A GENERAL MODEL RELATING EXTERNALLY APPLIED FORCES TO ENVIRONMENTALLY INDUCED STRESSES IN MATERIALS

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ABSTRACT

The structural degradation effects of temperature (T) and relative humidity (RH) are important considerations in the setting and maintaining of museum environments. An approach to determining the acceptable values for this environment would be useful if a general model could be developed from simple physical (mechanical) measurements. In this paper, a general model is developed in which the mechanical behavior of materials composing museum objects can be described using the easily determined parameters, stress development (force), environmental conditions and dimensional change.

INTRODUCTION

Changes in relative humidity (RH) cause changes in the moisture content of hygroscopic materials. Since all organic materials are hygroscopic to some extent, most traditional cultural materials are subject to swelling and shrinking with these moisture changes. These dimensional changes are largely responsible for environmental structural damage to objects. Temperature variations also cause dimensional changes in all materials and these environmental fluctuations can also be a very real source of damage.

The mechanism for inducing stress in materials from changes in temperature or RH involves restraint of the object. If hygroscopic materials are restrained and desiccated then they will develop tensile stresses. Materials that are restrained and then cooled will similarly develop stresses. This is, in part, a result of the inability to contract which is the normal behavior if the materials were unrestrained. It would be quite useful to relate this behavior to the mechanical properties of materials as measured by externally applied loads. This requires the equilibrium stress-strain plot as described in reference 1.

ALTERNATE PATHS TO STRESS DEVELOPMENT

Interesting observations were made during the materials testing program at a constant temperature. If any hygroscopic material specimen were allowed to swell from one RH to another; say RH_o to RH_1, restrained and then desiccated back to RH_o, it will attain the same state of stress (σ), length (L), and RH as a specimen subjected to an equilibrium tensile test at a constant RH, in this case RH_o. This is illustrated in figure 1, where the separate paths from one state of stress (σ_o), to another state of stress (σ_1), are described. In this diagram the coordinates are relative humidity (RH); length (L); and stress (σ). It simplifies the analysis if length is used instead of strain as the elongation coordinate. The paths from σ_o to σ_1, are either the equilibrium stress-length test or two separate tests: first, free swelling, and second, the desiccation of the restrained specimen. Paths in planes parallel to the stress-length plane are equilibrium tensile tests at constant temperature and relative humidity. Paths in planes parallel to the stress-relative humidity plane are restrained desiccation tests. The paths in the length-relative humidity plane are swelling isotherm measurements which can be either stress free or under a constant stress.

