HEURISTIC MODELS FOR MATERIAL DISCHARGE FROM LANDSCAPES WITH RIPARIAN BUFFERS

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Abstract. For landscapes with riparian buffers, we develop and analyze models predicting landscape discharge based on material release by an uphill source area, the spatial distribution of riparian buffer along a stream, and retention within the buffer. We model the buffer as a grid of cells, and each cell transmits a fixed fraction of the materials it receives. We consider the effects of variation in buffer width and buffer continuity, quantify the relative contributions of source elimination and buffer retention to total discharge reduction, and develop statistical relationships to simplify and generalize the models. Width variability reduces total buffer retention, increases the width needed to meet a management goal, and changes the importance of buffer retention relative to source elimination. Variable-width buffers are less efficient than uniform-width buffers because transport through areas of below-average buffer width (particularly gaps) dominates landscape discharge, especially for narrow buffers of highly retentive cells. Uniform-width models overestimate retention, so width variability should be considered when testing for buffer effects or designing buffers for water quality management. Adding riparian buffer to a landscape can decrease material discharge by increasing buffer retention and by eliminating source areas. Source elimination is more important in unretentive or wide buffers, while buffer retention dominates in narrow, retentive buffers. We summarize model results with simpler statistical relationships. For unretentive buffers, average width is the best predictor of landscape discharge, while the frequency of gaps was best for narrow, retentive buffers. Together, both predictors explain >90% of the variance in average landscape transmission for any value of buffer retentiveness. We relate our results to ecological theory, landscape-scale buffer effects, buffer management, and water quality models. We recommend more empirical studies of buffer width variability and its effects on material discharge. Landscape models should represent width variability and the nonlinear interactions between buffers and source areas.

Key words: grid cell; landscape ecology; landscape index; model; nonpoint source pollution; nutrient discharge; raster; riparian buffer; riparian management; scaling; sediment discharge; water quality.

INTRODUCTION

Landscape heterogeneity is one focus of the emerging discipline of landscape ecology (Forman 1982, Risser et al. 1984, Turner 1989), which seeks to understand spatial patterns, their causes, and their ecological effects (Forman and Godron 1987, Forman 1995). A body of theory for some aspects of landscape ecology has been derived from first principles (Forman 1995) or other theoretical constructs (Gardner et al. 1987, Milne 1988, Gardner and O’Neill 1990, Milne 1992, O’Neill et al. 1992). The effects of heterogeneity on material transport are also a major interest for landscape ecology (Risser et al. 1984, Jordan et al. 1986, Correll et al. 1992), but less progress has been made in developing an empirical and theoretical foundation in this area. This may restrict the application of landscape ecology principles to some practical issues, such as managing nonpoint-source pollution (Loehr 1974, Duda 1982).

Landscape with riparian buffers are one case where strong effects of landscape heterogeneity on material fluxes have been demonstrated. A riparian buffer is a strip of relatively undisturbed vegetation positioned along a stream and downhill from a source of material release. Riparian forests can take up large amounts of water, sediment, and nutrients from surface and groundwater draining uphill agricultural areas (Lowrance et al. 1984, Peterjohn and Correll 1984, Jacobs and Gilliam 1985, Jordan et al. 1993). Buffers can also retain materials from other sources (Chescheir et al. 1991, Schellinger and Clausen 1992, Hanson et al. 1994, Brem 1995). With effective retention, small areas of riparian buffer can greatly reduce land discharges of nutrients to aquatic systems (Jordan et al. 1986), so there has been strong interest in managing riparian systems to reduce nonpoint-source water pollution (Lowrance et al. 1985, Correll et al. 1994, Correll 1997, Haycock et al. 1997, Lowrance et al. 1997).

Riparian buffers have been studied by using transects to observe material uptake from water traversing the buffer (Lowrance et al. 1984, Peterjohn and Correll 1984, Jacobs and Gilliam 1985, Jordan et al. 1993),
and buffer models have also focused on the transect scale (Tollner et al. 1982, Williams and Nicks 1988, Flanagan et al. 1989, Phillips 1989, Nieswand et al. 1990, Altier et al. 1994). Success in scaling transect results to landscape-level analyses and models has been mixed. Some statistical comparisons among watersheds and buffer models have also focused on the transect scale (Tollner et al. 1982, Williams and Nicks 1988, Flanagan et al. 1989, Phillips 1989, Nieswand et al. 1990, Altier et al. 1994). Success in scaling transect results to landscape-level analyses and models has been mixed. Some statistical comparisons among watersheds have found little effect of streamside vegetation on water quality (Omernik et al. 1981, Hunsaker and Levine 1995, Johnson et al. 1997). Other statistical analyses (Osborne and Wiley 1988) and spatial models (Levine and Jones 1990, Hunsaker and Levine 1995, Soranno et al. 1996) have concluded that near-stream areas do have a disproportionate effect on water quality. The importance of the riparian zone within the whole catchment remains poorly understood (Johnson et al. 1997).

In this paper, we develop and analyze a suite of mathematical models for material discharge from landscapes with riparian buffers. The models predict landscape discharge based on material release from a source ecosystem, the spatial distribution of riparian buffer, and material retention within the buffer. We analyze the models to explore how variability in buffer width interacts with buffer retentiveness to yield overall landscape discharges. We consider the effects of buffer width and continuity, quantify the contributions of source elimination and buffer retention to total discharge reduction, and suggest some statistical simplifications for scaling results to larger landscapes. We relate the results to the theory of landscape ecology, to the design of riparian buffers for reducing nonpoint-source pollution, and to nonpoint-source models for predicting total discharge from real landscapes.

