# MECHANICAL BEHAVIOR OF PAINTINGS SUBJECTED TO CHANGES IN TEMPERATURE AND RELATIVE HUMIDITY

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ABSTRACT: The effect of changes in temperature and relative humidity on the mechanical behavior of paintings is examined with regard to assessing the potential for damage. While forces, such as those applied by vibration and impact, occur over brief time intervals, those caused by environmental change tend to be considerably longer. This study examines the effects of temperature and relative humidity on the mechanical properties of artists' materials. In addition, the physical response, such as swelling, of some of the materials is described. Using both the mechanical properties and the dimensional response of the materials, computer models of typical paintings correlate the magnitude of developed stresses to the environmental changes. Comparing these calculated stresses with the measured strength of the materials in the painting allows for the development of a risk assessment for damage caused by environmental change.

#### INTRODUCTION

Considerable experimental evidence exists indicating that changes in the environment induce internal forces (stresses) in the different layers of paintings.1 In fact, it was observed that moving the collection of the National Gallery of London to a quarry for protection against bomb damage during World War II reduced the active flaking previously found to occur while the collection was on exhibition at the Gallery. This observation prompted the idea that a stable climate offered long-term stability for the paintings. Since then, it has become almost standard policy for institutions to attempt to maintain environments with constant temperature and relative humidity, (usually around 50% RH and 24°C [75°F]) for the collections. This policy is sound, but unfortunately not always possible. Retrofitting many old historic sites with air-conditioning systems tends to dramatically alter the architectural quality of the spaces. In the winter, many old buildings suffer severe exterior wall damage caused by freezing water that permeated through the walls and crevices. In many of these cases, the installation of the proper vapor barriers is not possible or too costly. There are other circumstances where maintaining a constant environment may be problematic. One of these is the transportation of objects to and from exhibitions.

Curators, conservators, registrars, and collectors are continually attempting to assess the risk of lending a painting to an exhibition given the potential for the object being exposed to environmental changes and possibly suffer damage. This risk assessment is difficult since there is no real correlation between environmental deviations and damage to objects. Additionally, there is a seemingly infinite variation in the construction of paintings. Does this enormous variation, therefore, have an equally large influence on the painting's response to relative humidity and temperature? The problem, while large, can be systematically addressed using engineering mechanics and examining the mechanical responses of artists' materials and paintings to environmental factors such as temperature and relative humidity. These are the sources of stresses and stress-related cracking and flaking and will be encountered wherever there are changes in the ambient environment of the painting.

There are two fundamental properties that must be examined. The first is the dimensional response of the painting materials to both temperature and relative humidity. If these materials are restrained, as they are in a stretched painting, and are subjected to desiccation, they have a propensity to shrink. This produces a stress rise. The second mechanical properties are those that relate force and deformation (stress and strain). These properties are used to determine the magnitude of the stresses resulting from the restrained materials attempting to contract.

Using this information, a "structural" analysis of the typical paintings can be performed to determine how the objects as entities respond to the changes in environment. Specifically, the sources, magnitudes, and locations of stresses can be identified and compared with the measured strengths of the materials to correlate environmental change to damage of paintings. One of the most powerful analytical tools for structural analysis is the digital computer. For handling structures with complex shapes in three dimensions and constructed of a wide variety of materials, the technique of *Finite Element Analysis* (FEA) is most appropriate.

#### DIMENSIONAL RESPONSE OF ARTISTS' MATERIALS TO RELATIVE HUMIDITY

#### Oil Paints

Most artists' materials will expand with an increase in moisture content and conversely shrink upon its loss. The major mechanism for changes in the materials' moisture content is the ambient relative humidity. If a shipping case is not buffered, temperature will influence the internal relative humidity, causing a rise with cooling and a decrease with heating.

Various artists' materials respond differently to moisture content. Oil paints, for example, experience considerable variation in their dimensional response to relative humidity. Figures 1 and 2 (see *Appendix B* for

all Figures) illustrate two paints that differ considerably in this respect. In Figure 1, flake white, ground in safflower oil, swells a maximum of .18%, with the largest increase, about .6%, occurring between 70% and 95% RH. Burnt umber, in linseed oil, swells a total of 2.6% at 95% RH and, like flake white, the greatest increase occurs between 70% and 95% RH (Figure 2). These specimens, which were between .025 cm (.010 in.) and .038 cm (.015 in) thick, took at least forty-eight hours to reach equilibrium with the environment at which the data was recorded.

There are two basic regions of swelling for most of these paints, from 0% to 70% RH where the swelling rate is low, and from 70% to 95% RH, where there is a marked increase in swelling. Twelve paints were tested and their two swelling regions are listed in Table 1 (see *Appendix A* for all Tables). The paints listed in Table 1 were cast in March 1978 and tested in December 1982. Details of the paint sample casting, preparation, and testing are presented elsewhere in this publication.3 It was noted that the stiffer paints, the "fast driers" such as the white lead paints, showed considerably less dimensional response to moisture than the slow driers such as the earth colors. This suggests that with time, as paints become dryer, their swelling response to relative humidity diminishes. Both safflower and linseed oils seemed to respond similarly.

Using Table 1, it is now possible to calculate two different "moisture" coefficients of expansion that linearly approximate the effect of moisture on the dimensional properties of the paint. These coefficients include the ranges from 0% to 70% RH and from 70% to 95% RH and can be calculated by using the percent length changes in each of the listed relative humidity ranges. For example, the low relative humidity range, moisture coefficient,  $\alpha_a$ , for the flake white is:

$$\alpha_a = \frac{.18}{70} \times \frac{1}{100} = .0000257,$$

which is the change in strain per percent relative humidity. For the high relative humidity range:

the change in strain per percent relative humidity. For the high relative humidity range:

$$\alpha_b = \frac{.62}{25} \times \frac{1}{100} = .000248$$

which is nearly ten times the low range coefficient. These coefficients will be used to calculate the stresses resulting from the desiccation of restrained paint.

#### Rabbit Skin Glue and Gesso

One material, traditionally used in paintings, that is dimensionally responsive to relative humidity is rabbit skin glue. This material is used as a fabric size and a binder to gesso, when such grounds are used. The glue can absorb considerable water and in doing so, swells to over 3.5% of its dry length between 0% and 85% RH. Glue shrinkage during desiccation has been identified as one of the most important sources of relative humidityrelated stress development in paintings. Figure 3 shows the loss of length of two rabbit skin glue specimens when desiccated from 85% to near 0% RH. Similar to paint, there are two distinct regions of dimensional change, 0% to 70% RH and 70% to 85% RH, with the latter region having a considerably higher shrinkage rate than the former for the time that it takes glue to reach equilibrium (see Restrained Stress Development).

Gesso's response to relative humidity is similar to that of rabbit skin glue. The difference is that the total length change with comparable ranges of relative humidity is considerably less, and that 80% RH, not 70% RH, seems to mark the point of demarkation between the different swelling rates. Gesso swelling is influenced by the chalk-to-glue ratio. The higher this ratio, the smaller the total dimensional response to relative humidity. Table 2 gives the dimensional response for two different gesso mixtures when the filler was calcium carbonate.

#### Support Fabrics

Support fabrics' dimensional response to relative humidity is somewhat complicated because they respond differently at their initial relative humidity cycle from all subsequent cycles. At between 80% and 85% RH they shrink with desiccation, as well as shrink when the relative humidity increases. This behavior is illustrated in Figure 4, which shows the percent length changes of both the warp and weft directions of samples of Ulster Linen #8800<sup>5</sup>, with changes in relative humidity. At approximately 80% to 85% RH, shrinkage occurs irregardless of whether the relative humidity increases or decreases.

The high relative humidity shrinkage is a result of the textile weave. The transverse swelling of the yarns forces the mutually perpendicular yarns to increase in crimp, shortening the textile dimensions. The low relative humidity shrinkage (below 80% RH) is primarily a result of fiber shortening with desiccation, again with the weave again influencing behavior. The larger dimensional response in the warp direction, which has greater crimp in this textile, than in the weft substantiates this. The dimensional response occurring after the initial cycle is the most important.

The dimensional response of the fabric (Figure 4) is generally typical of woven linen textiles. The high relative humidity shrinkage results in high tension in a wetted, stretched linen, which is commonly observed by conservators preparing lining linens in painting conservation. This same stretched linen becomes completely slack upon drying and this has implications in the painting's response to environmental changes. Once a painting canvas is stretched and sized with glue, the resulting tautness upon drying is from the dried glue, not the canvas. At this stage the glue, not the canvas, is the primary support of the paint layers in the painting.

#### Wood

Wood is used as support panels in painting construction and as stretchers that support the canvas in fabric-supported easel paintings. The variety of woods used in making paintings is large, Wehlte lists at least nine hardwoods and nine softwoods. Additionally, there is a proliferation of literature on the dimensional response of paintings to rela-

tive humidity. Therefore, only a few pertinent features on the response of wood to relative humidity warrant discussion. Wood is orthotropic, that is, its physical (as well as mechanical) properties are different in the three mutually perpendicular directions. These directions are normally the longitudinal, tangential, and radial. Generally, the dimensional response of wood is considerably greater in the radial and tangential directions than it is in the longitudinal. Panshin and de Zeeuw<sup>8</sup> have shown that on the average, and over a broad spectrum of moisture contents, wood can dimensionally change approximately .3% in the longitudinal direction, 3.6% in the radial direction, and 5.9% in the tangential direction. Old woods still respond to changes in relative humidity and within the normal ranges encountered, the changes are still significant. Figure 5 shows the dimensional response to changes in relative humidity of three samples of oak that are at least one-hundred years old with approximate testing of two specimens in the radial direction and the third in the tangential direction. The total length changes in the specimens subjected to a change in relative humidity from 12% to 76% were .9%, .98%, and 1.64%, at 23 °C (73 °F).

These specimens were small; the end grain dimensions were between .099 cm (.039 in.) and .284 cm (.112 in.) and the widths of the test specimens were between .76 cm (.3 in.) and .89 cm (.35 in.). It took at least ninety-six hours for these specimens to reach equilibrium with the environment at which measurements were recorded. Estimating the total dimensional response of wood found in a panel painting requires not only the identification of the wood but its grain orientation relative to the dimensions of the panel.

It is significant that there is little response of the wood in the longitudinal direction. Materials applied to wood, such as glue, gesso, and paints will still attempt to shrink upon desiccation regardless of the substrate. If the wood responds little to relative humidity in the longitudinal direction, it must be considered a *restraint* to the different material layers applied to it. Upon desiccation, that restraint will cause stresses in the upper layers,

and if failure (cracks) occurs, they will generally appear perpendicular to the grain of the wood.

## DIMENSIONAL RESPONSE OF ARTISTS' MATERIALS TO TEMPERATURE

Relative humidity is not the only factor that affects the dimensional stability of artists' materials. Temperature, independent of relative humidity, can also have an effect. The material dimensional response to temperature is less dramatic than that of relative humidity when considering the temperature ranges encountered. Nevertheless, these effects, too, are significant.

For insight into the dimensional response of artists' materials to temperature without a serious influence from relative humidity, the thermal coefficients of expansion for Naples yellow paint and rabbit skin glue were determined while the ambient relative humidity was held at approximately 5%. The yellow paint was chosen because it is one of the stiffest paints examined. Figure 6 shows where the percent length change of the materials is plotted against temperature. Two tests of the Naples yellow were conducted to check for consistency of results. The results are remarkably linear in the temperature range from 23°C to -6°C as is shown by the fitted lines.

The total change in the length of the materials is small over the tested temperature range and the slope of these lines divided by one hundred will give the thermal coefficients (y) of expansion for these materials. This is the change in strain per degree Celsius. For the paint, the thermal coefficient,  $\gamma$ , is equal to .000052 per degree Celsius; the rabbit skin glue has a thermal coefficient of about half that or .000025 per degree Celsius. Wood has thermal coefficients that differ in the three directions. For both hardwoods and softwoods, the thermal coefficient of expansion ranges from .0000027 to .0000045 per de-Celsius in the parallel-to-grain direction. 10 This is about one-tenth that of the hide glue. Perpendicular to the grain, both radial and tangential, the coefficients are proportional to the density of the wood and can range between five to over ten times greater than the parallel-to-grain direction.

