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## A LONG-RANGE FORECAST OF UNITED STATES PRECIPITATION

By<br>C. G. ABBOT<br>Research Associate, Smithsonian Institution


(Publication 4390)

CITY OF WASHINGTON
PUBLISHED BY THE SMITHSONIAN INSTITUTION MARCH 23, 1960
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## A LONG-RANGE FORECAST OF UNITED STATES PRECIPITATION

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## FOREWORD

A hidden family of harmonic regular periods exists in weather. The periodic members of this family persist with unchanged lengths for scores of years. By determining their average forms and amplitudes for intervals of a thousand months, successful forecasts may be made for years to come; or backcasts may be made for former years and compared to former events. Agreement of such backcasts with the records warrants confidence in future forecasts.

These claims seem preposterous to most meteorologists. Therefore, before proceeding to explain the method and to give forecasts to 1967 for 32 cities of the United States, illustrative forecasts for the years 1950 to 1958 will now be shown and compared to the records of that interval graphically.

Figures I, 2, and 3 show forecasts (dotted) and the observed march of precipitation, 1950-1958. These curves represent 3 -month running means, and are expressed in percentages of normal precipitation. Figure 1 represents precipitation at Madison, Wis., and figure 2 at Nashville, Tenn. The curve at the top of figure 2 will be described later. Figure 3 shows forecast and observation for Sacramento, Calif.

I have computed for several cities coefficients of correlation between my forecasts and the observed precipitation for the years 1950 through 1958. They are as follows: Washington, D. C., 52.3 percent; Cincinnati, Ohio, 57.3 percent ; Nashville, Tenn., 59.0 percent ; Independence, Kans., 52.0 percent ; Madison, Wis., 56.6 percent; Sacramento, Calif., 69.0 percent.

These coefficients indicate that my forecasts are over halfway toward perfect long-range prediction of weather. There still remain undisclosed variables that produce the discrepancy of about 40 percent between my coefficients and perfect correlation.

Fig. I.-Madison, Wis. Forecast and event of monthly departures from normal precipitation, 1950-1958. Normal, heavy horizontal line; forecast, dotted line; event, full curve. All from 3-month running means.
 mal, heavy horizontal line ; forecast, dotted line; event, full curve. All from 3-month running means.

Fig. 3.-Sacramento, Calif. Forecast and event of monthly departures from normal precipitation, 1950-1958. Normal, heavy horizontal line;

## FORECASTS OF PRECIPITATION FOR 32 CITIES, 1950-1967

This project was sponsored by the Association for Applied Solar Energy of Phoenix, Ariz., and the Smithsonian Institution of Washington, D. C. Funds for the costs of electronic computations were supplied to the Association by the Valley National Bank and the Arizona Public Service Company. About 7,000 tables of precipitation were electronically computed by Jonathan Wexler, a student at the Arizona State College at Tempe. He ingeniously programmed the machine for this special purpose. Monthly records of precipitation at 32 stations from about the year 1870 were taken from publications generously furnished by the United States Weather Bureau.

## Table I.-List of stations

1. Abilene, Tex.
2. Albany, N. Y.
3. Albany, Oreg.
4. Augusta, Ga.
5. Bismarck, N. Dak.
6. Charleston, S. C.
7. Cincinnati, Ohio
8. Denver, Colo.
9. Detroit, Mich.

Io. Eastport, Me.
ir. El Paso, Tex.
i2. Helena, Mont.
13. Independence, Kans.
14. Little Rock, Ark.
15. Madison, Wis.
16. Montgomery, Ala.
17. Nashville, Tenn.
18. Natural Bridge, Ariz.
19. Omaha, Nebr.
20. Peoria, Ill.

2I. Port Gibson, Miss.
22. Rochester, N. Y.
23. Sacramento, Calif.
24. Salisbury, N. C.
25. Salt Lake City, Utah
26. San Bernardino, Calif.
27. Santa Fe, N. Mex.
28. Spokane, Wash.
29. St. Louis, Mo.
30. St. Paul, Minn.
31. Thomasville, Ga.
32. Washington, D. C.