**Models and Results**

**Conceptual model**

Our model considers a hypothetical landscape containing two ecosystems: an uphill source ecosystem that releases waterborne materials and a downhill riparian buffer that can take up those materials before they reach a stream. We implement the spatial model as a grid of buffer cells grouped along a stream, with water transporting materials downhill through the buffer and into the stream (Fig. 1). We model the retention of materials within the buffer by letting each cell discharge a fixed fraction, $t$, of the materials it receives. The discharge $d_i$ from a column of buffer cells of width $w$ would then be the fraction $t^w$ of the input, $i$, to that column:

$$d_i = it^w = ie^{iw}. \quad (1)$$

Thus, material uptake is a simple first-order process, and material flux decreases exponentially with the width of buffer traversed. Eq. 1 treats a buffer cell as a “black box” because we want to focus on the higher level interaction of ecosystem retention with landscape structure, not retention mechanisms. The linear retention function (Eq. 1) contributes to a mathematically tractable landscape model, and first-order response functions have been observed in buffer systems (Chesher et al. 1991, Haycock and Pinay 1993) and used in spatial models of material flux (Hunsaker and Levine 1995, Soranno et al. 1996).

**Assumptions and analyses**

We analyze a suite of related models that all share some assumptions. We assume that material release from a buffer receiving no source inputs is effectively zero, so we model only the fate of materials from the source system. We represent the landscape as a grid of cells. We assume that water enters a cell along its uphill edge and leaves along its downhill edge. We model total retention from both surface water and groundwater with a single, first-order function. We assume that all buffer cells have identical retention capabilities.

We supplement this basic framework to address specific questions. We explore the consequences of variability of buffer width by comparing uniform-width buffers to buffers that have a Poisson width distribution. In choosing the Poisson distribution, we assume that the riparian buffer is a minor component of the landscape and buffer cells are arrayed randomly and independently along the stream channel (Pielou 1977, Sokal and Rohlf 1981). We also assume that the output, $i$, from the source ecosystem is independent of buffer width. Then, we drop this last assumption and let source ecosystem area and material output decrease with increasing buffer width. This complicates the model, but allows us to evaluate the relative importance of source elimination and buffer retention in reducing total landscape discharge. Finally, we examine buffers that follow any width distribution to test the generality of conclusions about width variability from analysis of the Poisson distribution and to suggest some useful simplifications of the mathematical model.
F I G . 2. Landscape buffer transmission, $T$, vs. average buffer width for three values of cell transmission, $t$. The solid curves are for uniform-width buffers (Eq. 3), while the dashed curves are for buffers with Poisson width distributions (Eq. 10). $T$ is a fraction of the total material export from the source ecosystem to the buffer ecosystem.

**Uniform-width buffer**

For a riparian buffer of uniform width $w$, the average per column discharge from the entire landscape, $D$, is given by Eq. 1:

$$D = d_w = iw.$$  \hfill (2)

We can divide $D$ by the input from the source ecosystem to derive a relationship for the fraction of input discharged by the buffer. The average buffer transmission, $T$, for the landscape is then

$$T = D/i = t = e^{-iw}.$$  \hfill (3)

The quantity $T$ is a landscape index that summarizes the combined effects of buffer retentiveness and buffer width. This index is independent of material input from the source ecosystem and describes the fraction of any material load from the source ecosystem that would reach the stream. The fraction of materials transmitted to the stream decreases exponentially with buffer width, and the rate of decrease with increasing width is higher when individual buffer cells are more retentive (Fig. 2).

The width, $w'$, required to keep landscape discharge below some target threshold $D'$ is given by $D = iw' \leq D'$, which yields

$$w' \geq \frac{\ln D'/i}{\ln t} = \frac{\ln T'}{\ln t}.$$  \hfill (4)

where $T'$ is the target threshold expressed as average buffer transmission. Note that if the chosen threshold is zero, then cell transmission, $t$, must also be zero (L’Hopital’s rule for indeterminate forms shows that Eq. 4 approaches 0 as $t \to 0$). Values of $t$ between 0 and 1 allow landscape transmission to approach zero asymptotically with increasing width (Eq. 3). The width needed to remove half the materials entering the buffer (half-distance) is given by Eq. 4 with $T' = 0.5$:

$$w'_{1/2} = \ln 0.5/i = -0.693/\ln t.$$  \hfill (5)

**Buffer with Poisson width distribution**

**Width distribution.**—Natural riparian buffers seldom occur at uniform widths, and the widths could follow different statistical distributions. We explored the consequences of one particular distribution, the Poisson, which gives variable buffer width along the stream channel, including some zero-width gaps (Fig. 3). The distribution may be appropriate for many riparian buffers and provides a mathematically tractable way to explore the effects of variability in buffer width. The probability of any integer buffer width $w \geq 0$ is

$$p_w = \frac{\lambda^w e^{-\lambda}}{w!}.$$  \hfill (6)

where $\lambda \geq 0$ is the average width of the buffer. Initially, we let the output, $i$, from the source ecosystem be independent of buffer width. This is reasonable if buffer width is small relative to the width of the source eco-

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**Fig. 3.** The Poisson distribution. Frequency distributions of buffer width (Eq. 6) are shown for three values of average width, $\lambda$. 
system \((w_{\text{max}} \gg \lambda)\), as already assumed in choosing the Poisson distribution.

