval. Figure 2 shows a C-band radar image of the Mauna Ulu shield on Kilauea Volcano, Hawaii,



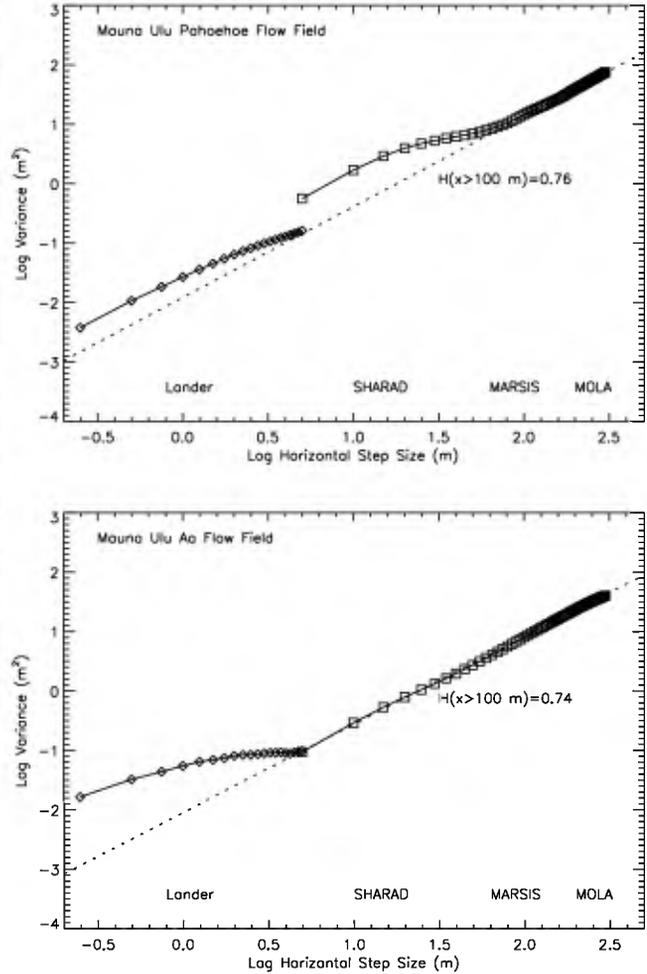
**Figure 2.** C-band TOPSAR radar image of Mauna Ulu shield and the Chain of Craters Road on Kilauea Volcano, Hawaii. White arrow marks 3.6-km pahoehoe flow field transect. Black arrow marks 6.3-km transect over a'a flows.

with the locations of two topographic transects. These transects cover 3.6 and 6.2 km, respectively, of smooth pahoehoe and rough a'a lava flow fields (Figure 3). Field data for both flow complexes were collected at 25-cm horizontal sampling interval, with a vertical accuracy of  $\sim 1$  cm [Campbell and Shepard, 1996]. The high-resolution topography is restricted to only a portion of the flows profiled using the TOPSAR data, so our comparison depends upon an assumption of homogeneity over 3–6 km scales.

[12] Figures 4a and 4b present variograms of the detrended TOPSAR and field topographic data for the two



**Figure 3.** Topographic profiles from TOPSAR data for the two transects noted in Figure 2. These data have not been detrended.



**Figure 4.** Variograms for Mauna Ulu (a) pahoehoe and (b) a'a flow fields, using detrended TOPSAR data for scales from 5–300 m (squares), and field topographic measurements for scales from 0.25–5.0 m (diamonds). The dashed lines are extrapolations based on the Hurst exponent for scales  $>100$  m. The approximate scales of interest for lander hazard assessment, SHARAD clutter, MARSIS clutter, and MOLA measurements are shown along the bottom axis.

test sites. The dashed line in each plot corresponds to the downward extrapolation of a fit to the TOPSAR variogram for horizontal scales  $>100$  m. The rms slope of the test surfaces at scales of 1 m, 15 m, 100m, and 300 m are noted in Table 2. For reference, data at the 1–15 m scale for a very smooth ponded pahoehoe surface are also listed [Shepard et al., 2001]. Note that the variance of the high-resolution topographic data for the pahoehoe flows falls below the TOPSAR-derived variance at scales of  $\sim 5$  m; this is likely

**Table 2.** Rms Slope Values for Test Surfaces<sup>a</sup>

Scale:	1 m	15 m	100 m	300 m
Ponded Flow	2.7°	0.6°		
Mauna Ulu Pahoehoe	9.3°	6.5°	2.2°	1.7°
Mauna Ulu A'a	13.2°	2.8°	1.6°	1.2°

<sup>a</sup>Rms slope, in degrees ( $\tan^{-1}s$ ), determined from Allan deviation of height at horizontal scale noted. Ponded flow data from Shepard et al. [2001]; profile length not adequate to define roughness above 15 m scale.

due to undersampling of the larger-scale flow undulations in the field-measured data.

