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A GENERAL METHOD FOR DETERMINING THE MECHANICAL PROPERTIES NEEDED FOR THE COMPUTER ANALYSIS OF POLYMERIC STRUCTURES SUBJECTED TO CHANGES IN TEMPERATURE AND RELATIVE HUMIDITY

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ABSTRACT

This study examines the methods for determining the effects of temperature and relative humidity (RH) on the dimensional and mechanical properties of artists' materials. Using both of these properties, typical cultural objects, such as paintings and photographs can be modeled on the computer to correlate the magnitude of developed stresses to environmental changes. Comparing these calculated stresses with the measured strength of the materials provides the opportunity to assess the potential risk from damage due to environmental change. Recognizing that the mechanical properties of polymeric materials are functions of the rate of the application of load, the time they are subjected to a load, the temperature and the relative humidity, it becomes a question of identifying those parameters that are the most important in determining the correct information needed for computer modeling.

INTRODUCTION

The stresses in a structure are usually determined with direct measurements using strain gages, stress coatings, or other systems that measure physical displacements of the structure.[1] These measurements and the mechanical properties of the materials used in its construction then help determine the stresses. Structures that are fully restrained and cooled do not exhibit these displacements and their material properties change. Paint film stresses, resulting from temperature or relative humidity fluctuations in paintings cannot be measured directly.[2] Traditional direct structural analysis requires displacements with moderately constant mechanical properties. Many structures, such as stretched paintings or photographic materials, exhibit little or no in-plane displacements and have highly variable mechanical properties. It then becomes necessary to find other techniques for analyzing such structures subjected to environmental changes.

One of the most powerful analytical tools for structural analysis is the digital computer with the technique of Finite Element Analysis (FEA).[3] FEA readily lends itself to complex structures composed of multiple materials. The fundamental principle of the method allows one to "sub-divide" a complicated structure into a "finite" number of simpler sub-structures called "elements". These elements are connected at points called "nodes". These simple elements are mathematically relatively easy to manipulate.

An example of a structural analysis using only a single element (Fig. 1) is a simple rod which is pulled with the force, F, at one end while the other end is stationary, or "fixed". The nodes, points 1 and 2, can be considered to be at the ends of the rod. In this analysis, the force, F, causes no displacement at node 1

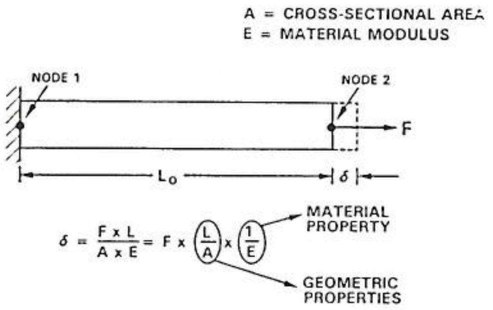

Figure 1. A simple structure modeled with a single element.

and a displacement, δ, at node 2. The magnitude of the displacement, δ, can be calculated as:

$$\delta = (F \times l)/(A \times E) \qquad \text{Eq. 1}$$

where
F is the applied force
l is the length of the structure, and in this case the length of the element,
A is the cross-sectional area of the structure (element) and
E is the modulus of elasticity, a material property.

The quantities, A and l, are geometric properties of the structure. In the example shown in figure 1, the geometry of the structure is quite simple and presents no mathematical complications. This is rarely the case in actual structures where the geometry can be complex (see fig. 2a). This complexity can be remedied by subdividing the structure into a "finite" number of "elements" as shown in figure 2b. These elements are geometrically simple and the structure is easily analyzed using a finite element program.

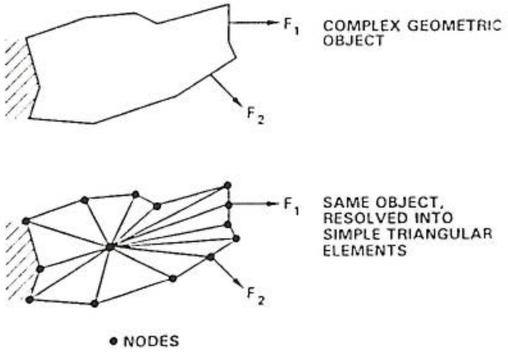

Figure 2. A complex structure modeled with several triangle elements.

In both the simple and complicated case, the mechanical and dimensional (physical) properties of the materials are of utmost importance, and are critical in correctly analyzing any structural problem. This paper concentrates on the mathematical and experimental procedures used in determining the physical and mechanical properties needed for the analysis of complex structures subjected to changes in temperature and relative humidity.

THE FUNDAMENTAL MECHANICAL PROPERTIES OF MATERIALS

From the tensile test of a material many fundamental properties of the material can be determined. The results of one of these tests is presented in figure 3. In this figure the different quantities of importance are labeled. Stress, σ, is equal to the applied force, F, divided by the test sample's cross-sectional area, A. Stated mathematically;

$$\sigma = F/A. \qquad \text{Eq. 1}$$

Engineering strain, ϵ, is the change in specimen length, δ, divided by the original length of the specimen, L_o, where;

$$\epsilon = \delta/L_o = (L_s - L_o)/L_o \qquad \text{Eq. 2}$$

and L_s is the "stretched" length of the specimen.