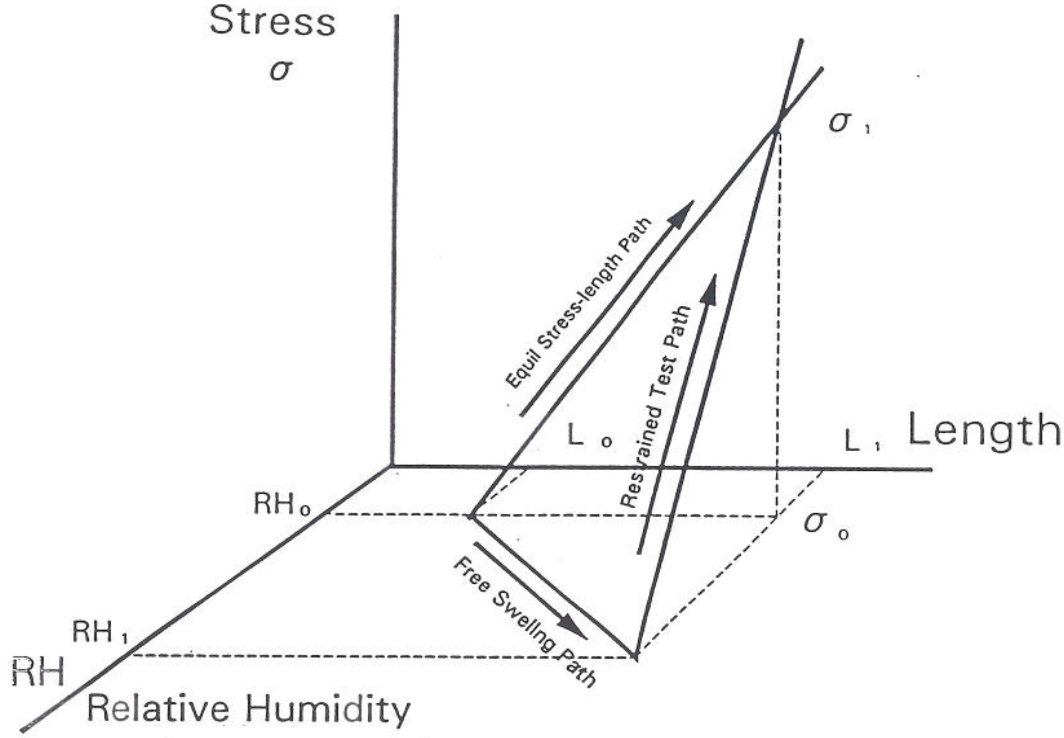

Figure 1. Diagram illustrating the different paths from one state of stress to another state of stress. One path is the equilibrium stress-length (strain) test and the other path consists of the free swelling test combined with the restrained desiccation test.

This, of course, implies that there is a link between the environmentally induced stress and the externally applied loads of a simple tensile test. It also implies that fundamental mechanical and dimensional properties of the materials govern the ultimate state of stress and strain whether the forces are externally applied or environmentally induced in the materials. In addition, this permits the modeling of environmentally induced structural behavior using traditional mechanical data.

Several different types of materials were tested at ambient temperature (23°C) to determine if this is a general concept. Some of the data of this test program are illustrated in figures 2-7. Figure 2 illustrates the different paths to two states of stress in a 13-year-old unsupported cadmium yellow alkyd paint film. The intersection points are indicated as I_1 and I_2. In figure 3, four intersection points, I_1-I_4 were established in a 13-year-old titanium dioxide pigment in safflower oil paint. Figures 4-6 are three different epoxy films. The first is a commercial product HYSOL™ EA 9394, which is modified with powdered aluminum and amorphous silica. This material has a very high moisture coefficient of expansion and has been stiffened with "inert" solids. As a result, it develops very high stresses with desiccation. Even with these modifications, it was a simple process to establish three intersection points as shown in figure 4. Figures 5 and 6 show Epoxy Adhesives 96 and Epoxy Adhesive 3H respectively, structural adhesives developed for a research program at the Martin Marietta Laboratories in Arbutus, Maryland. Several intersection points are shown in each of these adhesives. Finally, figure 7, shows rabbit skin glue with five actual intersections and with extrapolation, six additional points. For these materials, plots in a 3-D coordinate system describe surfaces generated by the parameters stress (or force), length, and RH.