**Landscape discharge.**—The fraction of average material discharge, \(D\), contributed by all columns of width \(w\) is the frequency of width \(w\) (Eq. 6) times the discharge width \(w\):

\[
p_w d_w = \frac{\lambda^w e^{-\lambda}}{w!}.
\]

Summing over all possible buffer widths gives average per column discharge:

\[
D = ie^{-\lambda} \sum_{w=0}^{\infty} \frac{(\lambda t)^w}{w!}.
\]

The summation is a power series in \(\lambda t\) and converges to \(e^{\lambda t}\) (Beyer 1976), so Eq. 8 simplifies to

\[
D = ie^{-\lambda (1 - t)}.
\]

and the average transmission \((T = D/i)\) for the entire buffer is

\[
T = e^{-\lambda (1 - t)}.
\]

For the linear Poisson model, the landscape index \(T\) (Eq. 10) summarizes the combined effects of buffer retentiveness and the buffer width distribution. The index describes the fraction of any source ecosystem discharge that would reach the stream. This fraction decreases exponentially with average buffer width, and the rate of decrease with width is higher when buffer cells are more retentive (lower \(t\), Fig. 2). Landscape transmission (Eq. 10) is equally sensitive to changes in average width and cell transmission. To illustrate this more clearly, define cell retention, \(r = 1 - t\), so that \(T = e^{-\lambda r}\). An equal change in \(\lambda\) or \(r\) would alter \(T\) by the same amount, but \(\lambda\) has a wider range \((\lambda \geq 0)\) than \(r\) or \(t\), which must be between 0 and 1.

**Comparison to uniform-width buffer.**—For a given average buffer width, the Poisson distribution will give some stream frontage with less than average buffer width (more material discharge), but also some frontage with greater than average buffer width (less discharge). How will these two effects balance? Eqs. 3 and 10 can be used to calculate the difference between landscape transmission for Poisson buffers \((T_p)\) and the uniform-width buffers \((T_u)\) of the same average width \(\bar{w}\):

\[
T_p - T_u = e^{-\lambda (1 - t)} - e^{-\lambda (1 - \bar{w})} = e^{-\lambda (1 - \bar{w})} - e^{-\lambda (1 - t)}.\]

The two exponential terms will be equal \((T_p = T_u)\) whenever \(-\bar{w} (1 - t) = \bar{w} \ln t\). This occurs for three parameter values: \(\bar{w} = 0\) (no buffer, \(T_p = T_u = 1\)), \(\bar{w} \to \infty\) (infinitely wide buffer, \(T_p = T_u = 0\)), and \(t = 1\) (completely unretentive buffer, \(T_p = T_u = 1\)). For any other parameter values, the Poisson buffer transmits more materials than the uniform-width buffer (Fig. 2), because \(1 - t\) is greater than \(\ln t\) (see Eq. 11) for all \(0 \leq t < 1\). The difference between the uniform-width and Poisson buffers is greatest for narrow (low \(\bar{w}\)) but highly retentive \((t \approx 0)\) buffers (Fig. 4). As average width increases but cell transmission is fixed, the difference between Poisson and uniform-width buffers first increases, then peaks and declines. The average width at which the difference \((T_p - T_u)\) is greatest decreases as cell transmission declines (Fig. 4).

We can also compare uniform- and variable-width buffers by asking what minimum buffer width \((\text{either } \lambda' \text{ or } w')\) is needed to keep landscape transmission \(T\) below some threshold \(T'\). For a Poisson buffer, the average width, \(\lambda'\), such that \(T = e^{-\lambda'(1 - t)} \leq T'\) is

\[
\lambda' \geq -\frac{\ln T'}{(1 - t)}.
\]

Dividing Eq. 12 by Eq. 4 gives the ratio of the minimum average width for a Poisson buffer divided by the minimum width for a uniform-width buffer:

\[
\frac{\text{minimum Poisson average width}}{\text{minimum uniform width}} = \frac{-\ln T'/\ln(1 - t)}{\ln T'/\ln t} = \frac{\ln t}{t - 1}.
\]

The ratio is greater than 1 for all values of \(0 \leq t < 1\), so a given material reduction would require a wider average width for a Poisson buffer than for a uniform-width buffer. The ratio of needed widths is greater for more retentive buffers (lower \(t\)). The Poisson buffer must average about 1.5 times wider at \(t = 0.4\), about two times wider at \(t = 0.2\), and about four times wider at \(t = 0.02\) (Fig. 5). L’Hôpital’s rule shows that the ratio \(\to 1\) as \(t \to 0\).

Variability in buffer width has important effects on landscape discharge. When buffer width varies within the landscape, the buffer retains less material than a uniform-width buffer of equivalent average width. Inferring landscape discharge from average width only would overestimate material retention, and the error
involved would be greatest for narrow but highly retentive buffers.