The strain times 100 is the percent elongation of the test specimen. The ultimate strength of a material, σ_{ult}, is the maximum stress the material can sustain, usually the point at which the material breaks. The modulus, E, is the measure of a material's ability to deform when subjected to stress, and may be viewed as a measure of the materials stiffness. Mathematically the modulus is:

$$E = \sigma/\epsilon. \qquad \text{Eq. 3}$$

The modulus is defined normally for the elastic region of a stress-strain plot where the material returns to its original

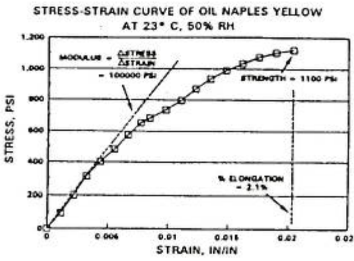

Figure 3. Typical stress-strain plot for an oil paint.

dimensions when the stress is removed. A material is said to exhibit plastic behavior if it remains permanently deformed after the removal of the stress. The yield point, σ_{yld}, is the stress where the material goes through a transition from elastic to plastic behavior. In many materials, this point is not clearly defined.

All of the properties defined above are altered with changes in temperature, relative humidity, and the speed with which the forces are applied.[4] In general, lowering the temperature or relative humidity, or increasing the speed of the applied force all increase the strength of the material, the stiffness of the material, E, and reduce the strain to failure. Raising the temperature, relative humidity, or slowing the speed of force application, lowers the strength and modulus, but increases the materials ability to deform, (strain to failure).

Because the time required for polymeric materials to equilibrate with new environments is relatively long, it is necessary to examine the mechanical properties of these materials under long-term or "equilibrium" conditions. The mechanical properties of painting materials are considerably different under these slow, long-term conditions than they are under rapid, dynamic conditions. Polymeric materials exhibit the ability to deform considerably when stresses are applied slowly. The maximum strengths attained are also quite low under these conditions when compared to rapid loading conditions.

STRESS DEVELOPMENT IN FULLY RESTRAINED MATERIALS RESULTING FROM CHANGES IN TEMPERATURE AND RELATIVE HUMIDITY

If any of the artists' materials are fully restrained and subjected to decreases in either temperature or relative humidity, they will experience an increase in tension. This results from the material's inability to contract while undergoing a loss of heat or moisture. The magnitude of the tensile stress the material experiences is a function of both the attempted shrinkage and the modulus of the material. The modulus in any of the artist's organic materials is dependent on both the temperature and relative humidity and must be taken into account in any calculation.

Temperature Effects

In general, if the thermal coefficient of expansion and the equilibrium modulus of a material are known, the stress levels resulting from cooling a restrained material can be predicted. The method of calculating this stress increase can be derived directly from the basic equation 3, repeated here;

$$E = \sigma/\epsilon \qquad \text{Eq. 3}$$

which can be rewritten as:

$$\sigma = E \times \epsilon. \qquad \text{Eq. 4}$$

To include the temperature effects, it must be recognized that both the modulus, E, and the strain, ϵ, are functions of temperature and therefore:

$$\sigma(T) = E(T) \times \epsilon(T). \qquad \text{Eq. 5}$$

Relative Humidity Effects

The effects of relative humidity on the artists' materials are analogous to those of temperature. Changes in relative humidity induce both dimensional and mechanical properties changes. In effect then, stress development in fully restrained materials subjected to changes in RH can be treated in a manner similar to restrained materials subjected to changes in temperature. The equation for relative humidity behavior would be:

$$\sigma(RH) = E(RH) \times \epsilon(RH) \qquad \text{Eq. 6}$$

STRAINS INDUCED BY TEMPERATURE OR RELATIVE HUMIDITY

Strains, as defined in Eq. 2, require a change in the length of the specimen. This presents an apparent paradox when discussing the strains developed in a material with fixed dimensions and subjected to either desiccation or cooling.

The strain, $\epsilon(T)$, as a function of temperature is derived by considering the shrinkage of the specimen as if it were free to do so. The specimen would contract upon cooling and it is effectively being "stretched" backed to its original restrained length. The "unstretched" length would be the free shrinkage length and the deformation would be the amount needed to stretch the contracted specimen to the restrained length. The amount of shrinkage is calculated from the thermal coefficient of expansion. Using the thermal coefficient of expansion, γ, the freely contracted length for the paint for any temperature is:

$$L_T = L_R + \gamma \times \Delta T \times L_R \quad \text{or}$$

$$L_T = L_R \times (1 + \gamma \times \Delta T) \qquad \text{Eq. 7}$$

where L_T is the free shrinkage length,
L_R is the restrained length,
ΔT is the change in temperature the material experiences,
γ is the thermal coefficient of the material, and for these calculations is considered to be a constant although that is not a requirement.

The strain at any temperature is now calculated as:

$$\epsilon(T) = (L_R - L_T)/L_T. \qquad \text{Eq. 8}$$

Equation 5 can also be written in terms of the thermal coefficient of expansion, temperature change, and the stretched length as:

$$\epsilon(T) = (L_R - L_R \times (1 + \gamma \times \Delta T))/(L_R \times (1 + \gamma \times \Delta T)) \qquad \text{Eq. 9}$$

or as

$$\epsilon(T) = -(\gamma \times \Delta T)/(1 + \gamma \times \Delta T). \qquad \text{Eq. 10}$$

The negative sign indicates that positive tensile strains are resulting from negative temperature changes, i.e. cooling. For changes in relative humidity, the equation for the strains developed in a fully restrained material and desiccated are:

$$\epsilon(RH) = -(\alpha \times \Delta RH)/(1 + \alpha \times \Delta RH) \qquad \text{Eq. 11}$$

and ΔRH is the change from the original RH to the new RH, with α the "moisture" coefficient of expansion.

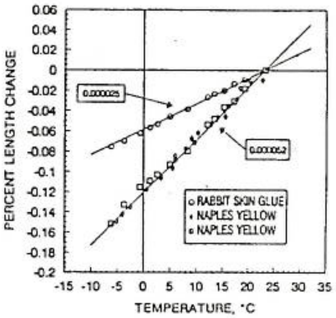

Figure 4. Plot of the percent length change versus temperature for rabbit skin glue and naples yellow oil paint. This data is used to determine the thermal coefficients of expansion for these materials.