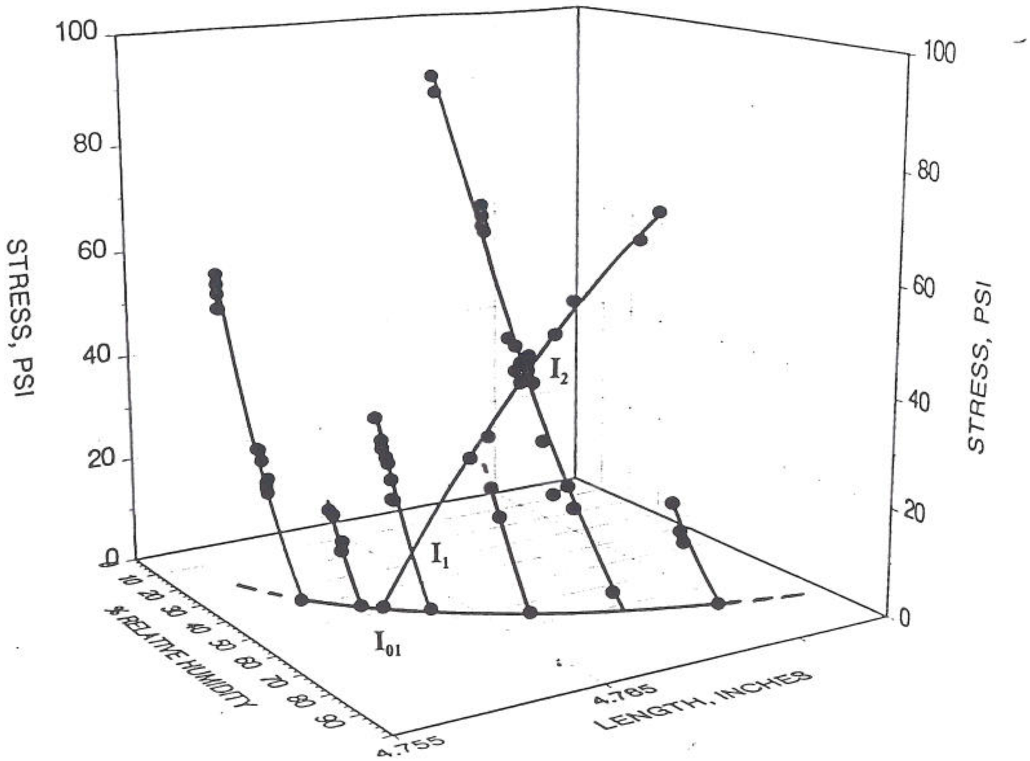

Figure 2. Stress-length-relative humidity plot for cadmium yellow alkyd paint showing mutual intersection points of stress starting at I_o, and going to I_1 and I_2.

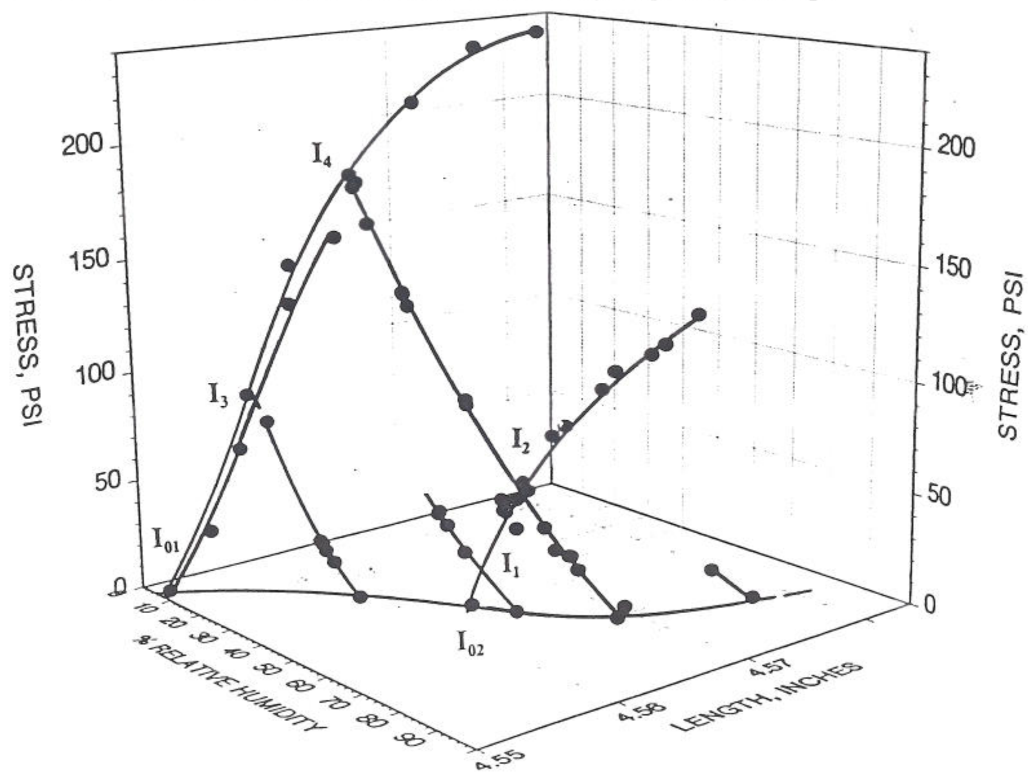

Figure 3. Stress-length-relative humidity plot for titanium dioxide oil paint showing mutual intersection points of stress starting at different points, I_{o1}, I_{o2} and going to four different stress states I_1, I_2, I_3, and I_4.

