

their purpose the stimulation, among the people taking part, of a common recognition, as of the gift of food, the value of tribal unity, of peace within or without the tribe. Many of the songs belonging to this class of ceremonies, although led by those who officiate in the rites, are sung by all present, men, women, and even children joining in these songs. The singing is always in unison, the personal emotion of each singer adds to the psychic force bred of a social accord, as all the people in one voice sing their thanks to the Unseen Giver of Life.

On a similar but lower plane are the songs belonging to vocations, by these not only are the hands strengthened for their task, but the unison singing of the song helps to development of social consciousness.

From what has been stated above it will be apparent that a wide field is opened by the study of Indian music not only concerning the beginnings of 'culture music' but in revealing the psychic influence of song upon the individual and upon his social development.

SOME RECENT ANTHROPOLOGICAL EXPLORATIONS

By Ales Hrdlička

DIVISION OF PHYSICAL ANTHROPOLOGY, U. S. NATIONAL MUSEUM, WASHINGTON

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In 1912 an arrangement was entered into by the Smithsonian Institution and the Panama-California Exposition at San Diego, by which it became possible to send out jointly a number of anthropological expeditions for the purpose of furnishing the Exposition with original material for its exhibits, and of serving the Smithsonian Institution and the science of anthropology by advancing knowledge in directions in which progress was especially desirable.

The objects particularly aimed at in the latter respect were the promotion of research into man's antiquity; the survey of certain parts of northern and eastern Asia in quest of possible traces of racial connection with America; and the accumulation of reliable data on child development in certain primitive races under widely differing environments. Owing to illness in the personnel of the expeditions, to the European war, and other untoward circumstances, only a part of these plans could be fully carried out; but the results are of sufficient importance to encourage further activity in the directions named.

The several expeditions and their results, briefly outlined, were as follows:

Ancient Man in Europe and Asia. Field work was done by J. Matiegka, head of the Anthropological Bureau of the Bohemian University

at Prague, on the subject of neolithic and later prehistoric man in central Europe. Other explorations were carried on by K. Stolyhwo, head of the Anthropological Institute of Warsaw in relation to: (a) ancient man of southwestern Russia; (b) ancient man of the Yenesei Valley, Siberia. And a journey was made by me for the purpose of making a survey of the various well authenticated ancient skeletal remains of man in the Museums of Europe and of the sites of their discovery.

The direct results of these expeditions were the acquisition of a series of valuable specimens, including a trephined and mineralized neolithic skull in excellent state of preservation, and of valuable data which in part have already been published¹ and in part are in preparation for publication.

The kourgans or burial mounds in northeastern Russia were found to range in age from the neolithic to the Scythic periods. They yielded some exceedingly interesting skeletal material of man as well as that of various animals, some of which are now extinct in these regions. The Yenisei caves, regrettably, gave few skeletal remains, though such could doubtless be obtained by further exploration; but they yielded objects of neolithic culture, which may be of value in future archeological comparisons with the more eastern portions of Asia and possibly even with America. The examination of the ancient skeletal finds in Europe showed, besides a series of more or less doubtful specimens, a wealth of thoroughly authenticated material relating to man's descent, and substantiating his gradual evolution, during the Quarternary, from lower primate forms.

Search in Asia for Traces of the Race that Peopled America. Besides the explorations in the Yenisei caves, above mentioned, two trips were made to northeastern Asia. The first of these, made by me, extended to the upper Yenisei and Selenga river territories and to outer Mongolia, while the second, by St. Poniatowski, head of the recently established Ethnological Institute at Warsaw, penetrated to the living tribes in the Primorskaia Oblast, Siberia. The results of these expeditions comprise anthropological data, a valuable series of photographs, and a collection of rare skeletal material. The evidence indicates that there still exist throughout extensive areas of northern and eastern Asia remnants of former prehistoric (rather than 'paleoasiatic') peoples closely resembling the American Indian in type, mentality and other important particulars. In southern Siberia, Mongolia, and the regions to the eastward, there are also extensive archeological remains in the forms of mounds and occupation sites, ranging in chronology from neolithic to historic times. The exploration of these has as yet scarcely been com-

menced, and may be expected, when properly entered upon, to reveal details of the greatest consequence to American anthropology, for here somewhere must lie the remains, cultural and skeletal, of the ancestors of those who peopled the American continent.²