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Thermal Coefficients of Expansion

The thermal coefficient of expansion for thirteen year-old oil paint and hide glues were measured. The total change in the length of these materials is linear and quite small over the tested temperature range (see fig 5). The slope of these lines divided by one hundred will give the thermal coefficients (γ) of expansion for these materials. This is the change in strain per degree C. For the paint, the thermal coefficient, γ, is .000052 per degree C and for the rabbit skin glue, a thermal coefficient of about half that or .000025 per degree C. For comparison purposes, three common metals, copper, steel, and aluminum have coefficients of .000017, .000011, and .000024 per degree C respectively.

Moisture Coefficients of Expansion

Oil Paints

Most artists' materials will expand with an increase in moisture content and conversely shrink upon its loss. The driving mechanism for changes in the materials' moisture content is the ambient relative humidity. Oil paints, for example, have a highly non-linear variation in their dimensional response to relative humidity. Figure 5 illustrates a paint's dimensional response to RH. In this figure, flake white, ground in safflower oil, swells a maximum of .18% between 0% and 70% RH, with a larger increase, about .6%, occurring between 70% and 95% RH. This specimen, which was about .038 cm (.015 in) thick, took at least forty-eight hours to reach equilibrium with each environment at which the data were recorded.

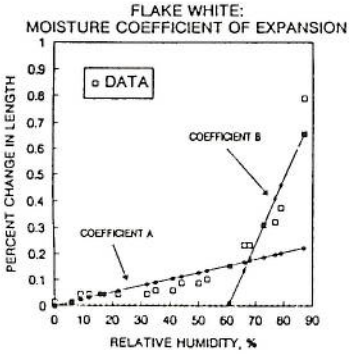

Figure 5. Plot of the percent change in length versus relative humidity for a typical oil paint. This data was used to determine the approximate "moisture" coefficients of expansion for this paint.

It is now possible to calculate two different "moisture" coefficients of expansion that linearly approximate the effect of moisture on the dimensional properties of the paint. These coefficients include the ranges from 0% to 70% RH and from 70% to 95% RH and can be calculated by using the percent length changes in each of the relative humidity ranges. For example the low relative humidity range, moisture coefficient, α_l, for the flake white is;

$\alpha_l = (.18/70)/100 = .0000257$, which is the change in strain per percent relative humidity. This coefficient will be used to calculate the stresses resulting from the desiccation of restrained paint. For the high relative humidity range;

$\alpha_h = (.62/25)/100 = .000248$, which is nearly ten times the low range coefficient. These are rarely going to be used since the equilibrium modulus of paints above 70% RH is nil. A non-linear treatment of the moisture coefficient is also possible.

Rabbit Skin Glue and Photographic Gelatin

One material, traditionally used in paintings, that is quite dimensionally responsive to relative humidity is rabbit skin glue (RSG). When further refined, hide glues are used as photographic gelatins found in the image and anti-curl layers of photographs.

As with the paints, only the low end moisture coefficients are useful. For the rabbit skin glue, α_l is .000264, over ten times that of the paints. For the more refined and processed photographic gelatin, α_l is .00044. This is significantly greater than the rabbit skin glue and remarkable considering that these are chemically very similar materials.

MECHANICAL PROPERTIES UNDER "EQUILIBRIUM" CONDITIONS

Temperature Effects

Depending on the material and the environment, artists' materials will "stress relax" over time. These times can be considerable.[5] For example, hide glues will stress relax to zero over a period of about 5 months at room temperature and low RH. Oil paints tested at low temperatures and moderate relative humidity give all indications of taking decades and even centuries to relax fully. On the other hand, the amount of time needed for these materials to equilibrate to a new environment is considerably shorter, for the paints, a matter of days and, for the glues, a matter of hours. Damage induced by environmental change occurs over these shorter time spans and the mechanical properties of the materials should be measured within similar time spans.

Direct Determination of E(T)

Several paints were mechanically tested under long-term loading conditions. Sample preparation and tests procedures are outlined in another paper.[6] The tests were conducted by applying a small strain to the test sample, about .007, and allowing it to stress relax fully. Once full stress relaxation was attained, a subsequent increment of strain was applied and the stress was again allowed to relax (Figure 6).

The time to relax fully the paints was typically between five and ten days, depending on the paint. This process was repeated until the specimen broke. Each test took several months to complete. The modulus was determined by taking the slope of the locus of relaxed points from the equilibrium test (see Figure 6). In this case, for Naples yellow at 23°C, 50% RH the modulus was 10,000 pounds per square inch (10 ksi), (68.9 Mega-pascals (MPa)).

The equilibrium modulus for the Naples yellow was measured at 23°C, 5% RH to be 47.5 ksi (327 MPa) and at -3°C, 5% RH was found to be 150 ksi (1,034 MPa). If it is assumed that E varies linearly with temperature (figure 7) then a linear function for E(T) can be fitted and stated as:

$$E(T) = 138{,}520 - 3770 \times T \qquad \text{Eq. 6}$$

where the units are psi for the modulus and the temperature is in degrees Celsius. That the function is linear is not unreasonable, since it has been shown that the thermal coefficient is linear over the temperature ranges considered here.

For the Naples yellow, the complete equation for a fully restrained sample of paint subjected to a drop in temperature at 5% RH is now:

$$\sigma(T) = -(138{,}520 - 3779 \times T) \times (.000052 \times \Delta T)/(1 + .000052 \times \Delta T).$$

where T is the current temperature and ΔT is the change from the original temperature to the current temperature.