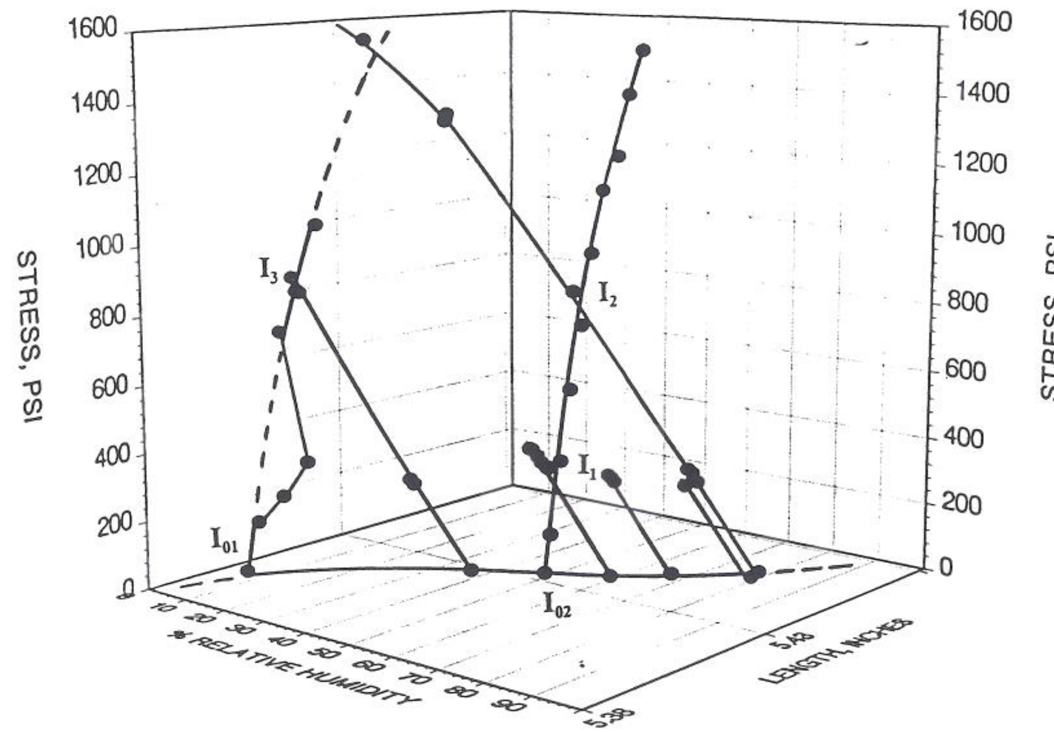

Figure 4. Stress-length-relative plot for epoxy adhesive HYSOL EA 9394 mutual intersection points of stress starting at different points, I_{o1}, I_{o2} and going to three different stress states I_1, I_2, and I_3.

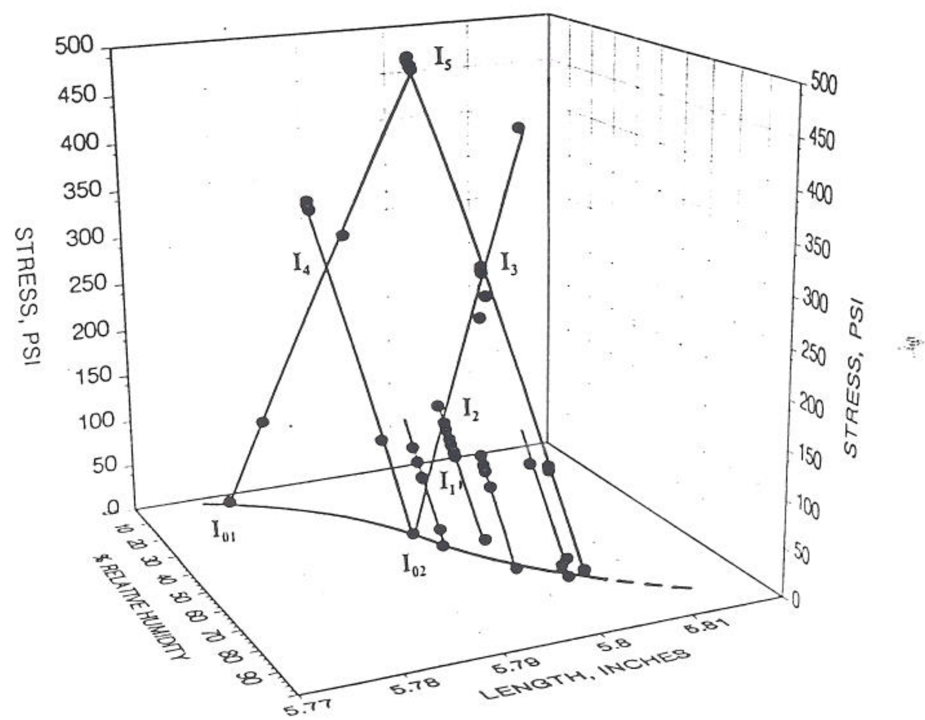

Figure 5. Stress-length-relative humidity plot for epoxy Adhesive 96 mutual intersection points of stress starting at different points, I_{o1}, I_{o2} and going to five different stress states I_1, I_2, I_3, I_4, and I_5.