**Width distribution of material loss.**—The extra material discharge through areas of below-average buffer width outweighs the extra material retention where the Poisson buffer is wider than average. We can examine this more completely by comparing the frequency distribution of buffer widths (Eq. 6) to the width distribution of material transmission. Material transmission through all columns of stream frontage of width \( w \) (Eq. 7) divided by input, \( i \), is

\[
\text{transmission through all columns of width } w = p_w \mu = \frac{\lambda \mu^w e^{-\lambda}}{w!} \quad \text{(14)}
\]

(note that \( \sum_{w=0} p_w \mu^w = T \)). For unretentive buffers (\( t \) near 1), the width distribution of material transmission (Fig. 6) is quite similar to the distribution of column width (Fig. 3, center), but becomes increasingly skewed as retentiveness increases (Fig. 6). When buffer cells are highly retentive (\( t \to 0 \)), almost all landscape discharge comes for areas with little or no buffer (Fig. 6). The frequency of gaps (zero buffer width) is given by Eq. 6 with \( w = 0 \), such that

\[
p_0 = e^{-\lambda} \quad \text{(15)}
\]

while Eq. 14 with \( w = 0 \) divided by Eq. 10 gives

\[
\text{proportion of landscape discharge through gaps } = e^{-\lambda}. \quad \text{(16)}
\]

For unretentive buffers, the proportion of discharge through gaps is the same as the proportion of gaps (Fig. 7). In contrast, almost all the discharge from retentive buffers comes through gaps (\( e^{-\lambda} \to 1 \)), even when gaps are a very small proportion of stream frontage (Figs. 3 and 6). As buffer retentiveness increases, gaps in the
buffer are increasingly the sites of material delivery to the stream (Fig. 7).

Source elimination vs. buffer retention

Adding a riparian buffer to a landscape can reduce material discharge in two ways. The buffer retains materials, and some of the source ecosystem is replaced with buffer ecosystem that does not release materials (Staver et al. 1988). The previous models did not include the trade-off between source area and buffer width, so they could not represent the effect of source elimination. Now, we enhance the model so that the buffer width in a column of stream frontage reduces the width and material release of the uphill source area.

Uniform-width buffer.—For unbuffered stream frontage, the width of source area is \( w_{\text{max}} \), and its material release is \( sw_{\text{max}} \), where \( s \) is material release per source ecosystem cell. When a buffer of width \( w \) is added, the width of source ecosystem drops to \( (w_{\text{max}} - w) \) and its material release drops by \( sw \) to \( s(w_{\text{max}} - w) \). Buffer retention further reduces this flux by \( t^\tau \) to give the final landscape discharge:

\[
D = s(w_{\text{max}} - w)t^\tau. \tag{17}
\]

Average buffer transmission, \( T \), for the landscape is average discharge (Eq. 17) divided by average input, \( I = s(w_{\text{max}} - w) \), from the source ecosystem. This gives the same expression for \( T \) derived earlier (Eq. 3). The expression for minimum buffer width to achieve a desired landscape discharge is Eq. 4. These correspondences between the enhanced and simpler models support the utility of the simpler representation.

The discharge reduction from source ecosystem replacement is the reduction in source output (\( sw \)), divided by \( sw_{\text{max}} \) to express the reduction as a fraction of the maximum possible discharge when \( w = 0 \):

\[
\text{source elimination} = \frac{sw}{sw_{\text{max}}} = \frac{w}{w_{\text{max}}} = f \tag{18}
\]

where \( f = w/w_{\text{max}} \) is the fraction of the landscape occupied by the buffer. This reduction occurs whether or not there is material retention in the buffer.

Discharge reduction by buffer retention is the input from the source ecosystem \( s(w_{\text{max}} - w) \) minus final landscape discharge \( s(w_{\text{max}} - w)t^\tau \), again divided by \( sw_{\text{max}} \) to obtain a fraction of maximum possible discharge:

\[
\text{buffer retention} = \frac{s(w_{\text{max}} - w) - s(w_{\text{max}} - w)t^\tau}{sw_{\text{max}}} = (1 - f)(1 - (t^\tau)^\gamma) = (1 - f)(1 - \tau') \tag{19}
\]

where \( w \) is replaced with \( fw_{\text{max}} \) (Eq. 18) and \( \tau = t^\infty \). Eq. 19 has a clear interpretation. The term \( (1 - f) \) is the fraction of land occupied by the source and also the fraction of maximum material export that is actually released by the source. This fraction falls linearly from 1 at \( f = 0 \) to 0 at \( f = 1 \) (Fig. 8). The second term, \( 1 - \tau' \), is the material retention capacity for a buffer of actual width \( w = fw_{\text{max}} \). The quantity \( \tau = t^\infty \) is the fraction of material transmitted through a buffer with cell transmission \( t \) and width \( w_{\text{max}} \). Thus, \( \tau \) is the minimum possible buffer transmission for the landscape, and \( 1 - \tau \) is the maximum possible retention. Raising \( \tau \) to the \( f \)th power gives the fraction of material transmitted when the buffer occupies only part of the landscape \( (w < w_{\text{max}}) \). Note that \( \tau' > \tau \) unless \( \tau = 0 \), \( \tau = 1 \), or \( f = 1 \). The difference \( (1 - \tau') \) is the realized retention capacity for a buffer of width \( w = fw_{\text{max}} \). This difference rises with increasing buffer width, starting at 0 when \( f = 0 \) (no buffer present) and curving upward to its maximum value of \( (1 - \tau) \) when \( f = 1 \) (Fig. 8). The initial rise is steeper and the curvature is greater for lower values of \( \tau \) (potentially more retentive landscapes). Multiplying the source export fraction \( (1 - f) \) by buffer retention capacity \( (1 - \tau) \) yields the actual reduction in discharge from retention (Eq. 19). As \( f \) increases from 0 to 1, buffer retention (Eq. 19) is zero at \( f = 0 \) (no buffer), rises to a maximum at intermediate \( f \), then drops back to zero at \( f = 1 \) (no source to release materials) (Fig. 8). The shape of this curve depends on \( \tau = t^\infty \). The curve rises more steeply with increasing \( f \) and reaches a higher peak at lower \( f \) when \( \tau \) is low (high retention potential) (Fig. 9).