Indirect determination of E(T)

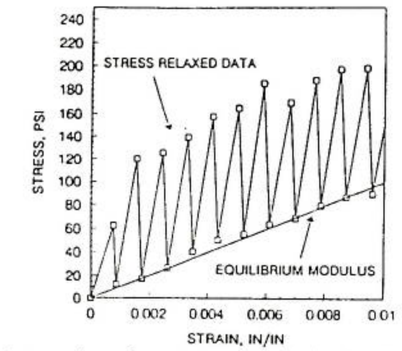

Figure 6. Direct determination the "equilibrium" stress-strain curve for the naples yellow paint by using the "stress-relaxation" method.

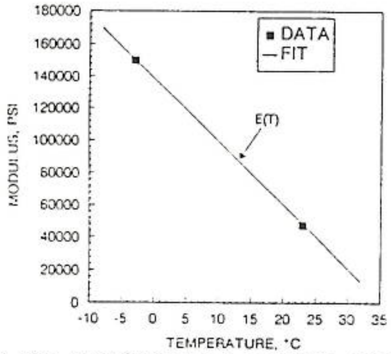

Figure 7. Plot of the "equilibrium" modulus data versus temperature, showing the fit used to determine E(T) for this oil paint.

Since the thermal coefficient was measured, it is possible to derive the equilibrium modulus from the restrained test itself. This can be done by rewriting Equation 5 as:

$$E(T) = \sigma(T)/\epsilon(T) \qquad \text{Eq. 7}$$

where $\sigma(T)$ is obtained from the restrained test and
$\epsilon(T)$ is obtained from the thermal coefficient of expansion.

At -3°C, the extrapolated value of the hide glue modulus is 705 ksi (4,860 MPa), which is only twelve percent less than the rapid loading modulus of 800 ksi (5,515 MPa). This is reasonable since the difference between the rapid loading modulus and the equilibrium modulus for the artists' materials diminishes with a reduction in temperature and relative humidity. E was measured directly at 23° C and 50% RH. Using these data points, E(T) was determined by assuming a linear fit.

Experimental verification

From an experimental point of view, it is not possible to measure easily the stress of a "fully" restrained specimen since the "load cell," the device that measures the stresses, is "compliant." This means the cell "gives" a bit as force is applied to it. This load cell compliance effectively relieves some of the restraint on the specimen and the stresses measured are actually less than if fully restrained. This compliance can, however, be included in the calculations that allow us to predict the behavior of the specimen subjected to restrained temperature changes. The compliance of the test device is a function of the total force and is measured during the test of the specimen. The value of the compliance, measured in units of length, is the raw data output, AO, times a compliance constant, KD. It is now possible to correct for the compliance of the load cell and calculate the expected stresses in the experimental restrained test where:

$$\sigma(T) = E(T) \times (\epsilon(T) - AO \times KD/L(T)) \qquad \text{Eq. 8}$$

This calculation was conducted for the Naples yellow using the measured modulus and the thermal coefficient of expansion of the material. The results of this calculation are presented in Figure 8 as the lower continuous line.

Also on that figure is the measured stress data (squares) for the Naples yellow paint. There is a substantial correlation between the predicted and the actual measured stress levels at the different temperatures while the relative humidity is held at 5%. This experiment was conducted with the temperature both increasing and decreasing. Also shown in figure 8 are the test data for rabbit skin glue and a fitted line resulting from the indirect determination of E(T).

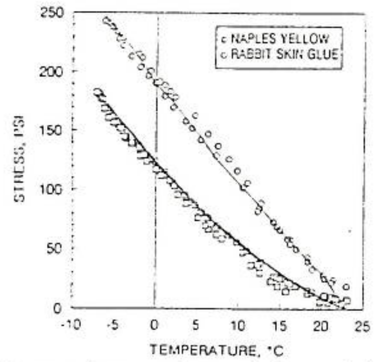

Figure 8. Stress versus temperature plot used to experimentally verify the mathematics developed computer modeling. The symbols are the measured data and the lines are the mathematical functions.

Calculation of the Fully Restrained Specimen

Using equation 5, it is now possible to calculate the stress development in Naples yellow and rabbit skin glue as if they were fully restrained. Figure 9 shows the expected stresses of these materials when fully restrained and chilled from 23°C to -6°C at 5% RH. The load cell compliance has a significant stress reduction effect since the fully restrained stresses are nearly twice for the glue and about fifteen percent higher for the paint.

The immediate conclusion to be drawn from Figures 8 and 9 is that while the coefficient of thermal expansion for hide glue is only one-half that of the Naples yellow paint, the hide glue still has a larger stress increase, not because of changes in dimension but because of a substantially higher modulus. It is important to recognize that both the modulus and the coefficient of expansion influence the magnitude of the resulting stresses in materials restrained and cooled.

Relative Humidity Effects

The effects of relative humidity on the artists' materials are analogous to those of temperature. Changes in relative humidity induce both dimensional and mechanical properties changes. In effect then, stress development in restrained materials subjected to desiccation can be treated in a manner similar to restrained materials subjected to decreases in temperature.

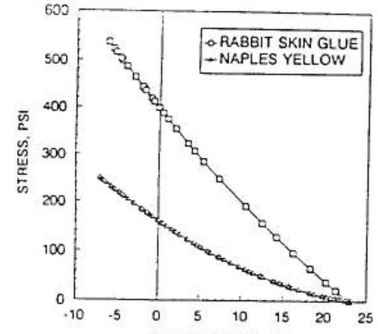

Figure 9. Predicted stress versus temperature for fully restrained rabbit skin glue and naples yellow oil paint.

Oil Paints

Two paints, Naples yellow and flake white, were restrained at 66% RH and 23°C, and desiccated to 58% RH where the stresses in the paints were .002 ksi (.014 MPa) and .004 ksi (.028 MPa) respectively. The paints were further desiccated to 5% RH and the stresses now reached were as high as .066 ksi (.455 MPa) and .092 ksi (.634 MPa) respectively (see Figure 10). The time for equilibration was forty-eight hours for .015 in (.0059 cm) thick films.