Secretary Leonard Carmichael of the Smithsonian Institution assigned Mrs. Lena Hill and Mrs. Isobel Windom to assist me in preparing forecasts. He approved grants from funds given for the study of solar radiation and weather by the late John A. Roebling. I am greatly indebted to Miss M. A. Neill for careful preparation of my manuscript.

I selected 32 cities distributed with approximate uniformity over the United States. The cities chosen are listed in table 1 .

## THE METHOD

As I suppose no one hitherto has ventured to predict values of precipitation, at definite places, for as much as 8 years in advance, I now indicate briefly how it is done. I quote apposite passages from
my former papers, ${ }^{3}$ with slight changes dictated by later experience.
Periods in sun and weather.-The sun's radiation which we see and feel, like that of many other stars, is variable. Solar output of radiation seldom exceeds 2 percent in its variation. However, its variation comprises as many as 60 regular periodic pulses, ranging from I month or less to 273 months or more. All are exact submultiples (or aliquot parts) of 273 months, as 91, 39, 7 months, and many more. They range in amplitude from $1 / 50$ to $I / 4$ percent. All go on simultaneously, like overtones of a musical note.

As many as 30 of these exact periods have been found in monthly weather records which have been kept from 1870 and earlier. They occur in records both of precipitation and temperature. Far from being confined to fractions of I percent, as in solar radiation, in precipitation they individually range from 5 to 35 percent of the normal average. In temperature they range from $\mathrm{I}^{\circ}$ to $3^{\circ} \mathrm{F}$., and these limits refer to 3 -month smoothed records. Owing to the large number of these weather periods, some in plus, some in minus phases at any one time, their combined influence is not usually startlingly great.
Normals.-Long records of weather ordinarily state "normal" monthly values found by taking the monthly averages of all the years tabulated. I have found considerable differences in normals if computed separately for years of high and low sunspot frequencies, respectively. I therefore compute separate monthly normals for years above and below an average of 20 Wolf numbers in sunspot frequency. From these normals I tabulate the departures in temperature, and the percentages of normal precipitation.

The monthly values have too wide jumps to be most useful. I smooth the record by 3-month consecutive means. Thus for February I use (January + February + March $) \times I / 3$, and similarly for other months.
Lags.-Supposing, contrary to meteorologists' opinion, that the variation of the sun is the real cause of the variation of the weather, since it has identically the same periods, I point out that well-known variations of insolation suffer variable lags in their weather influence, depending on place and time.

Lags of solar effects, as they differ with locality, indicate that the state of the atmosphere is an important factor. The atmospheric

[^0]condition varies not only with locality but with time of the year, prevalence of sunspots, and march of population. To partially meet these difficulties, I tabulate separately for three periods of the year: January-April; May-August; September-December; also with Wolf sunspot numbers above and below 20 ; also with lapse of time before and after the midpoint of the record. These divisions of the available monthly data lead to computing 220 tables at each station before undertaking a forecast.

Forecasts by periods.-My forecasts are made by adding the effects of 27 regular periodic cycles in precipitation. These cycles, like the harmonics of musical sounds, proceed simultaneously, and are integrally related to a fundamental cycle. This fundamental is 273 months. The harmonics employed are as follows:

Table 2.-Periods used for forecasting

| Fraction | Months | Fraction | Months | Fraction | Months |
| :---: | :--- | :---: | :--- | :---: | :---: |
| I/3 | 9 I | $\mathrm{I} / \mathrm{I} 2$ | $22-3 / 4$ | $1 / 27$ | $\mathrm{IO}-\mathrm{I} / 9$ |
| $\mathrm{I} / 4$ | $68-\mathrm{I} / 4$ | $\mathrm{I} / \mathrm{I} 4$ | $19-\mathrm{I} / 2$ | $\mathrm{I} / 28$ | $9-3 / 4$ |
| $\mathrm{I} / 5$ | $54-3 / 5$ | $\mathrm{I} / \mathrm{I} 5$ | $\mathrm{I} 8-\mathrm{I} / 5$ | $\mathrm{I} / 30$ | $9-\mathrm{I} / \mathrm{IO}$ |
| $\mathrm{I} / 6$ | $45-\mathrm{I} / 2$ | $\mathrm{I} / \mathrm{I} 8$ | $\mathrm{I} 5-\mathrm{I} / 6$ | $\mathrm{I} / 33$ | $8-3 / \mathrm{II}$ |
| $\mathrm{I} / 7$ | 39 | $\mathrm{I} / 20$ | $13-\mathrm{I} 3 / 20$ | $\mathrm{I} / 36$ | $7-7 / \mathrm{I} 2$ |
| $\mathrm{I} / 8$ | $34-\mathrm{I} / 8$ | $\mathrm{I} / 2 \mathrm{I}$ | I 3 | $\mathrm{I} / 39$ | 7 |
| $\mathrm{I} / 9$ | $30-\mathrm{I} / 3$ | $\mathrm{I} / 22$ | $\mathrm{I} 2-9 / 22$ | $\mathrm{I} / 45$ | $\mathrm{C}-\mathrm{I} / \mathrm{I} 5$ |
| $\mathrm{I} / \mathrm{IO}$ | $27-3 / \mathrm{IO}$ | $\mathrm{I} / 24$ | $\mathrm{II}-3 / 8$ | $\mathrm{I} / 54$ | $5-\mathrm{I} / \mathrm{I} 8$ |
| $\mathrm{I} / \mathrm{II}$ | $24-9 / \mathrm{II}$ | $\mathrm{I} / 26$ | $10-\mathrm{I} / 2$ | $\mathrm{I} / 63$ | $4-\mathrm{I} / 3$ |

The harmonic family referred to was discovered in the variation of the measures of the solar constant of radiation. Figure 4 shows 26 of over 60 periods discovered in solar variation. ${ }^{2}$ Identical cycles were later found in precipitation and temperature by study of longcontinued weather records. While the periods of the harmonics are invariable, both in the sun and weather, and their phases are invariable in solar radiation, their phases shift in weather, depending on atmospheric influences, as will be described below. On account of these phase changes, depending on several variables discovered in my studies of precipitation begun with Peoria, Ill., about io years ago, the harmonic family in weather is obscured and hidden, and is as yet unrecognized by most meteorologists. Nevertheless it is verified by an enormous mass of evidence, as will appear below.

No observations required.-Many meteorologists and others suppose that my method of long-range weather forecasting depends on solar observations, but this is not so. The harmonic family referred

[^1]
Fig. 4.-Twenty-six periods in solar variation, ranging from $273 / 67$ to $273 / 2$ months in length, all cleared of subordinate interfering integrally related periods. All from Smithsonian solar-constant observations of 1920-1952.
to was indeed discovered by the study of over 30 years of daily "solarconstant" observations of the Smithsonian Astrophysical Observatory. But now that the harmonic family has been found in weather, no observations of any kind are required. It is only necessary to employ a long record of monthly mean values of precipitation, or temperature, to make long-range predictions. These are approximately verified if no unusual alterations of atmospheric conditions make the averages from long records inapplicable.

Sports.-As my forecasts depend on the assumption that the average conditions of the periods over a thousand months will be projected into the future, it is important not to include wild "sport" values of precipitation in the thousand-month basis. Hence I have diminished sporadic very high values to about two times normal, and have raised sporadic drought values of less than 40 percent of normal to exceed that limiting low value. These limits refer to 3 -month smoothed records. For most of my 32 stations these changes are very rare. But in two or three of the desert stations possibly one value in ten was changed to avoid spoiling the representative character of the basis. The considerable measure of success of my forecasts is the main defense of the method used to produce them. If the degree of success is found to be valuable, no doubt those who in future will use the method will greatly improve it by modifications dictated by reason and experience.

Backcasts.-Since my forecasts are made by adding the average effects of 27 harmonic periods over an interval of about 1,000 months, the 12 months of record for any one year can produce only about I percent of influence on the forecast for that year, even if those 12 months are among the thousand months employed as a basis. Therefore all forecasts or backcasts are equally sound, whether they relate to time before, within, or after the thousand months of record. ${ }^{3}$

The preceding paragraph is important. The forecasts for 32 cities all extend from 1950 to 1967 . The degree of similarity between the forecasts and what happened up to 1958 is the index of their probable agreement from 1959 to 1967.