For comparison purposes, it is known that three common metals, copper, steel, and aluminum have coefficients of .000017, .000011, and .000024 per degree Celsius, respectively. A material having a coefficient of expansion of .000025 per degree Celsius and cooled from 20° to 0°C will shrink only .05%, which is small compared to the changes that occur with even modest changes in relative humidity.

## MECHANICAL PROPERTIES OF ARTISTS' MATERIALS UNDER LONG-TERM CONDITIONS

Because the time required for artists' materials to equilibrate with new environments is relatively long, it is necessary to examine the mechanical properties of artists' materials under long-term conditions. Paintings can hang on a gallery wall for years before being moved in preparation for transport to an exhibition in another institution. Under these circumstances it is probable that a painting will have equilibrated to the mean ambient environment of its location. This means that any dimensional changes are at a minimum if the environment is stable. It also means that any stresses in the different layers of the painting have fully relaxed, or diminished to a minimum. The mechanical properties of painting materials are very different under these long-term circumstances than they are under dynamic conditions. Artists' materials are able to deform considerably when stresses are applied slowly. The maximum strengths attained are also low under these conditions when compared to rapid loading conditions.

In defining some terms, stress,  $\sigma$ , is the applied force, F, divided by the cross-sectional area, A, of the test specimen. Stated mathematically:

(Equation 1) 
$$\sigma = \frac{F}{A}$$

Engineering strain,  $\epsilon$ , is the change in specimen length,  $\delta$ , divided by the original length of the specimen,  $L_o$ , where:

(Equation 2) 
$$\varepsilon = \frac{\delta}{L_o} = \frac{L_s - L_o}{L_o}$$
 and

 $L_{\rm s}$  is the "stretched" length of the specimen,  $L_{\rm o}$  is the original length of the specimen.

The ultimate strength of a material,  $\sigma_{ult}$ , is the maximum stress the material can sustain, and in artists' materials, it is usually the time at which the material breaks. The modulus, E, is the measure of a material's ability to deform when subjected to stress, and is:

(Equation 3) 
$$E = \frac{\sigma}{\varepsilon}$$

The modulus is defined normally for the elastic region of a stress–strain plot where the material returns to its original dimensions when the stress is removed. A material is said to exhibit *plastic behavior* if it remains permanently deformed after the removal of the stress. The yield point,  $\sigma_{yld}$ , is the stress where the material goes through a transition from elastic to plastic behavior.

# Oil Paints

Several paints were mechanically tested under long-term loading conditions. Sample preparation and test procedures are outlined elsewhere in this publication.<sup>12</sup> The tests were conducted by applying a small strain to the test sample, about .007, and allowing it to fully stress relax. Once full stress relaxation was attained, a subsequent increment of strain was applied and the stress was allowed to fully relax (Figure 7).

The time to relax fully the paints was typically between five and ten days, depending on the paint. This process was repeated until the specimen broke. The entire test took several months for each equilibrium test to be completed. Typically, three samples were tested to check for consistency of results. Figures 8 and 9 show the fully developed *equilibrium* stress-strain tests for Naples yellow and burnt sienna, both in linseed oil. Three samples were tested of these paints and the modulus, the initial slopes of the specimens, is consistent.

Of all the tested materials, Naples yellow showed the most scatter in the strain attained at breaking. All of the paints tested were cast in March 1978 and tested as free, unsupported films in 1989 and 1990. The modulus was determined by taking the slope of the locus of relaxed points from the equilibrium test (see Figure 7). In this case, for Naples yellow at 23 °C 50% RH, the modulus was 68.9 Mega-Pascals (MPa), (10,000 pounds per square inch [10 ksi]). This is only about one-tenth of the modulus of the same material when tested rapidly in the same environment (see Figure 10). The mechanical properties of the some of the paints tested in different environments are listed in Table 3.

#### Rabbit Skin Glue

Rabbit skin glue will not sustain a stress indefinitely. If a long enough period elapses, the glue will fully relax. The time for this complete relaxation to occur is at least 160 days for thin, .00059 cm (.0015 in.), specimens. If the glue is thicker, the relaxation time is considerably longer, at least 200 days for a .0043 cm (.011 in.) specimen. Its response time to environmental changes is considerably shorter and its mechanical properties can be measured in context of the time required to adjust to new environments. For the thickest glue measured, .0043 cm (.015 in.), it took about one-hundred hours to reach equilibrium with the new moisture content as measured by both stress development and swelling measurements. The thinnest glue sample, .00059 cm (.0015 in.), took only one hour to equilibrate. If ten days was used as the baseline to reach environmental equilibrium, then quasi-equilibrium mechanical properties can be determined for ten-day relaxation times for each loading increment. In Table 4, the ten-day equilibrium mechanical properties are shown for four different environments. The strengths reported in Table 4 are the approximate yield and not the maximum strengths. These values are used because the maximum glue stresses reached in a painting are based on their yield stresses. These strengths are much higher than those of the paints. These tests reveal that it is the glue layer in a sized painting that is responsible for most of the humidity-related damage.

# STRESS DEVELOPMENT IN RESTRAINED MATERIALS RESULTING FROM CHANGES IN TEMPERATURE AND RELATIVE HUMIDITY

If any of the artists' materials are restrained and subjected to decreases in either temperature or relative humidity, they will experience an increase in tension. This results from the material's inability to contract while undergoing a loss of heat or moisture. The *magnitude* of the tensile stress the material experiences is a result of both the attempted shrinking and the modulus of the material. The modulus in any of the materials is a function of both the temperature and relative humidity which must be considered in any calculation that attempts to predict the level of stress increase resulting from environmental changes.

# Temperature Effects

Generally, if the thermal coefficient of expansion and the equilibrium modulus of a material is known, the stress levels resulting in cooling a restrained material can be predicted. The method of calculating this stress increase can be derived from the basic Equation 3, repeated here:

(Equation 3) 
$$E = \frac{\sigma}{\varepsilon}$$

which can be rewritten as:

(Equation 4)  $\sigma = E \times \epsilon$ 

To include the temperature effects, it must be recognized that both the modulus, E, and the strain, ε, are functions of temperature and therefore:

(Equation 5) 
$$\sigma(T) = E(T) x \epsilon(T)$$
 where

T represents temperature.

The equilibrium modulus for Naples yellow was measured at 23 °C, 5% RH to be 327 MPa (47.5 ksi) and at -3 °C, 5% RH it was found to be 1,034 MPa (150 ksi). If it is assumed that E varies linearly with temperature (see Figure 11) then a linear function for E(T) can be fitted and stated as:

(Equation 6) E(T) = 955 - 26(T)

where the units are MPa for the modulus and temperature is in degrees Celsius. That the function is linear is not unreasonable, since it was shown that the thermal coefficient is linear over the temperature ranges considered here.

The strain, (T), as a function of temperature is derived by considering the shrinkage of the specimen as if it were free to do so. The specimen would contract upon cooling and it is effectively being "stretched" backed to its original restrained length. The "unstretched" length would be the free shrinkage length and the deformation would be the amount needed to stretch the contracted specimen to the restrained length. Using the thermal coefficient of expansion,  $\gamma$ =.000052, for the paint, the freely contracted length for the paint for any temperature is:

(Equation 7)  $L_T = L_R + \gamma x \Delta T x L_R$  or:  $L_T = L_R x (1 + \gamma x \Delta T)$  where:  $L_T$  is the free shrinkage length,  $L_R$  is the restrained length,  $\Delta T$  is the change in temperature the material experiences,

γ is the thermal coefficient of the material.

The strain at any temperature is now calculated as:

(Equation 8) 
$$\varepsilon(T) = \frac{(L_R - L_{T})}{L_T}$$

Equation 8 can also be written in terms of the thermal coefficient of expansion, temperature change, and the stretched length as:

(Equation 9) 
$$\varepsilon(T) = \frac{L_R - L_R \times (1 + \gamma \times \Delta T)}{L_R \times (1 + \gamma \times \Delta T)}$$

or as:

(Equation 10) 
$$\varepsilon(T) = -\frac{\gamma x \Delta T}{1 + \gamma x \Delta T}$$

The negative sign indicates that positive tensile strains are resulting from negative temperature changes, i.e. cooling. The stresses for a fully restrained specimen are now the product of Equations 6 and 10. From an experimental point-of-view, it is not possible to measure easily the stress of a *fully* restrained specimen as the *load cell*, the device

that measures the stresses, is compliant. This means it gives a little as force is applied to it. This load cell compliance effectively relieves some of the restraint on the specimen and the stresses measured are actually less than if fully restrained. This compliance can, however, be included in the calculations that allow us to predict the behavior of the specimen subjected to restrained temperature changes. The compliance of the test device is a function of the total force and is measured during the test of the specimen. The value of the compliance, measured in units of length, is the raw data output, AO, times a compliance constant, KD. It is now possible to correct for the compliance of the load cell and calculate the expected stresses in the experimental restrained test where:

(Equation 11) 
$$\sigma(T) = E(T) x [E(T) - \frac{AO x KD}{L(T)}]$$

This calculation was conducted for the Naples yellow using the measured modulus and the thermal coefficient of expansion of the material. The results of this calculation are presented in Figure 12 as the lower continuous line.

Also on Figure 12 is the actual test data (squares) for the Naples yellow. There is a substantial correlation between the predicted and the actual measured stress levels at the different temperatures while the relative humidity is held at 5%, and additionally, the nonlinear increase of the stresses with temperature in both the predicted and actual measurements. This experiment was conducted with the temperature both increasing and decreasing. Figure 12 also shows the data (crosses) for restrained rabbit skin glue. Since the thermal coefficient was measured, it is possible to derive the equilibrium modulus from the restrained test. At -3°C, the extrapolated value of the hide glue modulus is 4,860 MPa (705 ksi), which is only twelve percent less than the rapid loading modulus of 5,515 MPa (800 ksi). 14 This is the expected value since the difference between the rapid loading modulus and the equilibrium modulus for the artists' materials diminishes with a reduction in temperature and relative humidity. If, as before, the modulus is assumed to vary linearly with temperature, then a prediction for all of the temperature-related stresses is possible. This is shown on Figure 12 as the solid line overlaying the rabbit skin test data.

Using Equation 5, it is now possible to calculate the stress development in Naples yellow and rabbit skin glue if they were fully restrained. Figure 13 shows the expected stresses of these materials when fully restrained and chilled from 23 °C to -6 °C at 5% RH. The load cell compliance has a significant stress reduction effect since the fully restrained stresses are nearly double for the glue and about fifteen percent higher for the paint.

The immediate conclusion drawn from Figures 12 and 13 is that, while the coefficient of thermal expansion for hide glue is only one-half that of the Naples yellow paint, the hide glue still has a larger stress increase, not because of changes in dimension but because of a substantially higher modulus. It is important to recognize that both the modulus and the coefficient of expansion influence the magnitude of the resulting stresses in materials restrained and cooled.