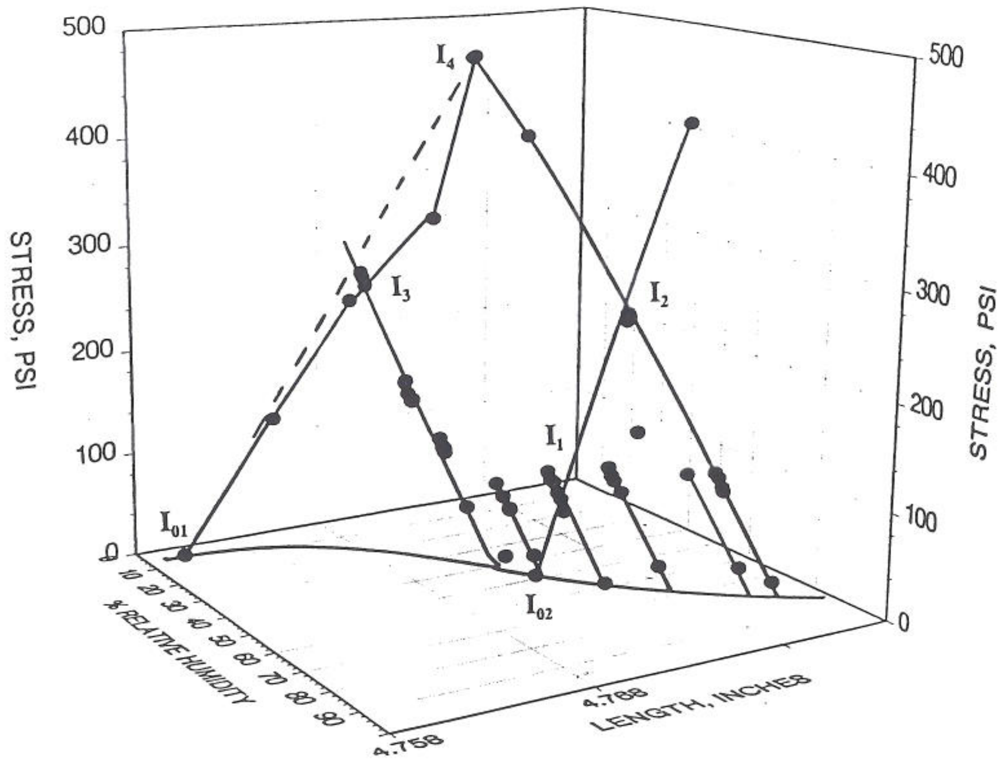

Figure 6. Stress-length-RH plot for epoxy Adhesive II 3H mutual intersection points of stress starting at different points, I_{o1}, I_{o2} and going to four different stress states I_1, I_2, I_3, and I_4.