Total material reduction is the sum of source elimination (Eq. 18) and buffer retention (Eq. 19):
Fig. 9. (Left) Discharge reduction from buffer retention in a uniform-width buffer for three values of minimum possible buffer transmission, $t$. The vertical axis gives fractions relative to the maximum possible material discharge from a landscape completely occupied by source ecosystem. The horizontal axis is the fraction of the landscape occupied by streamside buffer ($f$). Buffer retention cannot exceed source ecosystem export $(1-f)$; dashed line. (Right) Isopleths for the same three values of minimum possible buffer transmission as a function of cell transmission, $t$, and maximum landscape width, $w_{\text{max}}$.

Fig. 10. Fractions of material reduction from source elimination and buffer retention in a uniform-width buffer. The vertical axis gives fractions relative to the maximum possible material discharge from a landscape completely occupied by the source ecosystem. The horizontal axis is the fraction of the landscape occupied by streamside buffer ($f$). The solid curves show the fraction of material reduction from source elimination (Eq. 18), the fraction from buffer retention (Eq. 19), and their sum (Eq. 20). (Left) Landscape with relatively high retention potential ($t = 0.0001$). (Right) Landscape with low retention potential ($t = 0.5$).

The total discharge reduction is always 0 at $f = 0$ (no buffer) and 1 at $f = 1$ (no source). As the fraction of buffer, $f$, increases from 0 to 1, the importance of buffer retention to total reduction depends on the retention potential of the landscape. In landscapes with high retention potential (Fig. 10, left), buffer retention rises steeply as $f$ first increases above 0 so that buffer retention is a large fraction of total reduction. However, as $f$ increases further, buffer retention peaks and approaches a declining curve of slope $-1$. In this region, total material reduction changes little and further increases in buffer width merely raise source elimination while reducing buffer retention. For unretentive buffers (Fig. 10, right), buffer retention (Eq. 19) may be less than source elimination (Eq. 18) for all values of $f$. For such ineffective buffers, buffer retention is always a minor component of total reduction, and more of the benefit of adding the buffer comes from eliminating the source ecosystem.

Buffer with Poisson width distribution.—We also examined the relative importance of source ecosystem elimination and buffer retention in landscapes with buffers following a Poisson width distribution. We replaced the fixed source input term, $i$, in Eq. 7 with the expression, $s(w_{\text{max}} - w)$, for variable source input to obtain

$$p_w d_w = \frac{\lambda^we^{-\lambda}}{w!} s(w_{\text{max}} - w)^m.$$  (21)

As before (Eq. 8), we sum over all possible widths:

$$D = \sum_{w=0}^{w_{\text{max}}} p_w d_w = ne^{-\lambda} \left( w_{\text{max}} \sum_{w=0}^{\infty} \frac{(\lambda w)^n}{w!} - \sum_{w=0}^{w_{\text{max}}} \frac{(\lambda w)^n}{w!} \right).$$  (22)

The two summations are power series in $n!$ converging to $e^n$ and $n!e^n$, respectively (Beyer 1976). Eq. 22 simplifies to

$$\sum_{w=0}^{\infty} \frac{(\lambda w)^n}{w!} = e^{\lambda w}, \quad \sum_{w=0}^{w_{\text{max}}} \frac{(\lambda w)^n}{w!} = e^\lambda \left( 1 - \frac{e^{-\lambda w_{\text{max}}}}{e^{\lambda w_{\text{max}}}} \right).$$
FIG. 11. Graphical comparison of buffer retention in buffers with Poisson width distributions and uniform-width buffers. (Left) Buffer retention for the four combinations of cell transmission \( t \) and maximum width \( w_{\text{max}} \) indicated by the pairs of numbers on the figure. The vertical axis shows fractions relative to maximum discharge from a landscape completely covered by source ecosystem (no buffer). The horizontal axis is the fraction of the landscape occupied by streamside buffer \( f \). The upper curve in each pair is for a uniform-width buffer, while the lower is for a Poisson buffer. (Right) The difference between uniform-width buffering and Poisson-width buffering for each pair of curves in the left panel.

\[
D = s(w_{\text{max}} - \lambda t)e^{-\lambda(1 - t)}.
\]  

(23)

The average input to the buffer is

\[
I = \sum_{n=0}^{\infty} s(nw_{\text{max}} - w)p_n = s\left(\frac{w_{\text{max}}}{w_{\text{max}} - \lambda} \sum_{n=0}^{\infty} \frac{\lambda^n}{n!}\right).
\]  

(24)