The ambient temperature is not the only way in which paintings can be subjected to changes in temperature. Figure 14 shows a plot of the stress in a restrained specimen of Naples yellow that has been subjected to an acetone "cleaning treatment." The stress is plotted against the time of the solvent application. Within a short period of time, the stress in the paint film increases. There is a corresponding decrease in the paint film temperature from the evaporation of the acetone solvent and as the temperature returns to ambient, the stresses in the paint film slowly return to lower levels and eventually nearly to the original value. This cooling phenomenon also shows how rapidly temperature effects can influence the mechanical properties of a paint film. Other chemical and physical effects may be taking place as well. The removal of low molecular weight materials, which serve as plasticizers, may make the paint more brittle. Erhardt and Tsang have measured the loss of over one percent of the weight of a paint film as free palmitic acid after treatment with solvents. The loss of other free fatty acids, glycerides, and oligomers may total up to four to five percent of the weight of the film, which might be approximately fifteen percent of the weight of the medium.<sup>15</sup>

# Relative Humidity Effects

The effects of relative humidity on the artists' materials are analogous to those of temperature. Changes in relative humidity induce both dimensional and mechanical properties changes. In effect then, stress development in restrained materials subjected to desiccation can be treated in a manner similar to restrained materials subjected to decreases in temperature. The mathematics for relative humidity behavior would be:

(Equation 12)  $\sigma(RH) = E(RH) x \epsilon(RH)$  where:

(Equation 13) 
$$\varepsilon$$
 (RH) =  $-\frac{\alpha x \Delta RH}{1 + \alpha x \Delta RH}$  and  $\Delta$ 

RH is the change in relative humidity, with  $\alpha$  the *moisture* coefficient of expansion. This is an approximation as the change in dimensions with relative humidity is not linear. The simplest approximation would be to have two values of  $\alpha$ : one,  $\alpha$ , from 0% RH to 70% RH and another,  $\alpha$ <sub>b</sub>, from 70% RH to 90% RH. The values are shown as the slopes of the two straight lines in Figures 1 and 2. The change in the modulus, to a first approximation, could be handled as a linear function of relative humidity. Therefore, generally: (Equation 14) E(RH) = M(RH) + N

where: *M* is the slope of a linear *E* versus *RH* approximation and,

*N* is an offset constant which locates the *Y* intercept.

#### Oil Paints

Two paints, Naples yellow and flake white were restrained at 66% RH and 23°C, and desiccated to 58% RH where the stresses in the paints were .014 MPa (.002 ksi) and .028 MPa (.004 ksi), respectively. The paints were further desiccated to 5% RH and the stresses now reached were as high as .455 MPa (.066 ksi) and .634 MPa (.092 ksi), respectively (see

Figure 15). The time for equilibration was forty-eight hours for .0059 cm (.015 in.) thick films.

Using Equation 12 (corrected for load cell compliance), it was possible to calculate the expected stresses. The value,  $\alpha_{a=.0000257}$  was used for both the Naples yellow and the flake white. The equations (Eq. 14) for the modulus of the materials were:

for Naples yellow, E(RH) = 357.1 - 5.8(RH) and, for flake white, E(RH) = 440.0 - 5.8(RH) where the units are in MPa and percent relative humidity. These two equations have the same slope and differ only in the intercepts. The difference in the intercept values is the difference in the modulus of each material at 50% RH. The results of the calculations are shown in Figure 15 as the solid lines passing through the data points. The test data stresses are only about 15% less than that expected if the specimens were fully restrained.

One interesting aspect of these calculations is the remarkable accuracy using linear approximations of the actual material behavior. Equally interesting, for the Naples yellow, is the magnitude of stress reached, (.455 MPa [.066 ksi]), at a desiccation from 66% RH to 5% RH, considerably less than that reached (1.0 MPa [.145 ksi]), when cooled from 23°C to -3°C.

#### Rabbit Skin Glue

Until now, all of the discussions about the mechanical properties of the artists' materials and stress development in either cooled or desiccated materials have assumed that all behavior was elastic. This assumption seems to be valid because all of the elastic modulus equations result in successful stress predictions. For the rabbit skin glue, the behavior is almost never in the elastic range but exhibits a quasi-plastic behavior. The reason for this behavior is that the material wants to shrink extensively with desiccation. This results in strains, when the specimen is restrained and desiccated, that exceed the yield stress for any given relative humidity environment. Once these yield stresses are reached, the glue can have no further rise in stress no mater what strain level is attained. This presents

the complication that the elastic modulus is no longer valid, since any calculation using these values will result in stresses well in excess of the yield stresses.

What can be done is to determine an effective modulus,  $E_e(RH)$ , for all of the different relative humidity values. The effective modulus is:

(Equation 15)

 $E_e(RH) = 1,378.8 + 15.17 \times (75-RH.) + (15.17 \times RH)$  where:

RH<sub>i</sub> is the value of the starting or initial relative humidity,

RH is the relative humidity at any time.

If a moisture coefficient of  $\alpha_a$  = .000264 is used, which was determined from the data presented in Figure 3 including Equation 15 in Equation 12, the resulting restrained specimen desiccation stresses can be calculated for the rabbit skin glue. They are as presented in Figure 16, which is extremely close to the actual experimental data seen in Figure 17.

It is irrelevant that the load cell compliance is influencing the results, the stresses will become no higher for most of the testing since no additional stress is possible. Results that might be affected are the stresses resulting from the restrained desiccation starting at 30% RH, where it is possible that they are a result of elastic behavior.

The time for rabbit skin glue to reach equilibrium with the environment depends on the thickness of the sample. Three specimens, .028 cm (.011 in.), .013 cm (.005 in.), and .0038 cm (.0015 in.) thick were restrained at 60% RH, 23°C and the environment was rapidly desiccated to 5% RH. The increases in stress levels were measured against time as the relative humidity was quickly lowered to 5%. Figure 18 plots the results of the test for the first ten hours.

The thinnest specimen took only about sixty minutes to reach equilibrium at 27.9 MPa (4 ksi). The .013 cm (.005 in.) specimen took ten hours and the thickest specimen took about one-hundred hours to reach the maximum. All specimens achieved stress levels of nearly 27.9 MPa (4 ksi). After the specimens had reached their peak stress value, the hide glue films started to stress relax; Figure

19 shows the rate of relaxation of these films over several months. The thinnest hide glue film took over 150 days to relax to near zero stress. The thicker films took even longer but this may be a result of the longer time it took for it to reach its maximum stress. The data suggests that the actual relaxation rate is thickness independent.

An important point is that the whole process of stress increase caused by desiccation is much shorter than the time required to achieve stress relaxation. Stress resulting from restrained desiccation can linger for months. What was important was the glue's ability to reactivate itself if subjected to a high relative humidity level. On the 158th day, the relative humidity was raised to 90%, and maintained for forty-eight hours. The humidity was then lowered to 60% and the specimens that were sagging at 90% became tight. Finally, the relative humidity was again rapidly lowered to 5% and the dramatic stress rise reoccurred to nearly the same levels as before. The thinnest specimen glued itself to the test equipment at the very high relative humidity, and broke upon desiccation. For the two thicker specimens, stress relaxation proceeded at the same rates noted earlier.

In North America, extremely high relative humidity frequently occurs in the summertime and extremely low relative humidity occurs in heated, nonhumidified spaces in the wintertime. The magnitude of environmental changes discussed above can certainly occur as an annual event. It is difficult to envision such changes occurring on a daily basis, but the changes that occur might be sufficient to damage a painting. The stress levels reached in the hide glue tests are more than sufficient to damage a painting. At some time, however, the relative humidity must reach sufficiently high levels for the glue to reactivate. Otherwise it will stress relax to extremely low stress levels.

#### Fabric Canvas Supports

The mechanical properties of fabrics used as painting supports are effectively nonexistent. While it is possible to measure the modulus of a textile at different relative humidities, permanent creep caused by interfiber slippage at high humidity releases any ability to sustain a stress. In addition, when measured, the modulus of the textiles drops precipitously with desiccation. This is exactly the time when a stiff support is needed to prevent the in-plane displacements developed in a stretched painting at low relative humidities.

The only time a fabric develops a significant modulus is at an extremely high humidity, when all of the other materials of a painting become gellike as with the glue layer or extremely flexible as with the paints. The lack of fabric stiffness plays a significant role in the paintings' response to low relative humidities (see *Computer Analysis*).

## COMPUTER ANALYSIS OF PAINTINGS SUBJECTED TO CHANGES IN TEMPERATURE AND RELATIVE HUMIDITY

The experimental determination of the effects of cooling and desiccation on the mechanical behavior of an actual test painting is very limited. Normally, when a complex structure is experimentally evaluated during the application of forces, stress levels are determined by using strain gauges attached to the structure. 17 These gauges measure the strains applied to the structure while loaded and the stresses are calculated using the strain measurements and the modulus of the materials used in making the structure. A restrained structure, subjected to cooling or desiccation will develop either extremely small, or no strains, at the same time experiencing very high stresses. Here, strain gauges are of little value, as is any system that attempts to extrapolate an accurate assessment of the stress distribution of a painting by measuring the strains. Other researchers 18, have succeeded in showing that changes in temperature and relative humidity change the magnitude of the average forces as measured at the edges of a biaxially restrained painting. This is not, however, the same as determining the distribution of stresses occurring throughout the painting. What stresses develop at the edges

of a painting are not necessarily the same as those found in the center, or even a slight distance from the edges. Stresses can even vary along the edges. It is these variations in stresses that determine if a painting will crack at all during desiccation or cooling, and if it does, these variations determine the crack patterns that result.

It is not possible to use actual paintings for experiments, since this is a destructive test method. The evaluation of the overall behavior of painting is best approached by a thorough understanding of the mechanical properties of the artists' materials and a determinate analytical procedure. One of the best procedures to use is Finite Element Analysis (FEA) and the digital computer. 19 This method mathematically approximates the structure by assembling the structure from smaller, geometrically simple "elements" whose mechanical properties can be determined easily. (Consult Mecklenburg and Tumosa<sup>20</sup> for further information on this method and an example of its accuracy.) The program used in all of the computer modeling in this paper was ANSYS®, Version 4.4, run on a 386, 33 megahertz desktop computer with an expanded RAM of 4 megabytes.

# Trial Test of Computer Modeling

To assess the accuracy of the computer using the equations that relate the stress of a material to changes in temperature and relative humidity, the coefficients of expansion, and the modulus, a simple trial was conducted. In this test, a model of a sample of the Naples yellow paint specimen was numerically constructed and fully restrained. Equations 6 and 10 were programmed into the computer using .000052 per degrees Celsius as the thermal coefficient of expansion for the Naples yellow. The model was then mathematically subjected to changes in temperature from 23°C to: 18°, 10°, 5°, 0°, and -5°C. The results, which compare favorably, are shown in Figure 20, where stress versus temperature is plotted for the computer model results and the stress values calculated from the actual paint test data using Equation 5. This test verified that the equations were being programmed correctly as well as the computer model's ability to accurately analyze a material subjected to temperature changes that influence the modulus as well as the strains.

# Modeling the Effects of Cooling a 76 x 102 cm Painting

A 76 x 102 cm (30 x 40 in.) painting was modeled and subjected to changes in temperature while the ambient relative humidity was held at 5%. The layer of the painting consisted of a linen .064 cm (.025 in.) thick, modeled as a .0152 cm (.006 in.) layer<sup>21</sup>, a glue layer .00508 cm (.002 in.) thick and a Naples yellow (lead) oil paint layer .0076 cm (.003 in.) thick. The stretcher was not expanded. Each of the layers of the model painting were programmed with their respective material properties taken from the data tables presented earlier. For example, the paint layer was programmed as described in the computer trial section above. The glue layer was characterized using Equation 10 with a thermal coefficient of .000025 per degrees Celsius and a changing modulus that varied linearly from 4,860 MPa (705 ksi) at -3°C to 3,791 MPa (550 ksi) at 23°C. This data was used to develop the equation:

(Equation 16)  $E(T) = 4,756 - 51 \times T$  where: T is any temperature.

Equation 16 was the equation that allowed programming of the change of the rabbit skin glue's modulus with changing temperature at 5% RH. The fabric was programmed with an average constant modulus of 38.6 MPa (5 ksi) and a thermal coefficient of .00001 per degrees Celsius. The distribution of the principal stresses in the paint layer, calculated by the computer is shown in Figure 21. This stress distribution is uniform, ranging from 2.09 MPa to 2.11 MPa (.303 ksi to .307 ksi) and just reaches the measured breaking strength of Naples yellow paint at 5% RH and -2°C (see Table 3). This suggests that cracking of the paint layer will occur throughout the entire surface of the model painting when the temperature is decreased from 23°C to -3°C at 5% RH. It most certainly would have occurred if the starting environment was 23°C and 50% RH. This would be the consequence of the combined adverse effects of both desiccation and cooling.