For these two oil paints, E(T) was determined directly from paint film testing. The equations for the modulus of the materials were: for Naples yellow, E(RH) = 51,800 - 840xRH and for flake white, E(RH) = 63825 - 840xRH, where the units are in psi and percent relative humidity. These two equations have the same slope and differ only in the intercepts. The difference in the intercept values is the difference in the modulus of each material at 50% RH. Using equation 6 (corrected for load cell compliance), and the expansion coefficient, it was possible to calculate the expected stresses. The value, α_s equal to .0000257, was used for both the Naples yellow and the flake white. The results of the calculations are shown in Figure 10 as the solid lines passing through the data points.

One interesting aspect of these calculations is the remarkable accuracy using linear approximations of the actual material behavior. Equally interesting, for the Naples yellow, is the magnitude of stress reached, (.066 ksi (.455 MPa)), at a desiccation from 66% RH to 5% RH, considerably less than that reached (.145 ksi (1.0 MPa)), when cooled from 23°C to -3°C at 5% RH.

Rabbit Skin Glue and Photographic Gelatin

Until now all of the discussions about the mechanical properties of the artists' materials and stress development in either cooled or desiccated materials have assumed that all behavior was linear. This assumption seems to be valid since all of the elastic modulus equations result in successful stress predictions. For the rabbit skin glue, the behavior is almost never in the elastic range but exhibits a "quasi" plastic, non-linear behavior. The reason for this behavior is that the material wants to shrink extensively with desiccation. This results in strains, when the specimen is restrained and desiccated, that exceed the yield stress for any given relative humidity environment. Once these yield stresses are reached, the glue can have no further rise in stress no matter what strain level is attained. This presents the complication that the linear modulus is no longer valid, since any calculation using these values will result in stresses well in excess of the yield stresses.

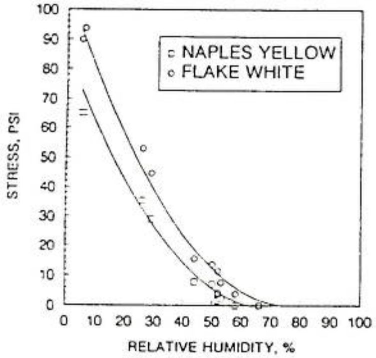

Figure 10. Paint stress versus relative humidity plots used to experimentally verify the mathematics developed for computer modeling. The symbols are the measured data and the lines are the mathematical functions.

What can be done is to determine an effective modulus, $E_\epsilon(RH)$, using Eq. 7 modified for RH, for all of the different relative humidity values. The effective modulus is for rabbit skin glue:

$$E_\epsilon(RH) = 200{,}000 + 2200 \times (75-RH_i) + (2200 \times RH)$$

where; RH_i is the value of the starting or initial relative humidity,
RH is the relative humidity at any time.
And a moisture coefficient of α_s = .000264 is as determined.
For the photographic gelatin:

$$E_\epsilon(RH) = 130{,}000 + 2500 \times (72-RH_i) + (1500 \times RH)$$

with moisture coefficient of .00044.

The results of the glue and gelatin modulus functions are compared to actual data in figures 11 and 12. The sample specimens were restrained at different relative humidities and desiccated. Both of the materials showed similar behavior and reach similar maximum stress values at slightly over 4 ksi (27.6 MPa). This is of particular interest since the coefficient of expansion of the glue is lower than the gelatin and the modulus of the glue is considerably higher than the gelatin.

COMPUTER ANALYSIS OF MUSEUM OBJECTS SUBJECTED TO CHANGES IN TEMPERATURE AND RELATIVE HUMIDITY.

Structural analysis was conducted using Finite Element Analysis (FEA) and the digital computer. The program used in the computer modeling in this paper was ANSYS®, Version 4.4, run on a 386, 33 megahertz desktop computer with 4 megabytes RAM.

Modeling The Desiccation of a Cibachrome Print.

There was interest in the stress development by desiccating the image layer of the photographic materials. Low relative humidity levels are currently recommended for long term preservation of photographic materials due to an incremental improvement in chemical stability. For example, 25 to 30% RH conditioning is recommended for color films prior to placing them in cold storage.[7] It is possible to model (using Ansys) a section of a cibachrome photograph. Cibachrome is manufactured by Ilford Photo Corporation and utilizes a unique silver dye bleach process. The finished print is structurally typical of a broad range of photographic materials coated on a polyester film base.[8] The image layer is gelatin and on the reverse there is an anti-curl layer which is gelatin with glass particles acting as a matting agent. The glass increases the stiffness of the material as well as reduces the moisture coefficient of expansion. So in effect we

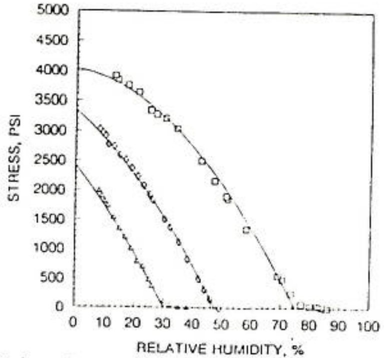

Figure 11. Rabbit skin glue stress versus relative humidity plots used to experimentally verify the mathematics developed for computer modeling. The symbols are the measured data and the lines are the mathematical functions.