The first summation (the Poisson distribution function) converges to 1, and the second is a power series converging to \( \lambda e^\lambda \). Average input then is

\[
I = s(w_{\text{max}} - \lambda)
\]  

(25)

and average landscape transmission is

\[
T = DI = \frac{(w_{\text{max}} - \lambda)e^{-\lambda(1 - t)}}{(w_{\text{max}} - \lambda)}. \]

(26)

Results for the Poisson model including source elimination do not differ greatly from the earlier Poisson model. For example, average buffer transmission \( T \) in the enhanced model is the expression from the simpler model (Eq. 10) times \( (w_{\text{max}} - \lambda)/(w_{\text{max}} - \lambda) \). The proportion of total discharge through gaps is the expression from the simpler model (Eq. 16) multiplied by \( w_{\text{max}}/(w_{\text{max}} - \lambda) \). Both of these multiplying ratios are very close to 1 as long as \( \lambda \ll w_{\text{max}} \), a constraint we have already accepted in choosing the Poisson distribution. The close correspondence between the simple and enhanced models again supports the utility of the simpler representation.

Discharge reductions from source elimination and buffer retention can be derived as before. The reduction from source elimination is the same as for a uniform-width buffer (Eq. 18) with \( w = \lambda \). The reduction from buffer retention is average input from the source ecosystem (Eq. 25) minus final landscape discharge (Eq. 23), divided by maximum possible discharge:

\[
\text{buffer retention} = \frac{s(w_{\text{max}} - \lambda) - s(w_{\text{max}} - \lambda t) e^{-\lambda(1 - t)}}{sw_{\text{max}}}
\]  

(27)

Total reduction is the sum of Eqs. 18 and 27:

\[
\text{total reduction} = 1 - (1 - f) e^{-\lambda(1 - t)} = 1 - (1 - f) e^{-\lambda w_{\text{max}}(1 - t)}. \]

(28)

Eq. 27 for Poisson buffer retention cannot be factored into source export and buffer retention capacity terms, and the effects of cell transmission and maximum width cannot be combined into a single parameter. These complexities preclude the simple interpretations achieved for Eq. 19, but graphical analysis demonstrates the similarities to and differences from uniform-width systems. Eq. 27 gives curves of buffer retention vs. buffer fraction like the curves for uniform-width buffers (Eq. 19, Fig. 8). Buffer retention is always less for a Poisson buffer than for a uniform-width buffer of equivalent average width (Fig. 11, left), except for special parameter values \((t = 1 \text{ or } f = 0)\) where buffer retention is zero. The decrease in buffer performance due to width variability is consistent with earlier observations (Figs. 2 and 4). The difference between Poisson and uniform-width systems depends on the buffer fraction, \( f \), and the landscape parameters cell transmission, \( t \), and maximum landscape width, \( w_{\text{max}} \). As the fraction of buffer increases from 0, the curves for Poisson buffer retention rise less steeply and reach a lower peak retention at higher \( f \) than do curves for uniform-width buffers with identical landscape parameters (Fig. 11, left). Higher cell retention (lower \( t \)) and narrower landscapes (lower \( w_{\text{max}} \)) increase the shortfall in Poisson buffer retention relative to uniform-width buffers (Fig. 11, right). The difference is greatest for values
of $f$ where buffer retention is most important (Fig. 11, right). For low values of $w_{max}$, Eq. 27 may give negative buffer retention at higher $f$. This unreasonable result occurs when terms for $w > w_{max}$ of the Poisson infinite series represent a significant fraction of stream frontage. Numerical analysis shows that more than 99.9% of the stream frontage will have buffer widths less than $w_{max}$ as long as average width, $\lambda$, is less than about half of $w_{max}$. In choosing the Poisson, we have already accepted $\lambda \ll w_{max}$, so avoiding $\lambda > 0.5 w_{max}$ does not further limit the model.

**Other width distributions**

Landscape models with buffers following uniform or Poisson width distributions were mathematically tractable, but many buffers may have less convenient distributions. We statistically analyzed models for other distributions to check the generality of our findings. We considered all the ways to distribute buffer up to 10 cells wide along a stream frontage 10 cells long. There are $11^6$ possible distributions, but some are redundant because the order of widths along the channel is unimportant. We wrote a computer program to identify the unique frequency distributions for stream lengths up to $l$ units long and buffers up to $w_{max}$ cells wide. The program uses a set of $w_{max}$ nested loops. The outermost loop steps through all possible values for the frequency of width $w_{max}$. This frequency can take values from 0 through $l$ where $l$ is the length of stream frontage. The next loop steps through all possible values (0 through $l$) for the frequency of width $w_{max} - 1$. The next loop does the frequency of width $w_{max} - 2$, and so on until the innermost loop handles the possible frequencies of width 0. Each loop is exited when the sum of frequencies exceeds the available stream frontage, so that invalid distributions are discarded. The innermost loop saves only those frequency distributions where the sum of frequencies equals available stream frontage $l$. The number of possible distributions increases geometrically with both length of stream frontage and maximum buffer width, and there are 184756 unique frequency distributions (ignoring order along stream) for the 10 by 10 landscape. For each distribution, we calculated “true” landscape transmission, $T$, from the full width distribution and discharge model by averaging estimates from $dtl = r^2$ for all widths in the distribution.