[13] Several points may be drawn from these results. First, the large-scale topography (i.e.,  $\Delta x > 50$  m) is very similar between the pahoehoe and a'a flow fields, reflecting a common pre-flow terrain. Second, the a'a variogram is linear (constant  $H$ ) over the scale range of 5–300 m, while the pahoehoe field has a dramatic shift in properties at a horizontal scale of  $\sim 50$  m. The pahoehoe surface is thus rougher at the 15-m scale than the a'a surface (Table 2), due to the presence of gently rolling surface billows and tumuli. Finally, the degree of correlation between 1-m scale surface roughness and that extrapolated from MOLA-scale features is very different between the two sites. The a'a flow is rougher at the 1-m scale, by a factor of 2–3 in  $s$ , than would be inferred by extrapolation. The pahoehoe surface is about 1.5 times as rough, at the 1-m scale, as the inferred value. These differences reflect the importance of decimeter-scale emplacement-related features (e.g., ropes, plates, rubble) to the final morphology of the lava flows.

#### 4. Conclusions

[14] Our results demonstrate two important points. (1) The profile lengths (100 points or less) often used to estimate Hurst exponents from MOLA data lead to significant uncertainties about the derived mean. This mean value, in turn, represents an underestimate of the true Hurst exponent when  $H > 0.5$ , and an overestimate when  $H < 0.5$ . The magnitude of the offset in the mean, and the width of the distribution of  $H$  values, increase with decreasing profile segment length. It is also important to note that any single estimate of  $H$  is one sample from the distribution about the mean, so interpretations of variability in surface properties over short length scales must be approached with caution. (2) The degree to which we may extrapolate surface roughness statistics for smaller scales based on self-affine scaling is uncertain. For example, roughness at the 15-m (SHARAD) and 1-m (lander) scales may be weakly correlated with topography

at  $>100$ -m scales, and we suggest that any extrapolation be regarded as a lower bound on the true terrain statistics.

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#### References

- Aharonson, O., M. T. Zuber, and D. H. Rothman, Statistics of Mars topography from the Mars Orbiter Laser Altimeter: Slopes, correlations, and physical models, *J. Geophys. Res.*, *106*, 23,723–23,736, 2001.
- Campbell, B. A., Radar backscatter from Mars: Properties of rock-strewn surfaces, *Icarus*, *150*, 38–47, 2001.
- Campbell, B. A., and M. K. Shepard, Lava flow surface roughness and depolarized radar scattering, *J. Geophys. Res.*, *101*, 18,941–18,952, 1996.
- Gameau, S., and J. J. Plaut, Topographic and roughness characteristics of the Vastitas Borealis formation on Mars described by fractal statistics, *Lunar Planet. Sci.*, [CD-ROM], XXXI, 2000.
- Kreslavsky, M. A., and J. W. Head, Kilometer-scale roughness of Mars: Results from MOLA data analysis, *J. Geophys. Res.*, *105*, 26,695–26,712, 2000.
- Orosei, R., R. Bianchi, A. Coradini, S. Espinasse, C. Federico, and A. Ferricconi, Characterization of the Martian surface scattering at decametric wavelengths, GeoMars Conference, *Cont. 1095*, Lunar Planet. Sci., Houston, Tex., 2001.
- Peebles, W. J., et al., Orbital radar evidence for lunar subsurface layering in Maria Serenitatis and Crisium, *J. Geophys. Res.*, *83*, 3459–3468, 1978.
- Plaut, J. J., R. Jordan, A. Safaeinili, G. Picardi, R. Seu, and R. Orosei, Subsurface sounding of Mars: The effects of surface roughness, GeoMars Conference, *Cont. 1095*, Lunar Planet. Sci., Houston, Tex., 2001.
- Shepard, M. K., B. A. Campbell, M. Bulmer, T. Farr, L. R. Gaddis, and J. Plaut, The roughness of natural terrain: A planetary and remote sensing perspective, *J. Geophys. Res.*, *106*, 32,777–32,795, 2001.
- Smith, D. E., et al., The global topography of Mars and implications for surface evolution, *Science*, *284*, 1495–1503, 1999.
- Turcotte, D. L., *Fractals and Chaos in Geology and Geophysics*, Cambridge Univ. Press, New York, 1992.
- Zebker, H. A., et al., The TOPSAR interferometric radar topographic mapping instrument, *IEEE Trans. Geosci. Remote Sens.*, *30*, 933–940, 1992.
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