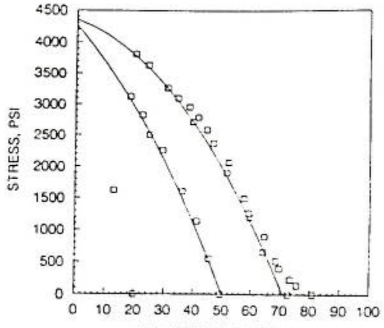

Figure 12. Photographic gelatin stress versus relative humidity plots used to experimentally verify the mathematics developed computer modeling. The symbols are the measured data and the lines are the mathematical functions.

have at least three material layers with distinct mechanical properties. The image layer is .000652 in. (.00166 cm) thick, the polyester base is .007 in. (.0178 cm) thick and the anti-curl layer is .00049 in. (.00125 cm) thick. The primary measurement was curling of the photograph as it dried out. If this calculation was right, then the rest of the stress analysis would be fairly reliable. In addition, it is possible to check the computed stress levels by comparing the results to the tested individual material layers. Figure 13 shows the dimensions of the model and the test specimen and the measured and calculated displacement. The actual test specimen was one inch wide and two inches long, held at the center. The test was started at 61.5% RH and deflections measured at 44.6%, 31%, 25% and 18.8% RH.

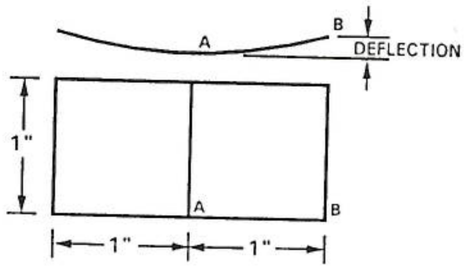

Figure 13. Dimensions of the Cibachrome experimental test specimen showing the displacement measured and computed.

The polyester layer was modeled with the measured moisture coefficient of expansion of .0000077 and a modulus that varied as a function of relative humidity (E(RH) = 695,000 - 1010xRH). Figure 14 shows the edge displacement results of the experimentally measured Cibachrome sample as well as the calculated results if the image layer and the anti-curl were the same gelatin material (labeled: model 2 material), and the calculated results if the model has an anti-curl layer as being stiffer and less responsive to dimensional change (labeled: model 3 materials). Notice that there is a rather distinctive plot for both the measured and correctly calculated displacements.

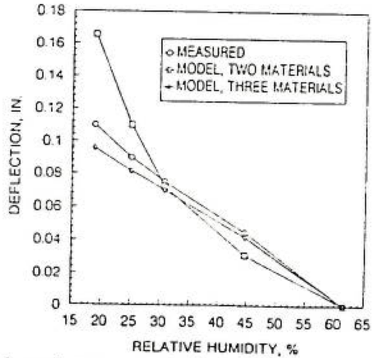

Figure 14. Computed and measured deflections of the Cibachrome computer model and test sample.

Figure 15 has the calculated maximum principal stress results which are as expected; higher than the simple experimental stress due to the biaxial stress field. From a conservation point of view, the stresses induced by cycling from commonly encountered environments to the presently recommended 25-30% RH level are undesirable. In the presence of defects such as cracks or at edges, the high stresses represent the source of delamination of the layers. Mechanical stability can now be quantified and should be considered along with improvements in chemical stability when storage environments are established for photographic materials.

The deflections in this test reach over ten times the thickness of the structure and over one tenth the gage length (1 inch.) of the specimen. This represents the successful modeling of a very large-displacement structure constructed essentially of polymers.

Modeling the Effects of Cooling a 30 in. x 40 in. (76 cm x 102 cm) Painting

A 30 in. x 40 in. (76 cm x 102 cm) painting was modeled and subjected to changes in temperature while the ambient relative humidity was held at 5%. The layer of the painting consisted of a linen .025 in. (.064 cm) thick, modeled as a .006 in. (.0152 cm) layer, a glue layer .002 in. (.00508 cm) thick and a Naples yellow (lead) oil paint layer .003 in. (.0076 cm) thick. The stretcher was not expanded. Each of the layers of the model painting were programmed with their respective material properties taken from the data presented earlier in the this paper. The glue layer was characterized using Equation 10 with a thermal coefficient of .000025 per degrees C and a changing modulus that varied linearly from 705 ksi (4,860 MPa) at -3°C to 550 ksi (3,791 MPa) at 23°C. This data was used to develop the equation:

$$E(T) = 689{,}875 - 7400 \times T \qquad \text{Eq. 16}$$

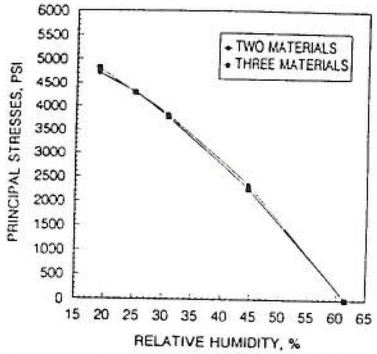

Figure 15. Calculated stresses in the image layer of the cibachrome computer model.

where T is any temperature. Equation 16 was the equation that allowed programming of the change of the rabbit skin glue's modulus with changing temperature at 5% RH. The fabric was programmed with an average constant modulus of 5 ksi (38.6 MPa) and a thermal coefficient of .00001 per degrees C. The distribution of the principal stresses in the paint layer, calculated by the computer is shown in Figure 16. This stress distribution is quite uniform, ranging from .303 ksi to .307 ksi (2.09 MPa to 2.11 MPa) and just reaches the measured breaking strength of Naples yellow paint at 5% RH and -2°C. This suggests that cracking of the paint layer is going to occur throughout the entire surface of the model painting when the temperature is decreased from 23°C to -3°C at 5% RH. It most certainly would have occurred if the starting environment was 23°C and 50% RH. This would be the consequence of the combined adverse effects of both desiccation and cooling.

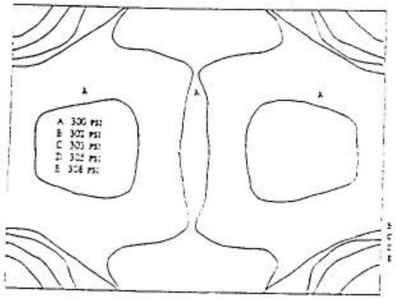

Figure 16. Computer calculated, maximum principal stress contours for a 30" x 40" traditional canvas supported oil painting subjected to cooling from 23$_\circ$C to -3$_\circ$C at low relative humidity.