The stresses have a directional bias as shown in Figure 22. In this figure, the principal stresses are indicated by the calculated directional vectors (arrows), and cracking that occurs will do so perpendicular to the vectors as shown by the continuous lines. This is a crack pattern that appears frequently in paintings in North America, particularly those that are on strainers, which are stretchers with fixed, nonexpandable corners. If the paint modeled was the flake white tested and reported in this paper, it would have also failed, but at a higher temperature, since this paint has a higher modulus but consistently less strength than the Naples yellow. The glue layer principal stresses reached levels ranging from 4.72 MPa to 5.13 MPa (.685 ksi to .745 ksi).

The model painting was again analyzed but with a much stiffer fabric (689.4 MPa [100 ksi]) such as a synthetic lining attached to the reverse of this model painting; the results show little difference. The calculated principal stresses for the paint film still reach 2.11 MPa (.306 ksi). This means that the individual layers of a painting are independently responding to the temperature change without any interlayer interference. As will be seen later, this is in marked contrast to the influences of linings on paintings subjected to deep desiccation. This temperature analysis has even greater implications. Paint, attached to any material that has little dimensional response to temperature, such as the parallelto-grain direction of wood, is capable of developing stresses sufficiently high enough to cause cracking when the temperature drops sufficiently. In the case of panel paintings, cracks will tend to form perpendicular to the grain of the wood. Conversely, if the painting is attached to a continuous support that has a thermal coefficient similar to the paint film, the problem of cooling is diminished significantly since the entire system is allowed to contract with cooling.

The question of why the stresses in the computer modeled paint layer achieve considerably higher levels than the fully restrained simple paint sample must be addressed. The paint layer in a painting is restrained in two mutually perpendicular directions, each developing considerable strains when cooled. The single paint specimen is restrained only in the test direction. It is readily shown that the stresses in one direction are influenced by the strains occurring in all directions.<sup>22</sup> For example, if the strains in three dimensions are nonzero, as with a painting, the stress in one dimension, say, *x*-direction is computed using: (Equation 17)

 $\sigma_{x} = \frac{E}{(1+v)x (1-2v)} x \left[ (1-2v)x \epsilon_{x} + vx (\epsilon_{x} + \epsilon_{y} + \epsilon_{z}) \right]$ where v is Poisson's ratio.

As with the simple paint samples, the stress in the painting does not increase linearly with decreasing temperature. If the painting is heated above the 23 °C starting point and the relative humidity is held constant, the painting will go slack and sag. The specific temperature depression that represents a danger to actual old paintings is unknown to date since the modulus and strength of the older materials is yet undetermined. It might be prudent to avoid temperatures below 10 °C if the relative humidity becomes low simultaneously.

# Modeling the Effects of Desiccating a 76 x 102 cm Painting

While seemingly similar to the effects of cooling, reductions in ambient relative humidity have somewhat different effects and they seem to be more readily controlled than the temperature effects. In starting the computer simulation of the effects of desiccation, the same 76 x 102 cm (30 x 40 in.) painting described in Effects of Cooling was used. In this case, the computer was programmed to alter the relative humidity instead of the temperature. Additionally, to obtain greater analytical detail, it was possible to take advantage of the painting's double symmetry and to model only the upper right-hand quadrant. This was done by programming the appropriate material properties—RH equations for each of the individual layers of the model painting as well as the proper boundary conditions. Again, in this case the paint layer

again was Naples yellow. For the changes in relative humidity, E(RH)=357.1-5.8(RH) was the equation used. For the strain calculations, Equation 13 was used with a moisture coefficient of .0000257 per percent relative humidity. For the glue layer, the modulus was computed using Equation 15 and for the strains, using Equation 13 with a moisture coefficient of .000264 per percent relative humidity. The fabric was assumed to be a minimally contributing material since glue sizing a stretched linen removes any residual stress in the fabric. In this case, the modulus was assigned a nominal constant 35 MPa (5 ksi) and the moisture coefficient was an average .0001 per percent relative humidity.

The model painting was desiccated from 70% to 10% RH at 23°C. The maximum calculated principal stress in the paint film was only .806 MPa (.117 ksi). This is insufficient stress to break the Naples yellow, which has a breaking strength of 2.5 MPa (.36 ksi) at 5% RH and 23°C, or perhaps even the flake white which is weaker. If changes in relative humidity are going to crack an unbroken paint film, other factors must be considered. As a paint dries longer, it get stiffer (a higher modulus), but in all probability it will also get stronger as all of the testing to date shows that the factors that stiffen a paint also increase its strength. Additionally, moisture coefficients of the paints seem to decrease with an increase in the modulus, making them less dimensionally responsive to changes in relative humidity. In this initial computer analysis it was noted that the calculated glue layer stresses exceeded 34.5 MPa (5.0 ksi). Perhaps the thickness of the glue layer is a factor to consider. In many poorly prepared, commercially available canvases, the linen is of poor, lightweight quality, and the glue size is applied freely to give the canvas body. Additionally, a painting lined by using traditional linen canvas and a hide glue adhesive increases the total glue thickness without significantly increasing the support from the canvas. Both of these cases can be simulated on the computer by increasing the thickness of the glue layer and without changing any other parameters. The model painting was modified to increase the glue thickness from

.0051 cm to .01 cm (.002 in. to .004 in.). The results of the new analysis are dramatic. The stresses in the paint layer increases to a new range of .698 MPa to 1.93 MPa (.1 ksi to .28 ksi). Doubling the glue layer more than doubled the stresses in the paint layer. The distribution of the stresses is shown in Figure 23. This distribution is not nearly as uniform as demonstrated from the cooling analysis. Cracking, if it occurs, will initiate mainly in the corner regions of the painting (Figure 24). In this figure, both the direction vectors of the principal stresses and the possible crack pattern are shown. If the equilibrium breaking strength of the paint at 5% RH and 23°C was low, e.g. only 1.38 MPa (.2 ksi), the cracks would go no farther than about 7.6 cm (3 in.) from the corner. It seems, therefore, that low temperature rather than low relative humidity has the greater potential for crack-

ing an undamaged painting.

Changing the thickness of the glue layer influenced the stresses in the paint layer when desiccating the painting. This indicates that the very high glue stresses cause an interlayer interaction. If this is true, then a stiff fabric material should reduce this effect of layer interaction. The model painting with the thick glue was again analyzed using a modulus of 689 MPa (100 ksi) for the fabric. The maximum principal stress in the paint layer was reduced to 1.44 MPa (.209 ksi) when the relative humidity was reduced from 70% to 10%. This shows that high glue stresses, (in all cases examined so far, it exceeded 34.5 MPa [5 ksi]), are capable of causing in-plane deformations in the painting and stiff supports are capable of reducing them. Whether these cracks will continue to grow once started is unknown, but in other brittle materials there is substantial evidence that stresses needed to cause further crack growth are considerably less than those needed to start the initial cracks.<sup>23</sup> In ductile materials, the stresses needed to increase crack length are no different from starting a new crack. As the paints get older and more brittle, perhaps this alteration of the mechanical properties is most manifest in crack extension and not new crack formation.

To date, it has been shown that traditional

glue linings can reduce the extremes in environments in which paintings might survive and stiff linings can expand that environment. See Figure 25 for a summary of these effects.

# Effects of Stretcher Expansion on a Painting Subjected to Changes in Relative Humidity

There are other factors that influence the paintings' response to relative humidity. Tightening the painting by expanding the stretcher is one of them. Traditionally, stretchers have been expanded when the paintings they are supporting have gone slack because of considerations such as increased relative humidity. Expansion has either been accomplished by driving in corner keys (wedges), or turning expansion bolts at the corners. In some situations, springs attached at the corners have been compressed to increase the tension in the painting. In all cases, the stretcher bars are displaced outward from the plane of the painting (Figure 26). In this figure, the dashed lines represent the original configuration of the painting and the solid lines show the newly displaced geometry. If the stretcher bars are each displaced outward by .1 cm (.04 in.) on the same  $76 \times 102 \text{ cm}$  (30 x 40 in.) painting used in the cooling analysis, and the painting is desiccated from 70% to 10% RH, the results are dramatic. This bar expansion is the same as expanding the corner gap at the key location only .142 cm (.056 in.). The result of the computer analysis (Figure 27) shows where the principal stresses reach a magnitude of 10.4 MPa (1.508 ksi). The paint film will fail as the strength of the Naples yellow is only 2.5 MPa (.36 ksi). Figure 28 shows the directions of the principal stresses and the expected crack pattern, which is different from the previous computer analysis, but nevertheless recognizable on actual paintings. Interestingly, any expansion of the stretcher so modifies the principal stress distribution due to desiccation that an entirely different set of cracks appear. A summary of the results of computer analysis at different degrees of stretcher expansion and different levels of desiccation appears in Figure 29. This figure suggests that even minimal stretching reduces the environmental changes that the painting can safely tolerate.

# Relative Humidity Effects on a Previously Cracked Painting

Until now, this study has concentrated on the environmental factors that contribute to the creation of new cracks in a painting. It is, however, important to examine the effects of changes in relative humidity on a painting with preexisting cracks. The model assumes that the crack runs from the surface of the paint layer to the interface of glue size and the fabric. The distance from crack to crack is .508 cm (.20 in.). The ground and paint layer are both lead-based oil paint (Naples yellow) .0076 cm (.003 in.) thick; the glue layer is .00508 cm (.002 in.) thick, and again, the fabric is .0635 cm (.025 in.) thick, modeled as .015 cm (.006 in.) thick. The strain and modulus equations used in programming the computer are the same as described in Modeling the Effects of Desiccation on a Painting. The section of the painting modeled is illustrated in Figure 30. The model was subjected to changes in relative humidity, starting from 70% and desiccating to 50%, 30%, and 10%. For a test basis, it is assumed that the glue layer responds as if it had been recently exposed to a very high relative humidity. Figure 31 shows the deformation (cupping) that occurs with desiccation from 70% to 10% RH. This is a result of the partial contraction occurring in the paint and glue layers. The principal stresses resulting from this desiccation are still substantial as shown in Figure 32, an enlarged view of the cracked part of the painting with stresses concentrating at the glue fabric interface. Figure 32 also shows the stresses as the paint glue interface reach levels of between 9.65 and 13.8 MPa (1.4 and 2.0 ksi). These calculated stresses in the paint film are over four times the breaking strength of the Naples yellow at this environment, indicating the onset of severe active flaking of the painting. The presence of a crack represents the source of stress concentrations and this model illustrates the effect of a crack. The stresses in the paint layer of the uncracked models never reached the levels observed here. Even if this model of the cracked painting was desiccated to only 50% RH, the stresses in the paint would reach 2.96 MPa (.43 ksi), which is still sufficient to cause flaking. Paintings don't always flake with variations in relative humidity of only 20% (70% to 50%, or 50% to 30%). If the fabric is thin or flexible, this can reduce restraint on the paint chip and thereby reduce stress development. Once a glue layer has been subjected to severe desiccation and stress relaxed, it goes slack upon returning to its starting environment. This residual slackness potentially reduces the total stress resulting from daily oscillations in relative humidity. Additionally, there appears to be a long-range stress reducing effect, still undetermined, that diminishes the glue's response to relative humidity. Hedley showed that high relative humidity reactivation of the stress-relaxed glue will at least triple its stress response to relative humidity when restrained in a painting over one-hundred years old.24 However, for the safety of the painting, it should be assumed that it has been recently subjected to high relative humidity due either to weather, a recent glue lining treatment, or as an attempt to reduce the cupping distortion in the painting's design layer. Depressions in relative humidity of 20% or more will probably cause incipient flaking in a substantial number of paintings.