The 273-month period.-Daily solar-constant observations proceeded from I920 to 1952 at Montezuma, Chile. This interval is not long enough to determine the master period accurately. But the IO-I/9-month period in weather is a strong one and has long been followed in Washington precipitation. I determined its amplitude for several periods differing slightly from $10-\mathrm{I} / 9$ months. For this pur-

[^2]pose I used about 790 monthly mean values of Washington precipitation, all observed when Wolf sunspot numbers exceeded 20. These values were smoothed by 3 -month consecutive means, which of course reduces the ranges of percentage departures from normal to about two-thirds of their actual monthly values. Table 3 and figure 5 show the results.

Figure 5 clearly shows that a value of the master period between 273 and 275 months is definitely indicated.

I have preferred 273 months rather than 275 months because it is an integral multiple of the strong periods $7,13,39$, and 91 months. It cannot be much more than $1 / 3$ percent from the true master period.

Table 3.-Percentage amplitudes of proposed periods

| Period Months | 105.7 | 103.4 | 102.5 | 100.7 | 100.9 | 96.3 | 97.3 | 97.9 | 98.0 | 97.7 | RangesPercent 9.4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 271.2 |  |  |  |  |  |  |  |  |  |  |  |
| 27 |  |  |  |  |  |  |  |  |  |  |  |
| 273.0 | 95.7 | 95.8 | 93.4 | 96.1 | 99.3 | 102.0 | 103.7 | 108.0 | 104.8 | 101.1 | 14.6 |
| 27 |  |  |  |  |  |  |  |  |  |  |  |
| 275.0 | 109.8 | 102.4 | 103.3 | 99.3 | 95.4 | 92.9 | 96.2 | 97.6 | 98.8 | 104.5 | 16.9 |
| 27 |  |  |  |  |  |  |  |  |  |  |  |
| 277.0 | 94.6 | 104.4 | 106.2 | 101.3 | 105.8 | 105.5 | 94.6 | 97.5 | 96.9 | 93.3 | 12.9 |
| 27 |  |  |  |  |  |  |  |  |  |  |  |

The subordinate periods.-Of the 27 periods used in forecasting, 12 exceed $15-1 / 6$ months in length. Owing to arrangements used to treat changes of phase, which will be described, 42 tabulations for each city are made of these 12 periodicities. Almost without exception the curves representing these 42 tables betray overriding harmonics of the period in question, from two to eight in number. These overriders must be evaluated and eliminated before the period in question stands free.

I show in table 4 and figure 6 the treatment of one only of the four tables representing the 39 -month period in precipitation at Helena, Mont. Eight tabulations of successive runs of this period over the interval of years 1891 to 1917 give the mean values and average deviations from the mean in percentages of normal precipitation. Then five harmonics of 39 months are successively removed, yielding the smooth-curve deviations from 100 percent given in column $S$, and its deviations from what remains after the five removals of harmonics. In the final column of table 4 , and the final smooth curve of figure 6, we see the real periodicity of 39 months. The average deviation from
curve $a$ is 29.6 percent, and that from curve $b$ is 2.I percent. The reduction of 93 percent in deviation is due to removing exact harmonics of 39 months.

Overriding periods.-As another example I quote from footnote $\mathrm{I}, \mathrm{g}$, cited above, showing figure 4 of that reference (here figure 7).


Fig. 5.-Demonstration of 273 -month master period in weather.
From the mean of 16 repetitions of the periodicity of $45-\mathrm{I} / 2$ months in Natural Bridge precipitation, the true $45-1 / 2-$ month period is cleared of four overriding harmonics. ${ }^{4}$ The reader will note what similarity to true sine curves is attained in both the above examples,

[^3]when overriding harmonics are computed and removed. From the examples given (out of about 10,000 cases available in my files) the following io exact harmonics of 273 months are exposed as follows:
$$
\mathrm{I} / 4, \mathrm{I} / 7, \mathrm{I} / 8, \mathrm{I} / \mathrm{I} 2, \mathrm{I} / \mathrm{I} 4, \mathrm{I} / 2 \mathrm{I}, \mathrm{I} / 28, \mathrm{I} / 35, \mathrm{I} / 49, \mathrm{I} / 56 .
$$


Fig. 6.-Helena, Mont. Thirty-nine-month period in precipitation as cleared of overriding subordinate integrally related periods. Original tabulation, $a$; cleared curve, $b$, with smoothcd curve above. Note approximate sine form. Range, 27 percent of normal precipitation.