We explored statistical models for predicting the “true” landscape transmission from simpler statistics of the width distribution. We considered the mean, median, mode, variance, standard deviation, coefficient of variation (CV), skewness, and kurtosis of the width distribution and the frequencies of all width classes. We also considered the evenness (Pielou 1977) of buffer width, $E$, calculated as

$$E = \frac{1}{l} \sum_{c=1}^{l} p_c \ln p_c, \quad (29)$$

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Cell transmission $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>Evenness</td>
<td>-0.909</td>
</tr>
<tr>
<td>CV</td>
<td>0.794</td>
</tr>
<tr>
<td>Frequency of gaps (width 0)</td>
<td><strong>0.995</strong></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.547</td>
</tr>
<tr>
<td>Median</td>
<td>-0.434</td>
</tr>
<tr>
<td>Mode</td>
<td>-0.410</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.198</td>
</tr>
</tbody>
</table>

where $l$ is the number of columns of stream frontage, $c$ is an index for those columns, and $p_c$ is the fraction of all buffer cells that are in column $c$.

The strength of correlation of landscape transmission $T$ with these predictors varied with cell transmission $t$ (Table 1), when the correlations were calculated separately for different values of $t$. Six of the predictors (mean, median, mode, evenness, CV, and the frequency of zero width) were highly correlated ($r > 0.7$) with landscape transmission $T$ for some values of $t$ (Table 1). The best predictor depended on the value of $t$. For retentive buffers ($t = 0.1$ in Table 1), landscape transmission $T$ was most strongly correlated with the frequency of gaps. For unretentive buffers ($t = 0.9$ in Table 1), mean buffer width was the best predictor of $T$, while evenness was best at predicting $T$ for moderately retentive buffers ($t = 0.5$ in Table 1). Because each predictor works best for a unique range of $r$ values, combining predictors in multiple regression models gave extremely good estimates of landscape transmission for any value of $t$. Multiple regression models predicting $T$ from mean buffer width and the frequency of gaps always gave $r^2 > 0.91$ for any value of $t$. Adding evenness further raised the minimum $r^2$ for any $t$ to 0.97 (Fig. 12). The results of the statistical analysis suggest that the mathematical model might be replaced by simpler statistical relationships for some applications. This should be useful in testing the model or in modeling larger landscapes.

**Discussion**

**Effects of variability in buffer width**

Variability in buffer width across a landscape has important effects on landscape discharge. A variable-width buffer retains less material than a uniform-width buffer of equivalent average width (Figs. 2 and 4). Conversely, to achieve a target efficiency, a variable-width buffer must have a greater average width than a uniform-width buffer (Fig. 5). The difference in effi-
cell transmission, distribution. Models were fit separately for different values of adds evenness.

frequency. The dashed line is for a three-variable model that using two independent variables, mean buffer width and gap frequency. The heavy solid line is for models univariate relationships. The dominance of smaller widths increases as buffer cell retention increases (Fig. 6). Gaps in riparian buffers are important sites of material delivery, particularly in narrower or more retentive buffers (Figs. 7 and 12, Table 1). Eliminating gaps should be a high priority for buffer management and will yield greater benefit than widening the buffer elsewhere.

Variability in buffer widths should be considered when evaluating or designing buffers for water quality management. Recommended widths have been developed from empirical transect studies or from models designed to represent transects (Phillips 1989, Nieswand et al. 1990, Barling and Moore 1994, Castelle et al. 1994). Real landscapes feature winding streams, variable topography, and complex boundaries, so it will generally not be possible to maintain buffers of truly uniform width. Inferring landscape discharge from the average buffer width only (ignoring width variability and gaps) overestimates material retention, and the error is greatest for narrow, retentive buffers (Figs. 4 and 11). Buffers designed without considering width variability will probably not meet management goals.

More empirical data are needed to quantify buffer width variability and its effects on material discharge. Many studies have considered how width affects material concentration in water traversing a single transect through a buffer (see review in Lowrance et al. 1997). However, we are aware of no published studies that have quantified the distribution in buffer widths across a landscape or determined the effects on watershed discharge of among-watershed variations in the buffer distribution. This lack of information limits efforts to test model predictions, such as the effects of mean buffer width and gaps on landscape discharge (Fig. 12, Table 1), or to verify the choice of distribution function (e.g., Eq. 6) for modeling width variation.

Some management models have added a complexity in modeling buffer width not considered in our model: buffer retention is assumed to vary with slope and soil properties. These models have been used to recommend the width needed at each streamside position to achieve a uniform retention at all positions (Phillips 1989, Xiang 1996). Even with such enhancements, it is still important to acknowledge nonuniformity in achieved buffer retention and to integrate that variability across the landscape.

Source reduction and buffer retention

We separated the reduction in material discharges from establishing a buffer into two components: elimination of source areas and buffer retention (Staver et al. 1988). Buffer retention is most important relative to source elimination when narrow bands of highly retentive buffer are present (Figs. 9–11), and variability in buffer width reduced the relative importance of buffer retention in total discharge reduction (Fig. 11). For unretentive buffers, much of the benefit of having a buffer comes from removing some of the source area that supplies materials rather than from retention within the buffer. Source elimination works to reduce landscape discharge even where buffer retention might fail, as when flow bypasses the riparian zone (Denver 1991, Jordan et al. 1993, Altman and Parizek 1995). Where material reductions are due to source elimination rather than buffer retention, adding other land uses that neither release nor retain materials would have the same effect on material discharge as adding buffer ecosystem. Understanding the importance of source elimination relative to buffer retention is necessary for correctly interpreting evidence of buffer retention at the landscape scale (Ömernik et al. 1981, Osborne and Wiley 1988, Hunsaker and Levine 1995) and for weighing the costs and benefits of different buffer designs.