# PAINTINGS ON WOOD PANEL SUPPORTS

The present discussion is limited to introducing some of the important considerations that relate relative humidity to wood response. There are two primary considerations when evaluating a wood panel's ability to withstand changes in relative humidity. The first is to determine the degree of a panel's constraint from dimensional response to relative humidity. If a panel is free to contract upon desiccation, stresses resulting from changes in relative humidity are at a minimum, if they exist at all. If a panel is fully restrained from motion, then relative humidity-related

stresses are maximized and can be destructive. A panel painting rigidly fixed in a frame, must be considered restrained. If there is a cradle attached to the reverse of the panel, even though this device was intended to allow in-plane motion, it is almost certain that panel warpage has locked up the cradle and motion is no longer possible. Battens, glued cross-grain to the panel, also act as a restraint to dimensional response. The second consideration, and possibly the more problematic, is the presence of cracks in the panel. There is difficulty because both the location and the length of the crack influence the panel's ability to safely accommodate changes in relative humidity and it is not always easy to determine either the existence or the true length of a crack.25

An analysis of a computer model of an oak panel, 76 x 102 x 1.27 cm (30 x 40 x .5 in.) was conducted to examine the potential for damage resulting from reductions in relative humidity. In this case, the panel was restrained at the upper and lower edges and the initial relative humidity of 50% as well as 70% was used in as in previous models. The parallelto-grain modulus was assumed to be a constant 6,204 MPa (900 ksi), the radial and tangential moduli were both assumed to be 689 MPa (100 ksi). The moisture coefficients used were .00002 parallel to the grain, .000219 in the tangential direction, and .000133 in the radial direction, which in this case is the restrained direction. The breaking strength of the oak was assumed to be 4.14 MPa (.6 ksi) in the radial direction though the actual strength of the wood can vary somewhat according to its density. The computer model and wood grain orientation with the representative panel painting are shown in Figure 33. Taking advantage of double symmetry, it was necessary to model only the upper righthand quadrant of the panel.

## Uncracked, Restrained 76 x 102 x 1.27 cm Oak Panel

The first series of analyses were conducted on a restrained panel where the changes in relative humidity were induced with the initial values at 50% and 70% RH. Figure 34 shows these results where the stresses rise in tension with desiccation from the starting points and rise in compression with increases in relative humidity. These analyses indicate that an average uncracked or flaw-free oak panel might withstand desiccation of 50% to 10% RH, but not from 70% to 10% RH if the breaking strength of the wood is 4.14 MPa (.6 ksi). If the panel had been restrained initially at 70%, a change to approximately 25% RH would have broken the panel. At higher levels of relative humidity, the wood actually becomes softer and crushing or *compression set* will occur. At the higher humidities, it is even possible to buckle thin panels.

Wood properties are not uniform even from trees of the same species, or even consistent throughout a single piece of wood, therefore, this analysis must be considered an average baseline from which other considerations such as the presence of a crack can be compared.

#### Cracked, Restrained 76 x 102 x 1.27 cm Oak Panel

Cracks can form anywhere in a panel, but it is probable that they will occur at defects in the wood or at the edges. Computer analyses were conducted on model oak panels with cracks occurring along the center line of the panel and originating at either the center or the edges of the panel. Figures 35 and 36 show the computer-generated principal stress distributions of cracked panels desiccated from 50% to 40% RH and the severe stress concentrations resulting from these cracks. Figure 35 also shows the results with a 20.3 cm (8 in.) center crack and Figure 36 shows the results with a 10.16 cm (4 in.) edge crack. For the desiccation from 50% to 40% RH, the edge crack has a maximum principal stress of 2.45 MPa (.358 ksi), or .7 MPa (.102 ksi) greater than the center crack, which is twice as long. This information, along with the summary results shown in Figure 37, indicate the substantial increase in potential hazard an edge crack presents over a center crack. More importantly, this figure shows the sharp increase in crack zone stresses resulting from the increasing crack length. This explains why a crack, once started, can run unchecked through the entire panel with no additional change in the environment. Further analysis showed that for desiccation from 50% to 30% RH, a 7.62 cm (3 in.) edge crack would grow, since stresses reached 5.25 MPa (.76 ksi). A desiccation from 50% to 20% RH would cause a 5.08 cm (2 in.) edge crack to grow as there was a stress rise to 5.03 MPa (.73 ksi), and for a change from 50% to 10% RH, a 2.54 cm (1 in.) crack would grow with a stress rise of 6.27 MPa (.91 ksi). None of the stresses discussed would have developed if the panel had been left unrestrained.

#### **SUMMARY**

To assess the risks to paintings associated with changes in temperature, relative humidity, shock, and vibration, an accurate determination of the stresses in the different layers of a painting is necessary. In turn, those stresses must be compared to the strengths of the artists' materials at the specific environments in question and rates of loading encountered by the painting. Computer modeling in the form of finite element analysis readily lends itself to performing the needed analysis. However, the ability to numerically model multiple-layered structures such as paintings largely depends on accurately determining the mechanical properties of the painting's constitutive materials. This includes assessing alterations to the mechanical properties brought about by time. This study has examined some of the materials typically found in a large proportion of canvas and panel paintings and concentrated on paints such as the lead-based oils that demonstrate brittleness. Additionally, the study confines the analysis to effects of temperature and relative humidity brought about over moderately long periods of time. In all of the analytical modeling, it was assumed that all of the materials were at equilibrium with the environment.

All of the materials respond dimensionally to the changes in temperature and relative humidity. The thermal coefficients of expansion for hide glue and lead paint were small, while their moisture coefficients were considerably higher. Restraining these materials caused stress development with desiccation and cooling, but the magnitude of the stress was a result of both the material's attempt to contract and its related modulus or stiffness.

Computer modeling of a typical canvas painting with glue size and lead-based oil paint layers showed that substantial drops in temperature at low relative humidity (from 23° to -3°C, at 5% RH) resulted in both uniform and high stresses in the paint layer. These stresses equalled or exceeded the measured breaking strengths of thirteen year-old paint. It could be inferred that an actual painting, constructed with the materials modeled, would exhibit severe and extensive cracking over the entire surface of the painting. The same computer analysis also showed that the glue layer reaches stresses slightly more than twice those of the paint layer. Additionally, subsequent modeling revealed that attaching a stiff lining material to reinforce the canvas support would do little to prevent the formation of new cracks in the design layer. It can be concluded that severe cooling results in the independent response of the various layers of the painting with little interlayer interaction. Paint and glue tested at -3°C and 5% RH, showed extremely brittle behavior as they shattered into multiple pieces when broken.

The same painting, when subjected to depressions in relative humidity (from 70% to 10% RH), showed on the computer model similar crack patterns as those formed during cooling; these patterns, however, were not as extensive. In fact, it was not until an additional layer of glue was added to the model that stresses in the paint layer were high enough to prompt cracking. Even then, the analysis suggested that it would cause considerably less damage than it incurred during cooling. Further analysis showed that the traditional lining technique of using linen and hide glue dramatically increased the potential for damage to paintings subjected to desiccation. Conversely, stiff lining supports considerably increased the painting's ability to withstand desiccation. This occurs because of the remarkable stress levels achieved by the glue during periods of low relative humidity and the almost total lack of canvas stiffness in the same environment. Unlike the cooling analysis, there appears to be considerable interlayer interaction in a painting during desiccation. Some glue layer contraction reduces the stresses in the paint layer in the central region of the painting while increasing the paint film stresses in the corner regions. Hence, almost no cracking occurs in the central region of the painting, most of the cracking is evident at the corners. This indicates that stiff lining supports attached with adhesives that are not responsive to relative humidity will reduce the formation of new cracks caused by severe fluctuations in relative humidity.

The expansion of paintings by any corner device, even seemingly small amounts, renders a painting more susceptible to damage due to either cooling or desiccation. This type of deformation of the painting expands its layers, including the grounds and paint layers. The stress distribution can result in cracks radiating from the corners in a pattern that is entirely different from those caused by cooling and desiccation alone. The advantage of a stiff lining material is to allow the painting to develop tension with the minimum of expansion, though this type of support does nothing to stop expansion-related stresses in the paint layers. It only minimizes them.

Paintings with existing cracks appear to be the governing case regarding restricting the allowable fluctuation in relative humidity. Computer modeling of a section of a painting containing preexisting cracks revealed intense stress concentrations in the crack tip region. With changes in relative humidity from 70% to 50% at 23°C, the resulting stresses in the paint-glue interface easily exceeded the paint's ultimate strength. This would be also true of drops from 50% to 30% RH. This analysis suggests that for paintings with an active glue layer (recently exposed to high relative humidity), 20% depressions in relative humidity can conceivably initiate active flaking of the paint layer. The analysis also suggests that the region of stress concentration is so localized and confined to the area of the crack, that nearly all lining techniques and materials will do little to retard this effect with the exception of those adhesives that actually retard moisture penetration to the area of the crack. For transportation risk assessment, traditional canvas paintings, with preexisting cracks should be protected against temperature and relative humidity fluctuations.

Relative humidity fluctuations present a very real hazard for paintings on wood panels with preexisting cracks if the panel is restrained in any way. While the computer analysis was confined to a model oak panel, it may be inferred that the presence of cracks in restrained panels of any wood presents a potential for relative humidity damage. Re-

straint can take the form of improper attachment in the frame, cradling, or cross-grain battens attached to the reverse of the panel. Panels free to dimensionally respond to changes in relative humidity will experience virtually no stress rise at cracks. Nevertheless, the lack of material uniformity of wood strongly suggests that dimensional response in a wood panel will vary in different areas of the same panel, resulting in localized stress development with desiccation. All panel paintings should be maintained in a very narrow relative humidity environment.

	PPENDIX A — TABLES	$\boldsymbol{A}$	
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TABLE 1 Relative humidity-related swelling of different 4.5 year-old paints, 20°-22°C (68°-72°F).

Paint Oil	% Length Change 0%-70% RH	% Length Change 70%-95% RH	% Length Change 0%-95% RH
Flake White Safflower	.18	.62	.80
White Lead Linseed	.23	.38	.61
Burnt Umber Linseed	.72	1.83	2.55
Burnt Umber Safflower	1.20	2.80	4.0
Alizerin Crimson Linseed	.59	.99	1.58
Cadmium Yellow Linseed	.65	2.75	3.4
Cerulean Linseed	.32	.4	.72
Prussian Blue Linseed	.53	1.07	1.6
Raw Sienna Linseed	.7	1.16	1.86
Titanium White Linseed	.2	.32	.52
Ultramarine Linseed	.65	2.50	3.15
Vermilion Safflower	.46	.56	1.02

TABLE 2 Dimensional response of gesso to relative humidity, 23°C (73°F).

Gesso Chalk- to-Glue Ratio by Weight	% Length Change 10% to 80% RH	% Length Change 80% to 95% RH	% Length Change 10% to 95% RH
3.15	.75	1.0	1.75
10.0	.25	.65	.90

TABLE 3 The mean "equilibrium" mechanical properties of oil paints tested at different environments.

Paint Oil	E MPa (KSI)	Max σ MPa (KSI)	Max ε	Test Temp, °C % RH
Naples Yellow	1034	2.1	*	-3
Linseed Oil	(150)	(.3)		5
Naples Yellow	327	2.5	.018	23
Linseed Oil	(47.5)	(.36)		5
Naples Yellow	68.9	1.2	.025	23
Linseed Oil	(10)	(.17)		50
Vermilion	51.7	.8	.025	23
Safflower Oil	(7.5)	(.115)		50
Flake White	155.8	.8	.008	23
Safflower Oil	(22.6)	(.115)		50
Burnt Sienna	110	.65	.03	23
Linseed Oil	(16.0)	(.095)		5
Burnt Sienna	25	.6	.05	23
Linseed Oil	(3.6)	(.087)		50
Burnt Umber	10	.5	.005	23
Linseed Oil	(1.45)	(.073)		5
Burnt Umber Linseed Oil	0	**	0	23 50

<sup>\*</sup> Test not completed \*\* Stress relaxed to zero

TABLE 4 Quasi-equilibrium mechanical properties of rabbit skin glue.