While most removals of harmonic riders are done to clear periods exceeding $15-\mathrm{I} / 6$ months, many curves representing periods between $9-\mathrm{I} / \mathrm{IO}$ and $\mathrm{I} 5-\mathrm{I} / 6$ months required removal of harmonics of $\mathrm{I} / 2$ or $1 / 3$ of their length. An algebraic theorem affords a check on mistakes of computation when clearing half periods.

Let a periodic curve be represented by equally spaced ordinates $\mathrm{a}, \mathrm{b}, \mathrm{c} \ldots \mathrm{k}, \mathrm{l}, \mathrm{m}$, and proceeding further, $\mathrm{n}, \mathrm{o}, \mathrm{p} \ldots \mathrm{x}, \mathrm{y}, \mathrm{z}$.

The mean form of the supposed overriding period of one-half length is:

$$
\frac{a+n}{2}, \quad \frac{b+0}{2}, \quad \frac{c+p}{2}, \ldots \frac{k+x}{2}, \quad \frac{l+y}{2}, \quad \frac{m+z}{2} .
$$

When this half-length curve is written twice, and subtracted, we have:

$$
\frac{a-n}{2}, \quad \frac{b-0}{2}, \quad \frac{c-p}{2}, \quad \ldots \quad \frac{k-x}{2}, \quad \frac{l-y}{2}, \quad \frac{m-z}{2},
$$

and following that:

$$
\frac{n-a}{2}, \quad \frac{o-b}{2}, \quad \frac{p-c}{2}, \quad \cdots \quad \frac{x-k}{2}, \quad \frac{y-l}{2}, \quad \frac{z-m}{2} .
$$

So the last half of the long curve, when cleared of the period of onehalf of its length, is exactly like the first half, but with reversed signs.

Grouping of periods.-All weather influences caused by changes in solar rays are subject to lags. For instance, June and noonday are times of highest solar altitudes, but the warmest months and hours occur later. The lag is longer the longer the period of the solar radiation change. These lags are due to atmospheric conditions, and vary from locality to locality, from month to month, from times of great sunspot activity to quiet solar times, and as population and forestation change. Hence, though the family of periods integrally related to 273 months proceeds with perfect regularity in measures of the solar constant, in weather the same family of periods is affected by changes of phase, depending on the locality, the population, the sunspot frequency, and the time of the year. The periods are the same in weather that they are in solar radiation, but owing to complex atmospheric influences on the lags the weather phases are so altered from time to time that these periods are unrecognizable without a segregation of the data, governed by consideration of these modifying influences.

It is not possible to anticipate and allow for these phase changes precisely. I content myself as follows:
(a) The year divided: January to April; May to August; September to December.
(b) Solar activity divided: Wolf numbers $>20$; Wolf numbers $<20$.
(c) Secular time divided: first half of tabulated records; second half thereof.

All these divisions of data hold for periods up to $15-1 / 6$ months, or 15 groupings for these periods. The segregation according to the Wolf numbers holds from $18-1 / 5$ months up to 39 months, but not the segregation for times of the year.


Fig. 7.-Natural Bridge, Ariz. Forty-five-and-one-half-month period in precipitation cleared of overriding subordinate integrally related periods. Range reduced ninefold by clearing.