Simplification and scaling

Our results suggest some simplifications that may be useful for predicting the effects of riparian buffers. The need to generalize more detailed data or models to larger systems (the “scaling” problem) has been emphasized in recent articles on the theory and practice of ecology (Meentemeyer and Box 1987, Turner et al. 1989, King et al. 1991, Rastetter et al. 1992). The effects of buffer distribution were predictable from simple statistical models, which explained much of the variance in landscape transmission without using the mathematical model or the complete width distribution.
from the source ecosystem and models. A riparian buffer receives a flux of materials and water-quality indices, average buffer transmission riparian buffers. We have also defined a new landscape attribute that may be useful for predicting the function of landscapes with riparian buffers. Our results suggest some indices, such as mean buffer width and the frequency of gaps, to be efficient in estimating water quality. Several studies have included riparian zones in multivariate statistical analyses relating water-quality characteristics to water quality in the associated streams. Some have reported that land use in riparian zones had no unusual effect on watershed discharges, while others concluded that streamside area did have a disproportionate influence on water quality. None reported the overriding effects of riparian buffers seen in transect studies.

Landscape indices

The development of indices for describing landscapes has been an active focus of landscape ecology. Most of this work has sought to describe the spatial patterning of ecosystems or habitats with measures like dominance, diversity, contagion, or fractal dimension. Some of these indices have been compared to water-quality data, and there is a need for new methods to characterize landscape attributes that influence water quality. Our results suggest some indices, such as mean buffer width and the frequency of gaps, that may be useful for predicting the function of landscapes with riparian buffers. We have also defined a new landscape index, average buffer transmission, that differs from other indices in combining information on landscape pattern and ecosystem function in a single index. It may be possible to develop more such hybrid indices for describing other landscape functions.

Water-quality modeling

Our analysis has implications for effectively including riparian buffers in discharge and water-quality models. A riparian buffer receives a flux of materials from the source ecosystem and modifies that flux before it reaches a stream. Models that add the separate outputs from landscape elements cannot capture such important nonlinear interactions between ecosystems. Simple loading models may yield incorrect predictions or inferences in landscapes with riparian buffers. More complex, spatially lumped simulations of nonpoint-source pollution (e.g., Knisel 1980, Haith and Shoemaker 1987, Bicknell et al. 1993) also share this limitation. In theory, spatially distributed models (see review in Tim and Jolly 1994) could represent the source-buffer transfer, but have not been applied to the issues considered in this paper. Moreover, it is often difficult to parameterize, interpret, and verify complex distributed models. Some simulations have modeled spatial interactions along lines of flow (Levine and Jones 1990, Hunsaker and Levine 1995, Soranno et al. 1996). Enhancement and further integration of such models with landscape maps and water-quality data could incorporate some of the results and hypotheses generated in this paper.

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Landscape theory

The present analysis follows others that have used simple mathematical models to develop the body of theory and general principles underlying landscape ecology (Gardner et al. 1987, Milne 1988, Gardner and O’Neill 1990, O’Neill et al. 1992). Many important complexities of riparian buffers were not considered in our models, including: differences between surface and subsurface retention (Peterjohn and Correll 1984); effects of cover type (Correll 1997); retention mechanisms (Altier et al. 1994, Weller et al. 1994, Gold and Kellogg 1997); saturation of retention over time;
(Weller et al. 1994): variations with slope, soil type, and surface characteristics (Phillips 1989, Xiang 1996); and channelized or deep subsurface flow (Staver et al. 1988, Dillaha et al. 1989, Denver 1991, Jordan et al. 1993, Altman and Parizek 1995, Bohlke and Denver 1995, Lowrance et al. 1997). Instead, we used a simple, first-order model of retention and focused on the effects of landscape structure and the interactions among landscape elements. The approach of analyzing the behavior and practical implications of simple models has been instrumental in developing ecological theory in other areas, such as population and community ecology (May 1976). We used our model to explore one of the basic themes of landscape ecology, the effects of pattern on process (Forman and Godron 1987, Turner 1989, Forman 1995). In particular, we considered how the structure of the landscape (the distribution of source and buffer ecosystems) interacts with ecosystem function (the release or retention of materials) to give overall landscape function (net release of materials from the landscape). Our model was simple enough to focus on this interaction and yield some basic understanding and testable hypotheses. Our analysis adds simple models for material transport to the body of general theory describing the ecological effects of spatial patterning.

ACKNOWLEDGMENTS

We thank Zhi-Jun Liu, Michelle Coffee, two anonymous reviewers, and Ecological Applications editor Monica Turner for helpful comments on the manuscript. This research was supported by the Smithsonian Institution, the Smithsonian Environmental Sciences Program, National Science Foundation grants BSR-9085219 and DEB-9317968, and the National Oceanic and Atmospheric Administration’s Coastal Oceans Program Office.

LITERATURE CITED


San Francisco, California, USA.


model to account for spatial patterns of land use. Ecological Applications 6:865–878.