The stresses have a directional bias as shown in Figure 17. In this figure, the principal stresses are indicated by the calculated directional vectors (arrows), and cracking that occurs will do so perpendicular to the vectors as shown by the continuous lines. This is a crack pattern that shows up frequently in paintings in North America, particularly those that are on strainers, i.e. stretchers with fixed, non-expandable corners. If the paint modeled was the flake white tested and reported in this paper, it would have also failed, but at a higher temperature, since this paint has a higher modulus but consistently less strength than the Naples yellow. The glue layer principal stresses reached levels ranging from 4.72 MPa to 5.13 MPa (.685 ksi to .745 ksi).

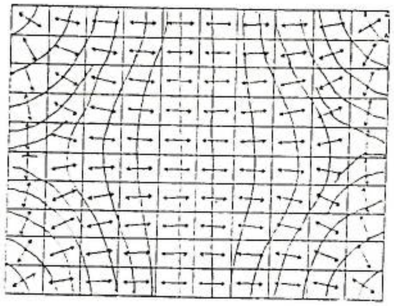

Figure 17. Principal stress directions and the projected crack pattern of a 30" x 40" oil painting subjected to cooling from 23$_0$C to -3$_0$C at low relative humidity.

Modeling the Effects of Desiccating a 30 in. x 40 in. (76 cm x 102 cm) in.) Paintings

While seemingly similar to the effects of cooling, reductions in ambient relative humidity have somewhat different effects. In starting the computer simulation of the effects of desiccation, the same 30 in. x 40 in. (76 cm x 102 cm) painting as described in the section on cooling was used. In this case the computer was programmed to alter the relative humidity instead of the temperature. Additionally, in order to obtain greater analytical detail, it was possible to take advantage of the painting's double symmetry and model only the upper right hand quadrant. This was done by programming the appropriate material properties-RH equations for each of the individual layers of the model painting as well as the proper boundary conditions. In this case, the paint layer again was Naples yellow. For the changes in relative humidity; E(RH) = 51,800 - 840xRH was the equation used. For the strain calculations, a moisture coefficient of.0000257 per percent relative humidity was used. For the glue layer, the modulus was obtained as discussed previously under RH and the strains were calculated using a moisture coefficient of.000264 per percent relative humidity. The fabric was assumed to be a minimally contributing material since glue sizing a stretched linen removes any residual stress in the fabric. In this case, the modulus was assigned a nominal constant 5 ksi (35 MPa) and the moisture coefficient was an average .0001 per percent relative humidity. The model painting was desiccated from 70% to 10% RH at 23°C. The model painting was modified to increase the glue thickness from .002 in. to .004 in. (.0051 cm to .01 cm). The results of the analysis showed the stresses in the paint layer ranging from .1 ksi (.698 MPa) to .28 ksi (1.93 MPa). Doubling the glue layer more than doubled the stresses in the paint layer. The distribution of the stresses are shown in figure 18. This distribution is not nearly as uniform as resulting from the cooling analysis. Cracking, if it occurs will initiate mainly in the corner regions of the painting as shown in figure 19. In this figure, both the direction vectors of the principal stresses and the possible crack pattern are shown. If the equilibrium breaking strength of the paint at 5% RH and 23 C was low, only .2 ksi (1.38 MPa), the cracks would run no farther than about 3 in. (7.6 cm) from the corner. Clearly it seems that low temperature has the greater potential for cracking an <u>undamaged</u> painting than low relative humidity. Changing the thickness of the glue layer influenced the stresses in the paint layer when desiccating the painting. This indicates that the very high glue layer stresses cause an interlayer interaction.

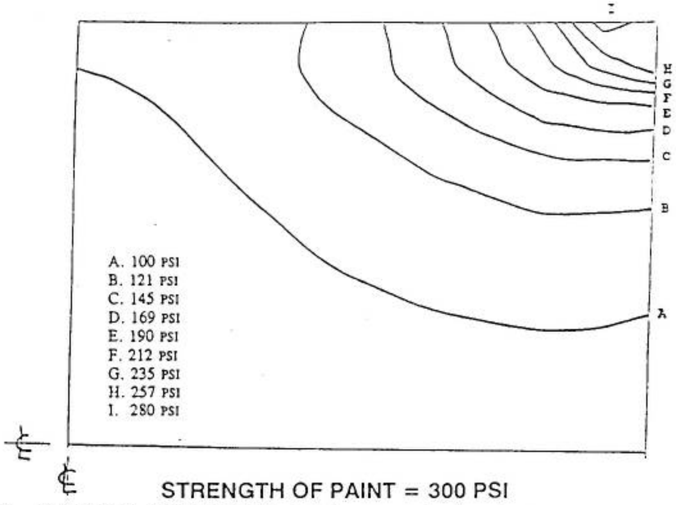

Figure 18. Computer calculated, maximum principal stress contours for a 30" x 40" traditional canvas supported oil painting subjected to desiccation from 70% to 10% relative humidity at 23$_0$C.