Table 4.-From three-month running means of precipitation, Helena, Mont. 39 -month period $=p$

Elimination of subordinate periods. Interval 1891-1917
Mean percentages of the normal. Original mean and departures after removing subordinate periods

| Original 8 deter-mina- | Average deviaPercent of normal | Removed periods |  |  |  |  | Smooth | ${ }_{\Delta}^{p / 8}-S$ | $\begin{aligned} & \text { Final } \\ & \text { mean } \\ & \text { cleared } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| tions | normal ppt | p/2 | p/3 | p/5 | \$/7 | $p / 8$ |  |  |  |
| 102 | 30 | 0 | +2 | -9 | -13 | -17 | -12 | -5 | 88 |
| 99 | 40 | -8 | -8 | -10 | -II | -II | -13 | +2 | 87 |
| 92 | 29 | -24 | -26 | -18 | -17 | -17 | -14 | -3 | 86 |
| 82 | 35 | -35 | -32 | -20 | -15 | -12 | -14 | +2 | 86 |
| 82 | 29 | -37 | $-27$ | -15 | -15 | -17 | -I4 | -3 | 86 |
| 89 | 32 | -16 | -9 | -9 | -11 | -15 | -13 | -2 | 87 |
| 112 | 34 | -2 | 0 | -9 | -13 | -13 | -13 | 0 | 87 |
| III | 31 | +2 | 2 | -12 | -13 | -13 | -12 | -I | 88 |
| III | 29 | -6 | -4 | -15 | -14 | -II | -10 | - | 90 |
| 98 | 38 | -9 | -II | -13 | -8 | -10 | -8 | -2 | 92 |
| 109 | 23 | 0 | -7 | +1 | +1 | -3 | -6 | +3 | 94 |
| 109 | 30 | -8 | -12 | 0 | -4 | -4 | -4 | - | 96 |
| 112 | 40 | -I | -II | +1 | o | o | -1 | +1 | 99 |
| 110 | 40 | -9 | -7 | -7 | -6 | -3 | 0 | -3 | 100 |
| 107 | 18 | +9 | +9 | o | +5 | +3 | +2 | +1 | 102 |
| III | 19 | +11 | +9 | +4 | +4 | 0 | +4 | +4 | 104 |
| 112 | 34 | +16 | +19 | +8 | +6 | +6 | +6 | - | 106 |
| 119 | 26 | +7 | +17 | +15 | +II | +II | +7 | +3 | 107 |
| 97 | 34 | -13 | -6 | +2 | +1 | +4 | +9 | -5 | 109 |
| 100 | 23 | -6 | -4 | +8 | +9 | +7 | +9 | -2 | 109 |
| 102 | 27 | 0 | -4 | +8 | +13 | +9 | +II | -2 | III |
| 116 | 20 | +9 | +II | +1I | +11 | +II | +11 | 0 | III |
| 146 | 16 | +25 | +13 | +14 | +10 | +10 | +12 | -2 | 112 |
| 152 | 20 | +35 | +28 | +18 | +17 | +20 | +13 | +7 | 113 |
| 156 | 33 | +37 | +33 | +22 | +23 | +19 | +13 | +6 | 113 |
| 122 | 34 | +17 | +7 | +5 | +10 | +10 | +13 | -3 | 113 |
| 117 | 21 | +3 | +5 | +13 | +13 | +13 | +12 | +1 | II2 |
| 108 | 16 | -I | -1 | +11 | +9 | +12 | +II | +1 | III |
| 123 | 27 | +6 | +4 | +16 | +12 | +10 | +9 | +1 | 109 |
| 117 | 21 | +10 | +13 | +13 | +12 | +8 | +8 | 0 | 108 |
| 109 | 28 | $\bigcirc$ | +10 | +1 | +2 | +2 | +4 | -2 | 104 |
| 125 | 37 | +8 | +15 | -3 | +2 | +2 | +2 | 0 | 102 |
| 115 | 26 | +2 | +4 | -7 | -7 | -4 | -I | -3 | 99 |
| 128 | 32 | +9 | +5 | +3 | -1 | -3 | -3 | - | 97 |
| 89 | 32 | -9 | -7 | +1 | 0 | -4 | -4 | $\bigcirc$ | 96 |
| 90 | 33 | -10 | -12 | - | +1 | +1 | -6 | +7 | 94 |
| 81 | 30 | -15 | -24 | -12 | -7 | -7 | -7 | 0 | 93 |
| 106 | 50 | -6 | -10 | -10 | -10 | -7 | -9 | +2 | 91 |
| 123 | 30 | +13 | +3 | -6 | -8 | -10 | -10 | 0 | 90 |