Comparative Study of Child Growth among Primitive Tribes. It was planned to conduct researches in this especially important field among the aboriginal Australians, the Negrito, the African Pygmies, the Zulu, the Eskimo, and the Chinese; but illness of the available explorers, the war in Europe, and other unforeseen conditions interfered with the proposed excursions to the Australians, Pygmies, and Chinese. The Negrito of the Philippines were studied by P. Newton, demonstrator in Anatomy of Georgetown University; and four hundred individuals, including many children, were measured. Particular care was taken to reach the full-bloods, which involved some perilous journeys. A series of photographs and some interesting skeletal remains were obtained, but casting became impossible, the dampness of the climate (the journey had to be undertaken partly during the rainy season) being such that no plaster could be preserved in dry condition. The Eskimo of St. Lawrence Island, Alaska, were examined by Riley D. Moore, aid in the Division of Physical Anthropology of the National Museum; 180 individuals, including a good proportion of children and adolescents, were measured; 30 facial casts and numerous photographs were made; and an important collection of skeletal material was gathered. The data secured and the collections from this island are of particular interest, as they relate to a group of people intermediary between the American and Asiatic Eskimo. Finally, an expedition, occupying fourteen months, was made by V. Shuck, an able anthropologist of Prague, to the Zulu, the Bushmen, and some British East Africa negroes, resulting in the acquirement of facial casts of 24 Zulus and 20 Bushmen, numerous photographs and other material, and anthropometric observations on upward of 800 children and adolescents of known age. The scientific data have already been partly utilized in charts for the Exposition, and are to be reported on as soon as conditions in Europe permit. They should form an excellent basis for eventual comparisons with observations of similar nature on the American negro, and also serve, of course, for the study of further contrasts or resemblances between the black and the white man. Duplicates of all the African measurements are preserved in the National Museum.

In addition to the above, the first four months of 1913 were spent by me on an expedition to Peru, the objects of which were to conduct a greatly needed extension of former anthropological explorations in that

country, and to obtain a collection of material for illustrating prehistoric aboriginal pathology and surgery. Considerable success was met with in both directions. The anthropology of the coast was mapped out for the distance of approximately 600 miles, and some insight was obtained into that of the highlands. It was ascertained that important separate political and cultural coastal groups, such as the Chimu and the Nasca people, were no special units, anthropologically, but belonged to the same physical type as the rest of the coast population. The collections made on this trip, being selections from nearly 5000 burials, are especially valuable. Finally, the exploration made possible rich original exhibits at San Diego, covering practically the whole field of pre-Columbian Indian pathology, to which are added 60 crania showing all the forms of ancient Indian trephining. The general results of this expedition have already been published,³ but the material collected offers a rich opportunity for further investigation.

¹ Hrdlička, A., The most ancient skeletal remains of man, *Smithsonian Rept. for 1914*, pp. 491-552, pls. i-xli.

² For preliminary reports on this work, see *Smithsonian Inst., Misc. Collect.*, 60 (1912); *Compte-Rendu XIV Cong. Intern. d'Anthropologie et d'Archéologie Préhist.*, Genève, 1913; and *Trans. XVIII Intern. Cong. Americanists*, London, 1914.

³ Hrdlička, A., Anthropological work in Peru in 1913, with notes on the pathology of the ancient Peruvians. *Smithsonian Inst., Misc. Collect.*, 61, no. 18 (Publication 2264), 1914, pp. i-v, 1-69, 26 pls.

THE SECOND DERIVATIVES OF THE EXTREMAL INTEGRAL FOR A GENERAL CLASS OF PROBLEMS OF THE CALCULUS OF VARIATIONS

By Arnold Dresden

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF WISCONSIN

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1. In an earlier paper,¹ I obtained expressions for the second derivatives of the extremal-integral for the problem of minimizing the integral

$$\int F(x, y, x', y') dt, \quad (1)$$

in terms of fundamental solutions of the Weierstrass form of Jacobi's differential equation for that problem.² In the same paper these expressions were used for deriving necessary conditions which must be satisfied by a curve which is to minimize the integral (1), if one or both endpoints are allowed to vary along a curve,³ or if curves are admitted whose slopes possess a finite number of finite discontinuities.⁴

2. The method of differentiation employed in arriving at these re-