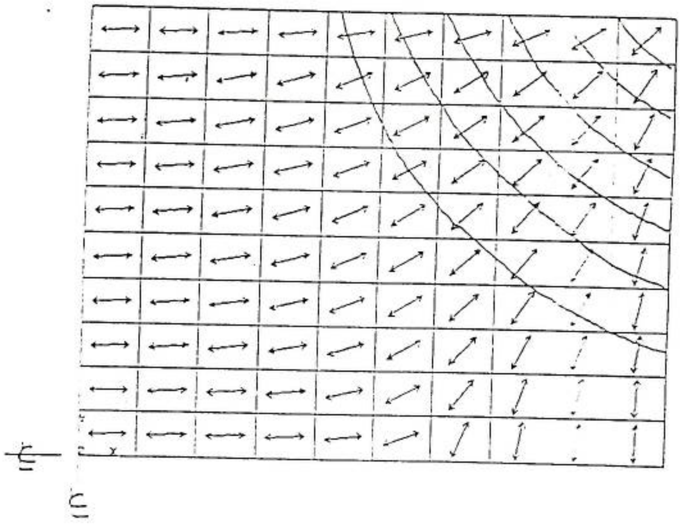

Figure 19. Principal stress directions and the projected crack pattern of a 30" x 40" oil painting subjected to desiccation from 70% to 10% relative humidity at 23$_0$C.

SUMMARY

In order to assess the risks to photographic prints and paintings associated with changes in temperature or relative humidity an accurate determination of the stresses in the different layers of a painting is necessary. In turn, those stresses must be compared to the strengths of the artists' materials at the specific environments in question and rates of loading encountered by the painting. Computer modeling in the form of finite element analysis, readily lends itself to performing the needed analysis. However, the ability to numerically model multiple layered structures such as paintings is extremely dependent on an accurate determination of the mechanical properties of the paintings constitutive materials. This includes assessing alterations to the mechanical properties brought about by time. This study has examined some of the materials typically found in a large proportion of canvas and panel paintings and concentrated on those paints such as the lead based oils that have shown to be the most brittle. Additionally the study confines the analysis to effects of temperature and relative humidity brought about over moderately long periods of time. In all of the analytical modeling, it was assumed that the materials were at equilibrium with the environment.

All of the materials respond dimensionally to the changes in temperature and relative humidity. The thermal coefficients of expansion for hide glue and lead paint were small, while their moisture coefficients were considerably higher. Restraining these materials caused stress development with desiccation and cooling, but the magnitude of the stress was a result of both the material's attempt to contract and its related modulus or stiffness.

Computer modelling of a typical canvas painting with glue size and lead oil paint layers showed that deep drops in temperature at low relative humidity (from 23° to -3°C, at 5% RH) resulted in both uniform and high stresses in the paint layer. These stresses equalled or exceeded the measured breaking strengths of thirteen year-old paint. It could be inferred that an actual painting, constructed with the materials modeled, would exhibit severe and extensive cracking over the entire surface of the painting. The same computer analysis also showed that the glue layer reaches stresses slightly more than twice those of the paint layer. It can also be concluded that severe cooling results in the independent response of the various layers of the painting with little interlayer interaction. Paint and glue tested at -3°C and 5% RH, showed extremely brittle behavior, shattering into multiple pieces when broken.

Computer modelling of the same painting, subjected to drops in relative humidity (from 70% to 10% RH), shows similar crack patterns as those found in cooling could develop but not nearly as extensive. In fact, it was not until an additional layer of glue was added to the model, that stresses in the paint layer were high enough to suggest cracking. Even then the analysis suggested that changes in RH would cause considerably less damage than cooling.

Computer modeling of the cibachrome print subjected to desiccation successfully calculated both the deflection and images layer stresses even though the deflection was large for such a thin film structure.

FOOTNOTES

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THE EFFECTS OF TEMPERATURE AND RELATIVE HUMIDITY ON THE MECHANICAL PROPERTIES OF MODERN PAINTING MATERIALS

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ABSTRACT

The mechanical properties of strength, modulus, and elongation to break were studied for artists' acrylic and alkyd paints under varying conditions of temperature and relative humidity (RH). In the ambient environment, 23° C, 50% RH, acrylic paints are very flexible and are able to sustain large deformations (>50%). Alkyd paints are much stiffer and stronger, and they cannot sustain deformations nearly as dramatic as the acrylics. Acrylic paints at 5% RH are stiffer and stronger than at 50% RH and their ability to stretch is lessened. At temperatures below 15° C at 50% RH, the strength and stiffness of acrylic paints begin to rise rapidly. Some were found to be brittle at 5° C, and by -3° C, all were brittle. At a lower RH, some acrylic paints became brittle at a temperature near 11° C. These temperatures and relative humidities may be found in the transport environment of art objects, and may render them subject to possible damage.

INTRODUCTION

The measured mechanical properties of oil paint films in different relative humidities (RH) and temperatures have recently been used to predict potential damage to oil paintings during transit. Two sets of mechanical properties are useful, those under rapid loading conditions, such as shock and vibration, and those under slow-loading conditions, in which the paintings stay equilibrated to environmental conditions. By using the experimentally determined mechanical properties of traditional oil paint, a structural analysis computer program accurately predicted cracking patterns due to vibration, edge, and corner impacts of oil paintings.[1] Stress fields and crack patterns in oil paint films subjected to dry and cool environments were also determined.[2]

Experimental techniques are being extended to determine the mechanical properties of other artists' materials, in particular modern acrylic and alkyd paints. Acrylic paints are pigmented polymer emulsions in water. In general, they are more flexible than traditional oil paints. Alkyd paints, on the other hand, are oil paints modified so that there is much more cross-linking in the paint film when dry. As a result, alkyd paints tend to be stiffer and stronger than traditional oil paints.

In this paper, the mechanical properties under rapid loading conditions of acrylic and alkyd paints in different relative humidities and temperatures are reported. These properties will be used in further studies of the sources of cracking and damage in modern paintings. Other experiments are currently being

Mat. Res. Soc. Symp. Proc. Vol. 267. 1992 Materials Research Society

MATERIALS RESEARCH SOCIETY SYMPOSIUM PROCEEDINGS VOLUME 267

Materials Issues in Art and Archaeology III

Symposium held April 27-May 1, 1992, San Francisco, California, U.S.A.

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ISBN 1-55899-162-X