Mean da 29.6 percent.
Average deviation before clearance 29.6 percent.
Mean $\Delta$ 2.I percent.
After clearance 2.1 percent.
Note.-Thus the removal of overriding harmonics reduces the average deviation by 93 percent. Of about 10,000 such removals of overriding harmonic periods, probably 4,000 gave fully as satisfactory end results as the 39 -month curve at Helena did for the years 1891 to 1917.

Hence for these longer periods there are about four divisions to a period. The secular time segregation holds beyond 39 months, two divisions each for four periods.

The grouping just indicated leads to computing many tables for each station:

> Up to $15-\mathrm{I} / 6$ months, $15 \times 12=180$ tables
> Thence to 39 months, $8 \times 4=32$ tables
> Thence to 91 months, $4 \times 2=8$ tables
> Total $\ldots \ldots \ldots \ldots \ldots \ldots . \ldots 220$ tables

Shifts of phases.-The numerous groups used for the shortest 15 periods leads to tabulations with so few columns that the mean values of individual periods are of little weight. To remedy this defect, I assume that the forms and amplitudes of periods up to I5-1/6 months in length, and in the same grouping as regards Wolf numbers, will be similar, though in different phase relations. I therefore make superposed graphs of the six tables of one period for each of the two stated conditions of sunspot activity. From inspection checked numerically I am then able to shift the individual curves of the graphs to the same phase relations. Then I take a mean for all six tables and use that generalized mean in forecasting. But when using it in forecasting, I must shift back the generalized mean to the proper phase, as will appear by an example later. Figure 8 gives an example of these shiftings in phase.

## NOMENCLATURE, SYMBOLS, AND TIME

As stated above, 27 periods, all aliquot parts of 273 months, are to be used in the forecasts. But, as just stated, these are used in several groups, depending on the length of the periods. Lags, depending on atmospheric conditions, dictated tabulations of 12 independent groupings for the periods of shortest length, that is $a_{1}, b_{1}, c_{1}, a_{2}, b_{2}, c_{2}$, as tabulated for the period of $9-\mathrm{I} /$ Io months of $\mathrm{SS}>20$ in tables 5 and 6. Besides these, there are six tables $a_{1}^{\prime}, b_{1}^{\prime}, c_{1}^{\prime}, a_{2}^{\prime}, b^{\prime}{ }_{2}, c^{\prime}{ }_{2}$, for $\mathrm{SS}<20$. However, for periods above $15-\mathrm{I} / 6$ months this extended grouping brings too few columns into the tables to be capable of yielding satisfactory mean values. Hence for periods $273 / 15$ to $273 / 7$, the distinction between months of the year is dropped, thus reducing the number of groupings from i2 to 4 for these 8 periods. For the remaining 4 periods, $45-\mathrm{I} / 2$ to 9 I months, the distinction $\mathrm{SS}>20$ or $\mathrm{S} S<20$ is also dropped, reducing their groupings to 2 . So there are three different arrangements of assembly, as just explained ( $12 \times 15$ )
$=180+(8 \times 4)=32+(4 \times 2)=8$, making 220 separate tabulations in all.
In tables of periods $1 / 18$ to $1 / 63$ of 273 months, there are many cases when the number of columns for $a_{1}, b_{1}, c_{1}, a_{2}, b_{2}, c_{2}$, and $a_{1}^{\prime}, b_{1}^{\prime}$,


Fig. 8.-Sixfold grouping of periods to form generalized means.
$c^{\prime}{ }_{1}, a_{2}^{\prime}, b^{\prime}{ }_{2}, c^{\prime}{ }_{2}$ are too few to give a trustworthy mean. Accordingly, as I stated above, I have made the assumption that in form and amplitude groups of $\mathrm{SS}>20$ will be fairly similar, though of different phases, and in form and amplitude groups for $\mathrm{SS}<20$ will also be
fairly similar, though differing in phases. Making this assumption, I combine into one table $a_{1} b_{1} c_{1} a_{2} b_{2} c_{2}$, merely changing the phases to give best accord, and similarly I combine $a_{1}^{\prime} b^{\prime}{ }_{1} c^{\prime}{ }_{1} a^{\prime}{ }_{2} b^{\prime}{ }_{2} c^{\prime}{ }_{2}$ into a single

Table 5.-Grouping of six tables when $S S>20$. Period of 9-I/io months. Eastport, Me.