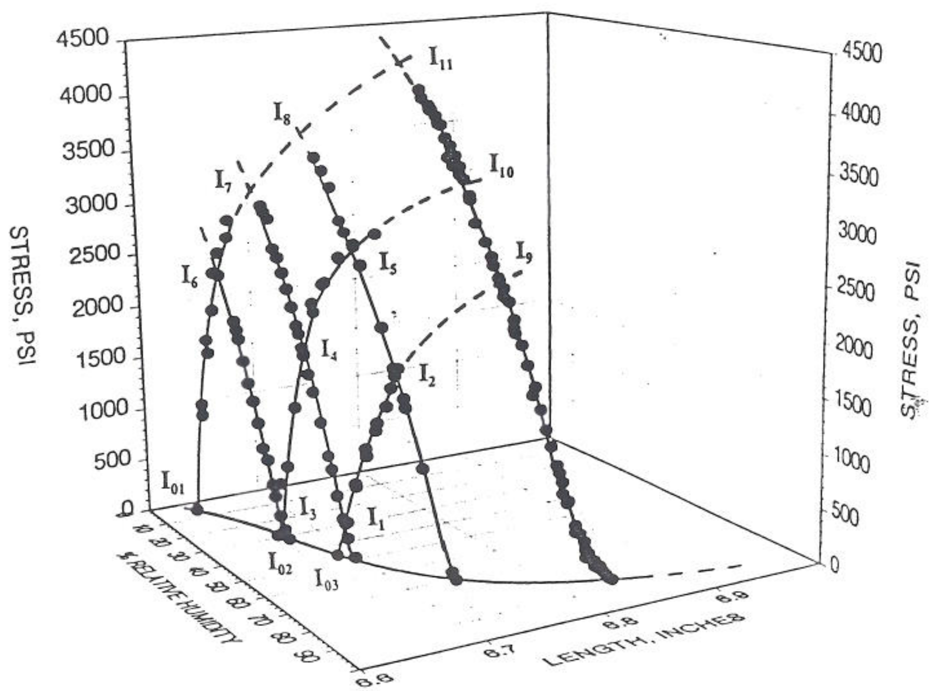

Figure 7. Stress-length-RH plot for rabbit skin glue mutual intersection points of stress starting at different points, I_{o1}, I_{o2}, and I_{o3} and going to five different stress states I_1, I_2, I_3, I_4, and I_5. Six additional projected mutual stress points are shown at I_6, I_7, I_8, I_9, I_{10}, and I_{11}.

These materials represent both cross-linked and non-cross-linked polymer networks, yet the basic behavior is the same. So far no material tested has failed to follow this pattern. For these materials and perhaps many more, a fundamental relationship between the mechanical, dimensional and environmental behavior can be established.

GENERAL MATHEMATICAL CONSIDERATIONS

The preceding data can be discussed in mathematical terms and it is possible to construct a theoretical model. In figure 8 the hypothetical paths are illustrated in the force, length, and relative humidity (F,L,RH) three dimensional coordinate system. The slope of the force-length equilibrium plot, (analogous to the stress-strain plot) at any point can be defined as $(\partial F/\partial L)_{RH,T}$. The slope of the length-relative humidity plot can be defined as $(\partial L/\partial RH)_{F,T}$, which is analogous to the coefficient of expansion when the force is zero, i.e., F=0. Finally the slope of the force-relative humidity plot is defined as $(\partial F/\partial RH)_{L,T}$. The subscripts refer to the fixed conditions when the partial differentials are taken. For any small increment of force (dF), the following holds:

$$dF = (\partial F/\partial L)_{RH,T} dL \qquad (1)$$

and

$$dF = -(\partial F/\partial RH)_{L,T} dRH. \qquad (2)$$

The corresponding increment of length dL can be described as:

$$dL = (\partial L/\partial RH)_{F,T} dRH \qquad (3)$$

Combining equations 1 and 3 will give:

$$dF = (\partial F/\partial L)_{RH,T} (\partial L/\partial RH)_{F,T} dRH \qquad (4)$$

This equation can be combined with equation 2 such that:

$$(\partial F/\partial L)_{RH,T} (\partial L/\partial RH)_{F,T} = -(\partial F/\partial RH)_{L,T}, \qquad (5)$$

and can be written as:

$$E^* \times \alpha^* = -\phi^*, \qquad (6)$$

where:

$$E^* = (\partial F/\partial L)_{RH,T}$$
$$\alpha^* = (\partial L/\partial RH)_{F,T}$$
$$\phi^* = (\partial F/\partial RH)_{L,T}$$

This equation relates the externally applied forces in a material to the environmentally induced forces. In addition, the stiffness, the equilibrium modulus, of a material as measured by externally loaded processes in a constant environment is the same stiffness that determines the forces developed by changes in relative humidity for a restrained material.