Relative Humidity 27°C	E MPa (KSI)	Yield Strength MPa (KSI)
5%	3792 (550)	27.6 (4.0)
30%	3192 (463)	19.3 (2.8)
50%	1551 (225)	12.4 (1.8)
75%	0 * (0)	0 (0)

<sup>\*</sup> Fully stress relaxed



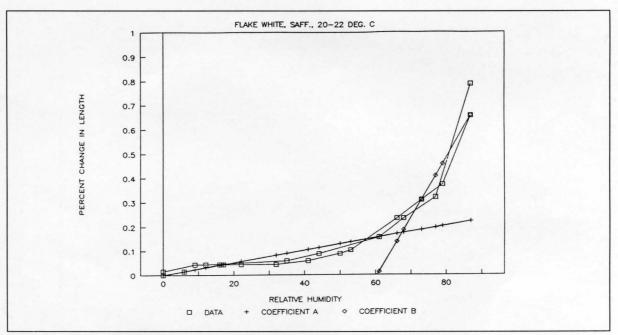


FIGURE 1 Percent length change of flake white (lead carbonate) ground in safflower oil versus the percent relative humidity. This paint shows relatively low dimensional response to relative humidity compared to burnt umber in linseed oil. The straight lines are the approximations used to determine the moisture coefficients of expansion  $\alpha_a$  and  $\alpha_b$ .

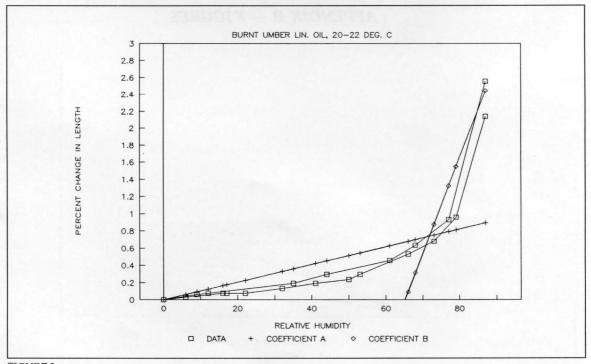


FIGURE 2 Percent length change of burnt umber ground in linseed oil versus the percent relative humidity. This paint shows relatively high dimensional response to relative humidity compared to flake white in safflower oil. The straight lines are the approximations used to determine the moisture coefficients of expansion,  $\alpha_a$  and  $\alpha_b$ .

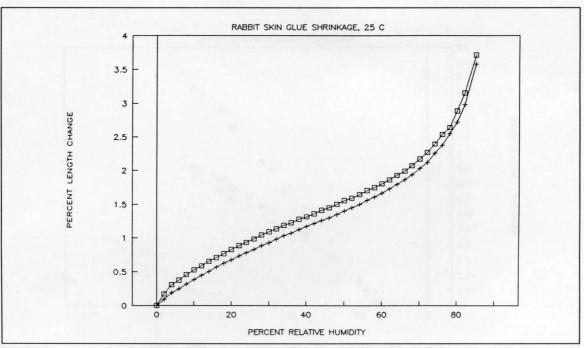


FIGURE 3
Percent length change of two samples of rabbit skin glue versus percent relative humidity during desiccation. Glue's dimensional response to relative humidity is greater than any of the other artists' materials.

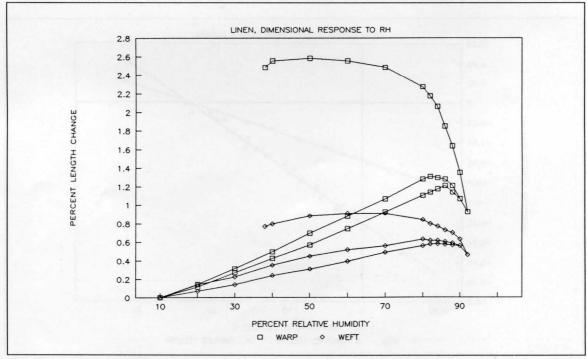


FIGURE 4
Percent length change of samples of a linen textile in the warp and weft directions. After an initial contraction from about 50% RH, the samples settle into a repeatable cycle as shown in the lower portions of the plots. Starting from approximately 80% RH, shrinkage occurs from either decreasing or increasing the relative humidity.

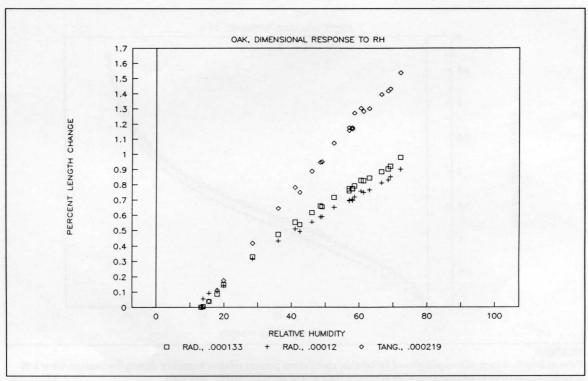


FIGURE 5
The dimensional response of three oak samples to increasing relative humidity. The samples are over one-hundred years old. The two lower plots are samples measured in the radial direction, the upper plot is in the tangential. The numbers next to the legends are the moisture coefficients of expansion for these samples, in strain per percent relative humidity.

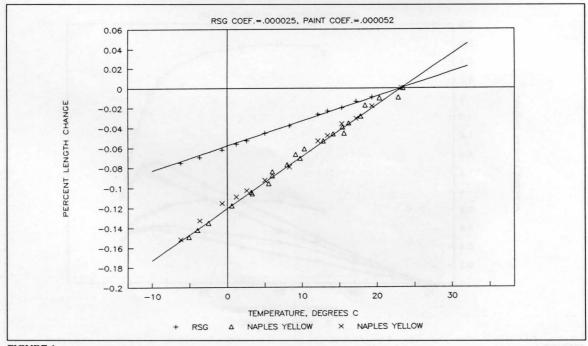


FIGURE 6
The dimensional response of rabbit skin glue and Naples yellow oil paint to changes in temperature. The thermal coefficients of expansion listed at the top of the figure are in strain per degree Celsius.

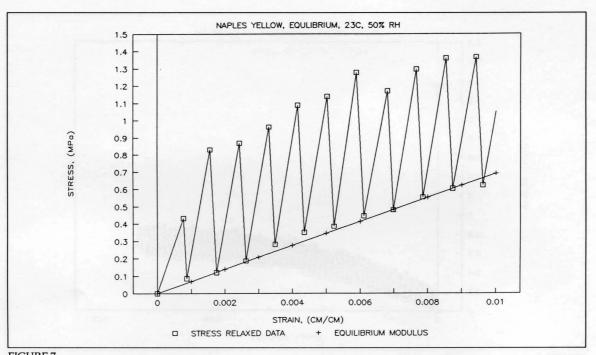
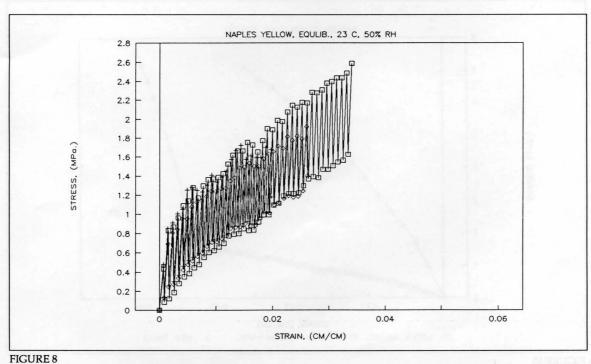


FIGURE 7
The initial portion of the equilibrium stress-strain plot for Naples yellow at 50% RH and 23° C showing the line used to determine the equilibrium modulus, E. This line is the locus of stress relaxed points measured from the long-term test.



The equilibrium stress-strain plots for three separate samples of Naples yellow paint at 50% RH and 23°C. The slopes of the specimens were nearly identical while the breaking strengths were scattered. This paint showed the largest amount of scatter in the materials test program.

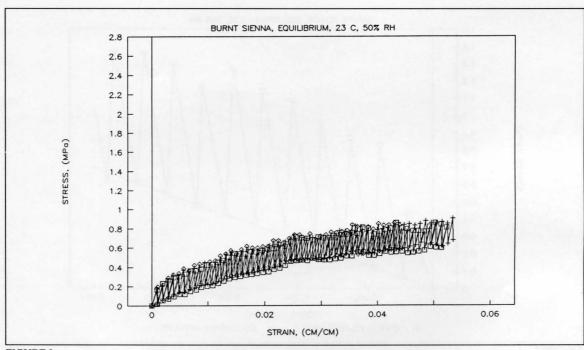


FIGURE 9
The equilibrium stress-strain plots for three separate samples of burnt sienna paint at 50% RH and 23°C. The slopes and breaking strengths of the specimens showed considerable consistency. This paint, while reaching a lower strength than the lead-based paints, showed considerable elongation to failure.

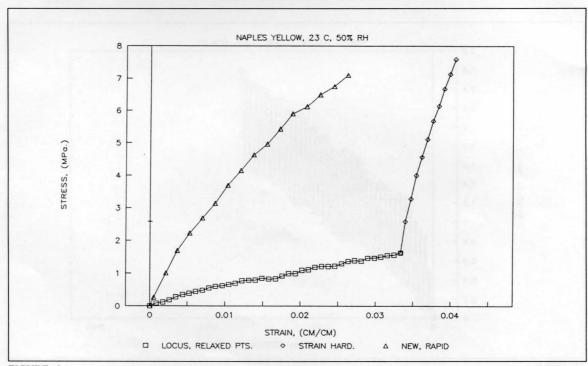


FIGURE 10
The stress-strain plots for Naples yellow paint tested at different rates of straining. The steep plots had strain rates of .001348 per second, while the lower plot reflects months of testing. The difference between the rapid loading tests reflects the strain hardening that long-term testing imparts to the material.

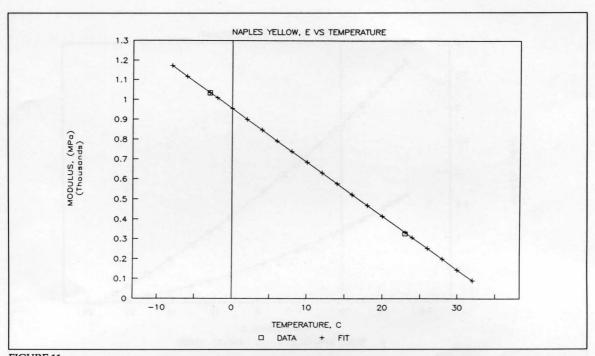


FIGURE 11
The change in the equilibrium modulus, E, of Naples yellow paint with change in temperature. The line labeled fit was used to determine Equation 6 in the text.

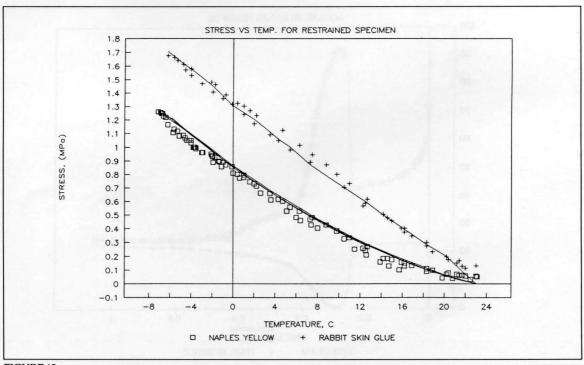


FIGURE 12
Stress versus temperature for restrained samples of Naples yellow paint and rabbit skin glue. This figure shows the data (symbols) and the predicted stress developed (lines) for these materials using Equation 11 which includes the partial stress release due to the load cell compliance.

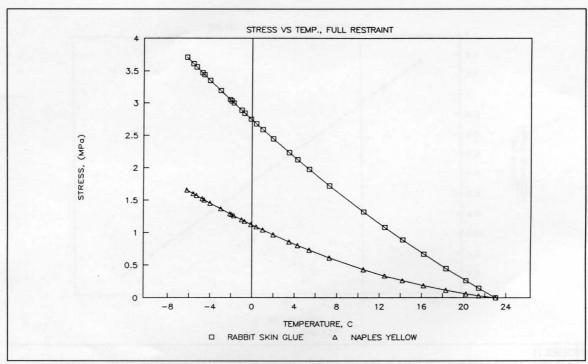


FIGURE 13
The stress versus temperature plots for fully restrained Naples yellow and rabbit skin glue using Equation 5.

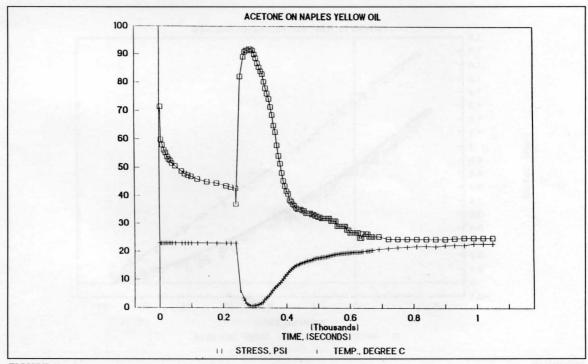
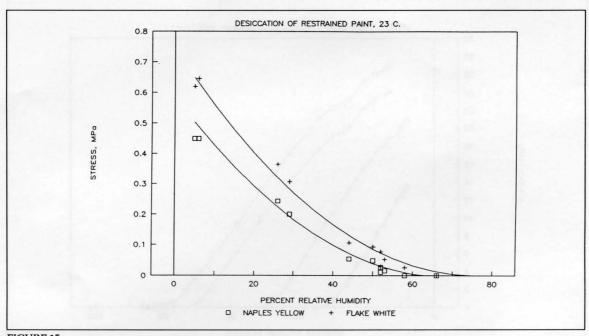


FIGURE 14 The effect of solvent evaporative cooling on a restrained samples of Naples yellow paint. The units of stress, plotted in pounds per square inch (psi) were used to simplify the graphics. For conversion, 1 ksi (1,000 x psi)=6.894 MPa. The time for the paint to respond to the temperature change is short.



Stress versus percent relative humidity for restrained samples of Naples yellow and flake white paint. This figure shows the data (symbols) and predicted stress developed (lines) for these materials using Equation 12 with a correction for the stress release due to the load cell compliance. The primary difference in these two paints is the higher modulus in the flake white.

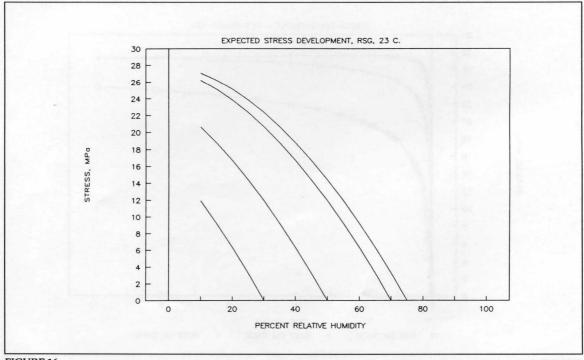


FIGURE 16
The computed predicted stress versus relative humidity for restrained rabbit skin glue with the length fixed at: 75%, 70%, 50%, and 30% RH and desiccated. Equation 12 was used for this calculation where the modulus was calculated using Equation 15.

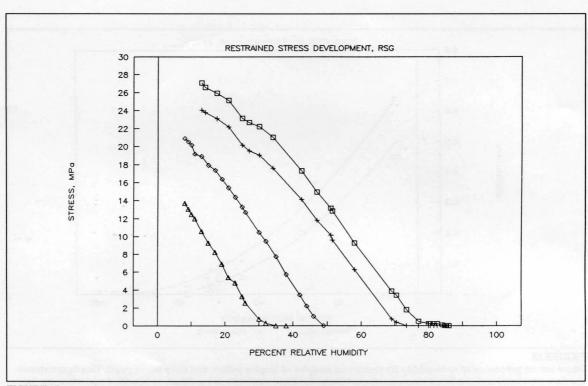


FIGURE 17
Measured stress versus relative humidity for samples of rabbit skin glue restrained at different values of relative humidity and then desiccated. This test data can be compared with the calculated values shown in Figure 16.

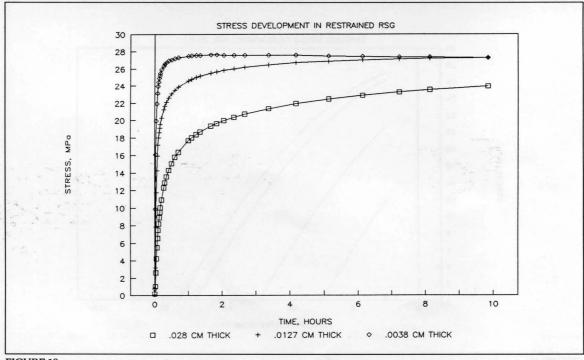


FIGURE 18
Stress versus time data for three different thicknesses of rabbit skin glue subjected to rapid desiccation to 5% RH after being restrained and equilibrated at 66% RH. All samples ultimately reached a maximum stress level of 27.6 MPa (4 ksi), though it took the thickest sample 100 times longer than the thinnest.

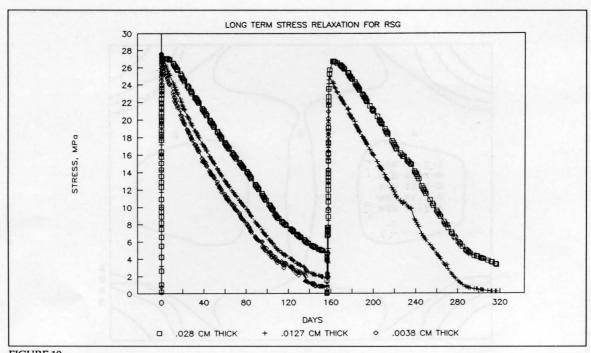
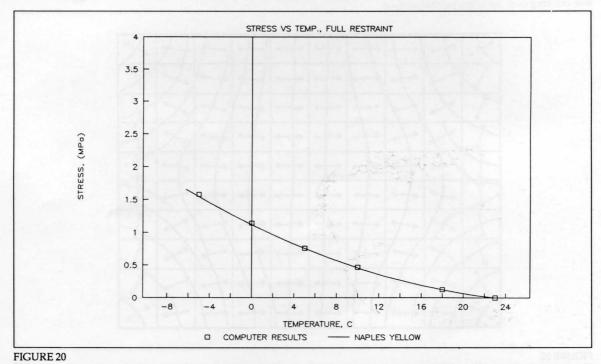


FIGURE 19
The stress relaxation versus time for the three samples of rabbit skin glue shown in Figure 18. All of the glue samples show the stress "reactivation" after raising the relative humidity to 90% and then rapidly desiccating to 5% for a second time. The time required for full stress-relaxation is considerably longer than the time needed for even the thickest specimen to fully react to a change in relative humidity.



The stress versus temperature plots for a computer model results of a simple restrained Naples yellow paint sample and the results of Equation 5, using the Naples yellow paint test data. This was a programming check to verify both the modeling accuracy and the correct computer input equations for the dimensional and mechanical properties of the material. The computer model correlates well with the experimental test results.

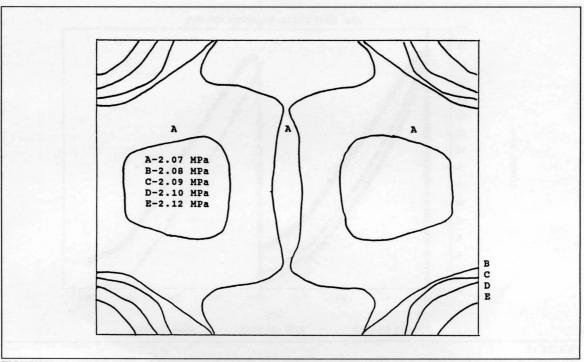


FIGURE 21

The computer calculated principal stress contours for a numerical model of a typical canvas supported oil painting 76 x 102 cm (30 x 40 in.), cooled from  $23^{\circ}$ C to  $-3^{\circ}$ C at  $5^{\circ}$ RH. The stress distribution is uniform, varying only by  $2^{\circ}$  and reaches a magnitude that exceeds the tested breaking strength of the paint used in the model. This model suggests extensive cracking over the entire painted surface.

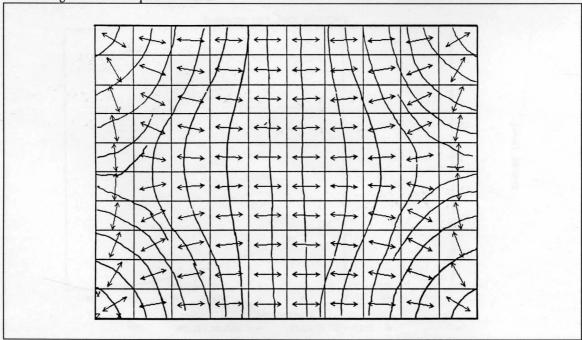


FIGURE 22

The directions vectors (arrows) of the computer calculated principal stresses in the model painting described in the text and Figure 21. Cracks that form will do so perpendicular to the direction vectors. Superimposed over the vectors is the expected crack pattern from this computer analysis. This crack pattern occurs frequently in North America.

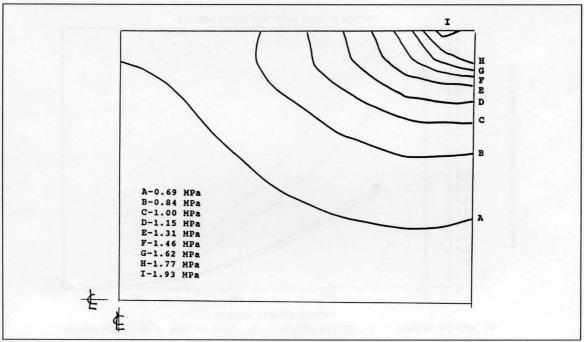


FIGURE 23

The computer calculated principal stress contours for a numerical model of a typical canvas supported oil painting  $76 \times 102 \text{ cm}$  ( $30 \times 40 \text{ in.}$ ), desiccated from 70% to 10% RH at 23%C. The stress distribution is not as uniform as found in the cooling analysis, varying by 64%. The stresses don't reach the magnitude of the tested breaking strength of the paint used in the model. This model suggests minor cracking occurring at the corners only if the strength of the paint was less than 1.7 MPa (.28 ksi).

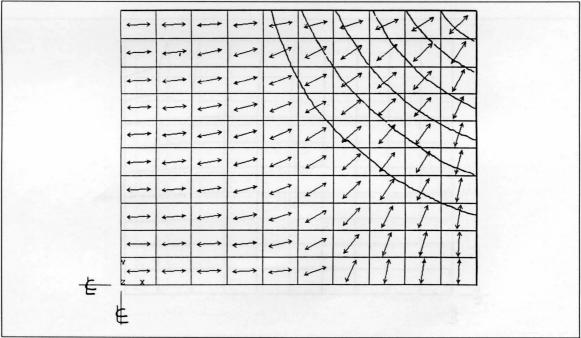


FIGURE 24

The direction vectors (arrows) of the computer calculated principal stresses in the model painting described in the text and Figure 23. Cracks that form will do so perpendicular to the direction vectors. Superimposed over the vectors is the expected crack pattern from this computer analysis only if the strength of the paint is less than 1.7 MPa (.28 ksi). This pattern occurs in paintings throughout the world.