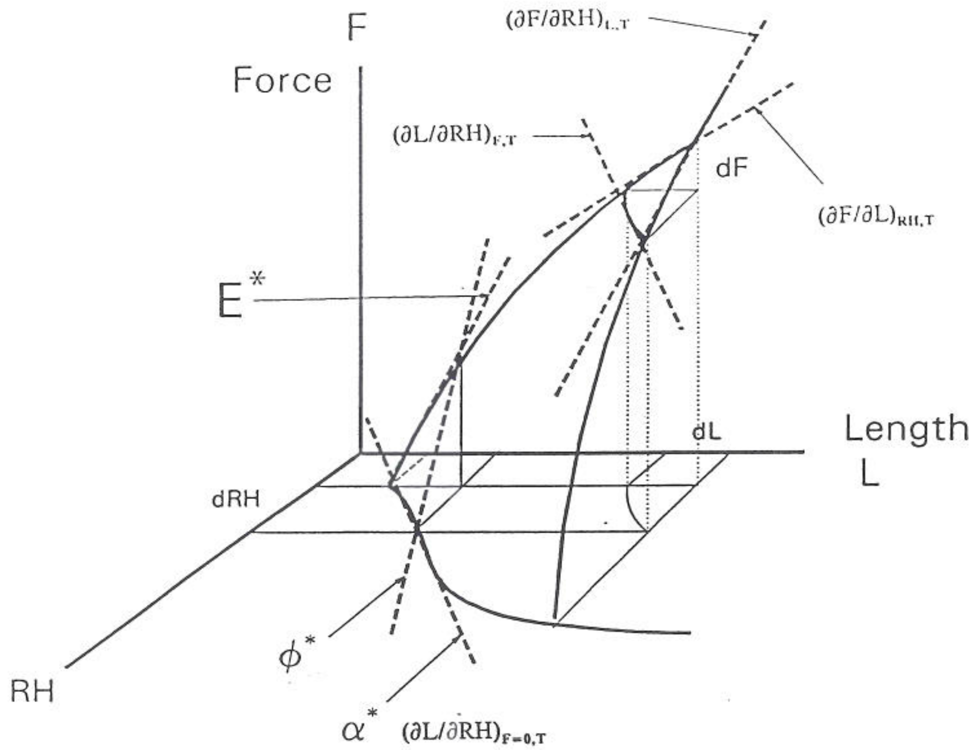

Figure 8. Theoretical force-length-relative humidity (F-L-RH) diagram illustrating the mathematical definitions and relationships.

ENGINEERING APPLICATION OF THE CASE

It would be useful to apply these concepts to the engineering problems that confront the researcher. The immediate question that can be answered is, do environmental fluctuations induce plastic deformations? This can be approached by rewriting equation 5 into engineering terms. This conversion is accomplished by first dividing both sides of the equation by A, the cross-sectional area of the test sample:

$$(\partial F/\partial L)_{RH,T}\ (\partial L/\partial RH)_{F,T}\ (1/A) = -(\partial F/\partial RH)_{L,T}\ (1/A) \qquad (7)$$

the equation then becomes:

$$(\partial \sigma/\partial L)_{RH,T}\ (\partial L/\partial RH)_{F,T} = -(\partial \sigma/\partial RH)_{L,T} \qquad (8)$$

and if L_o is the initial length of the specimen, and by multiplying the left side of the equation by (L_o/L_o), i.e., identity (i.e. 1) the equation becomes:

$$(\partial \sigma/\partial \epsilon)_{RH,T}\ (\partial \epsilon/\partial RH)_{F,T} = -(\partial \sigma/\partial RH)_{L,T} \qquad (9)$$

For linear elastic behavior of a material the term, $(\partial \sigma/\partial \epsilon)_{RH,T}$, is in fact the modulus of elasticity (E) or Hooke's Law. The term $(\partial \epsilon/\partial RH)_{F,T}$ is α, the moisture coefficient of expansion when $\sigma=0$ and the term, $-(\partial \sigma/\partial RH)_{L,T}$, is ϕ, the change in stress with the change in relative humidity when the length is fixed. This last term describes the restrained specimen stress development.

Looking at this specific relationship in symbolic form becomes:

$$(E)_{RH,T}\,(\alpha)_{\sigma=0,T} = -(\phi)_{L,T} \qquad (10)$$

This equation was found to be applicable to all classes of materials studied such as, hide glues, photographic gelatin, epoxy adhesives, oil and alkyd paints.

CONCLUSIONS

A series of experiments were performed on several classes of museum materials. These described the dimensional response with respect to temperature and relative humidity, the stress-strain curve under rapid and equilibrium conditions, and stress development under restrained conditions with variation in temperature or relative humidity. From this data one unifying observation emerged.

From the changes in force with respect to length, the changes in length with respect to relative humidity, and the changes in force with respect to RH, a general equation for the behavior of materials under environmental changes was developed. A similar treatment relating temperature to force, dimensional change and temperature is being developed.

REFERENCES

1. Mecklenburg, M.F. and Tumosa, C.S., In Art in Transit, Studies in the Transport of Paintings, Edited by. M. F. Mecklenburg, (NGA, Washington, DC, 1991) pp 173-216.