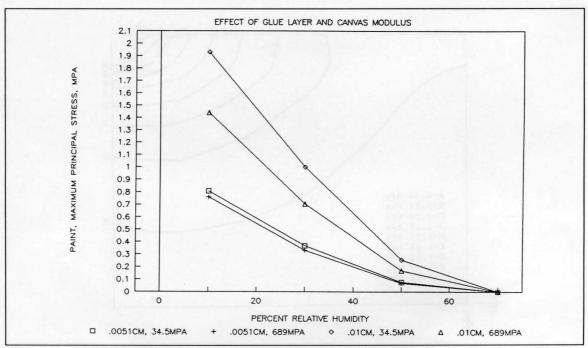
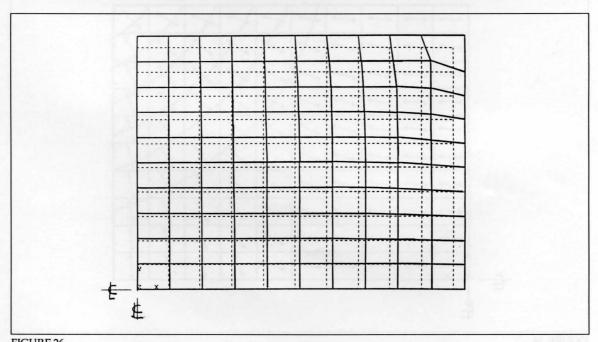


FIGURE 25
This figure shows the computer generated results reflecting the effects of the glue thickness and the support canvas stiffness on the stresses in the paint layer when the painting is desiccated from 70% to 10% RH. The two numbers given in each legend are the glue thickness and the modulus of the support. This analysis shows that moisture responsive lining adhesives such as hide glue, reduce the allowable relative humidity range a painting might safely sustain. Stiff lining materials increase the range.



The computer calculated distortion for a numerical model of a typical canvas supported oil painting  $76 \times 102$  cm ( $30 \times 40$  in.), keyed out at all corners .14 cm (.056 in.) and desiccated from 70% to 10% RH at 23°C. The solid lines display the distorted shape and the dashed lines represent the original configuration of the painting. The corner of the painting is highly distorted.

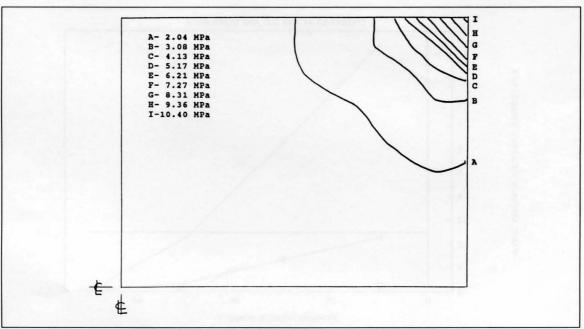


FIGURE 27

The computer calculated principal stress contours for a numerical model of a typical canvas supported oil painting,  $76 \times 102 \text{ cm}$  ( $30 \times 40 \text{ in.}$ ), keyed out at all corners .14 cm (.056 in.) and desiccated from 70% to 10% RH at 23%C. The stress distribution varies as much as 500% and reaching magnitudes five times greater than the measured breaking strength of the paint used in this model. Expanding a stretcher considerably reduces the range of relative humidity the painting is able to withstand.

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FIGURE 28

The direction vectors (arrows) of the computer calculated principal stresses in the model painting described in the text and Figure 27. Cracks that form will do so perpendicular to the direction vectors. Superimposed over the vectors is the expected crack pattern from this computer analysis. The cracks radiate approximately 8 in. from the corners. This pattern occurs in paintings throughout the world.

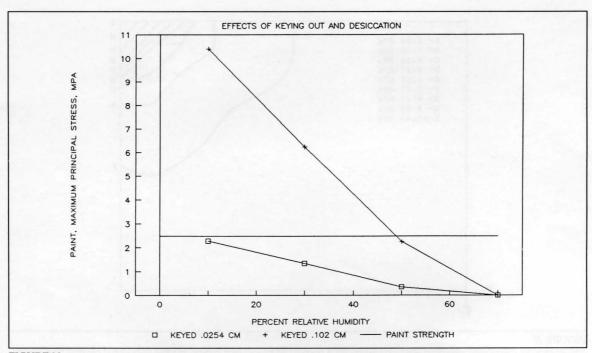


FIGURE 29
This figure shows the computer generated results reflecting the effects of different magnitudes of corner expansion on principal stresses in the paint layer when the painting is desiccated from 70% to 10% RH. The paint strength is included as a reference.

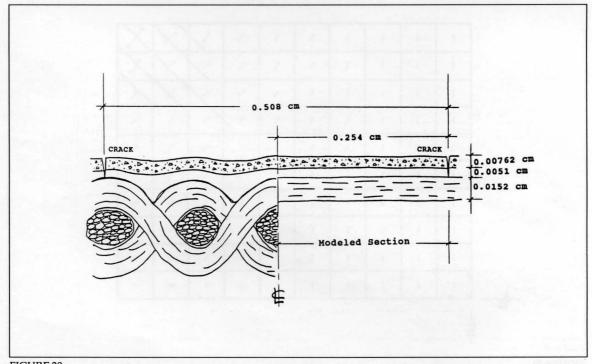


FIGURE 30
Detail section view of a painting from crack to crack, illustrating the difference between the actual structure and the computer model of the same structure. The reduction in thickness of the fabric is necessary to account for the voids in fabric and hence correct for the fact that the in-plain stiffness of a canvas is considerably higher than the bending striffness, though both are low. Details of the correction factors are in reference 3.

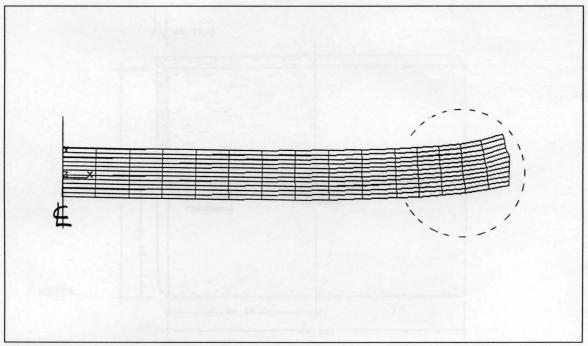


FIGURE 31

The computer calculated distortion of "cupping" for a numerical model of a typical canvas supported oil painting with existing cracks as illustrated in Figure 30. The model section was numerically desiccated from 70% to 10% RH at 23°C. The left-hand edge of the model is the center of the painting section detail and the right-hand edge is the crack location. The circle encloses the area shown in greater detail of Figure 32.

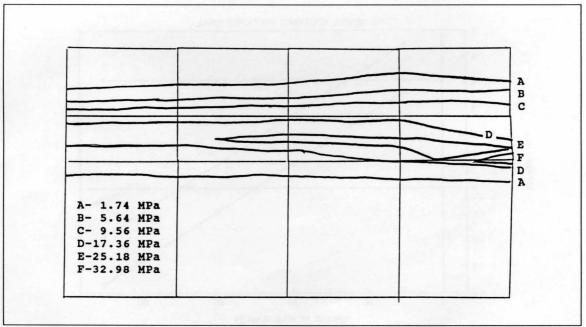


FIGURE 32

The computer calculated principal stress contours for a numerical model of a typical canvas supported oil painting with existing cracks as illustrated in Figure 30. The model was numerically desiccated from 70% to 10% RH at 23°C. The stress distribution shows intense concentrations at the glue fabric interface with stresses in the paint layer well in excess of the measured breaking strength of the modeled paint. This model suggests major flaking of the design layer will be a consequence of this magnitude reduction in relative humidity.

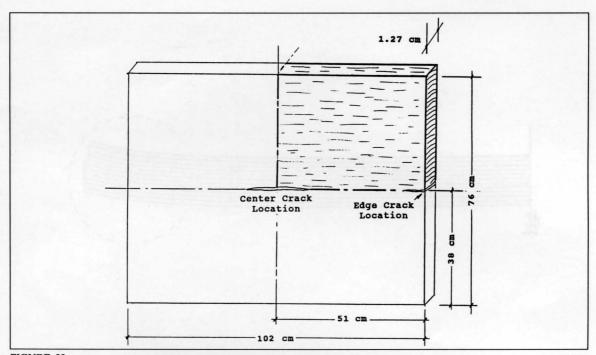


FIGURE 33

This figure illustrates a typical oak panel, showing the location of the modeled cracks, grain orientation, and panel restraints. Due to the double symmetry of the panel, only the upper right-hand section needed to be modeled.

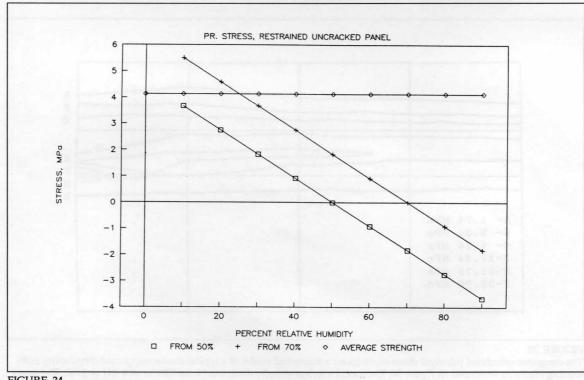


FIGURE 34
This figure shows the computer generated maximum principal stresses in the uncracked wood panel, resulting from changes in relative humidity when the panel is restrained at 50% and 70% RH. The average strength of the wood is included as a reference. Cracking of the panel might occur at 25% RH if the panel had been restrained at 70% RH.

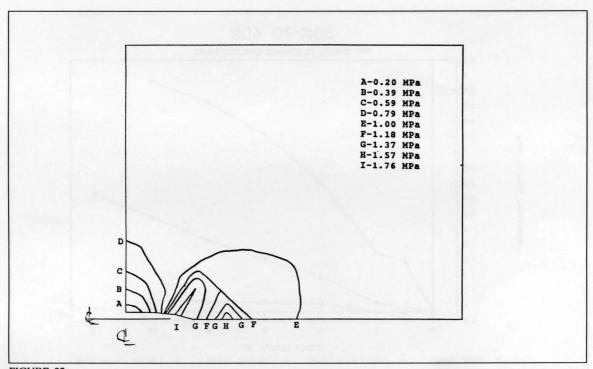
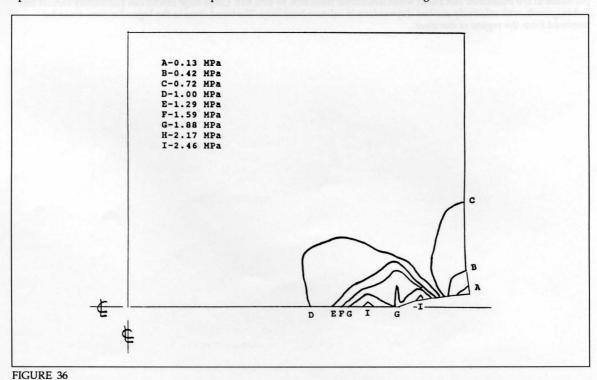


FIGURE 35
The computer calculated principal stress contours for a numerical model of a restrained oak panel with an 8 in. center crack and desiccated from 50% to 40% RH. The stress distribution shows intense concentrations in the regions of the crack tip. The maximum stress calculated in the presence of a center crack is about four times greater than without the crack.



The computer calculated principal stress contours for a numerical model of a restrained oak panel with a 4 in. edge crack and desiccated from 50% to 40% RH. The stress distribution shows intense concentrations in the regions of the crack tip. The maximum stress calculated in the presence of a center crack is about six times greater than without the crack.

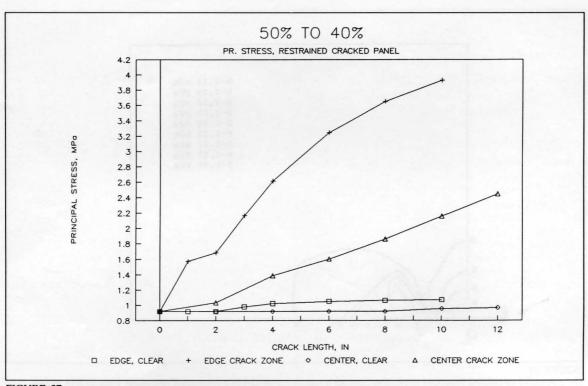


FIGURE 37
This figure shows the computer generated results reflecting the effects of different crack lengths on the maximum principal stress in the restrained oak panel when desiccated from 50% to 40% RH. Large edge cracks can potentially exhibit unstable growth with relatively modest depressions in relative humidity. The word clear in the legend means areas far removed from the region of the crack.

#### ACKNOWLEDGMENT

We wish to thank the Scholarly Studies Program of the Smithsonian Institution for its generous support of this research project.

#### NOTES

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