First half of the records, 189 I to 1920

| 1893 Apr. | ${ }_{\text {l }}{ }^{1894}$ Jan. | 1896 Apr. Aps | $\begin{aligned} & { }_{\text {Jan. }}^{897} \end{aligned}$ | ${ }_{\text {Feb }}^{1906}$ | 1909 ${ }_{\text {Mar. }}$ | Mrs. | J916. | $\begin{aligned} & 1918 \\ & \text { Apr. } \end{aligned}$ |  | $\underset{a_{1}}{\text { Means }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 64 | 92 | 55 | 62 | 130 | 158 | 79 | 84 | 70 | 72 | 87 |
| 71 | 47 | 56 | 92 | 164 | 127 | 91 | 66 | 85 | 80 | 88 |
| 79 | 54 | 98 | 104 | 180 | 87 | 127 | 79 | 96 | 78 | 98 |
| 103 | 62 | 98 | 179 | 156 | 81 | 129 | 69 | 95 | 109 | 108 |
| 103 | 89 | 106 | 173 | $1{ }^{1} 6$ | 80 | 120 | 96 | 108 | 93 | 108 |
| 91 | 78 | 123 | 166 | 64 | 133 | 92 | 123 | 108 | 101 | 108 |
| 51 | 72 | 144 | 108 | 55 | 120 | 66 | 127 | 107 | 86 | 94 |
| 89 | 47 | II9 | 88 | 70 | 134 | 54 | 109 | 77 | II4 | 90 |
| 88 | 59 | 80 | 61 | 81 | 78 | 78 | 79 | 70 | 116 | 79 |
|  |  |  |  |  |  | 75 |  |  |  |  |


| ${ }_{\text {June }}^{1892}$ | ${ }_{\text {July }}{ }^{1805}$ | ${ }_{\text {July }}^{1808}$ | 1904 Aug. des | ${ }_{\text {May }}^{1905}$ | ${ }_{\text {Aug. }}^{1907}$ | $\begin{aligned} & 1908 \\ & { }_{3} 900 \end{aligned}$ | ${ }_{\text {July }}^{1917}$ | ${ }_{\text {July }}^{1920}$ | $\underset{\substack{\text { Means } \\ b_{1}}}{ }$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 86 | 115 | 72 | 160 | 81 | 132 | 70 | 167 | 64 | 105 |
| 114 | 120 | 67 | 162 | 103 | 131 | 76 | 94 | 79 | 105 |
| 85 | 94 | 134 | 103 | 90 | 128 | 82 | 115 | 88 | 102 |
| 86 | 94 | 164 | 66 | 102 | 126 | 103 | 77 | 85 | 100 |
| 59 | 98 | 159 | 77 | 75 | II4 | 77 | 89 | 79 | 92 |
| 65 | 94 | 117 | 93 | 94 | 128 | 108 | 72 | 75 | 94 |
| 69 | 65 | 94 | 82 | 83 | 126 | 130 | 100 | 84 | 92 |
| 75 | 70 | 109 | 53 | 109 | 119 | 168 | 96 | 86 | 98 |
| 71 | 72 | 86 | 57 | 95 | 112 | 178 | 94 | 103 | 97 |
| 75 |  |  |  |  | 78 |  |  |  |  |


| ${ }^{1891}$ | ${ }^{1894}$ | ${ }^{1897}$ | ${ }^{1903}$ | ${ }^{1006}$ | ${ }^{1916}$ | ${ }^{1919}$ | Means |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 128 | 67 | 80 | 96 | 123 | 77 | 118 | 98 |
| 100 | 72 | 85 | 94 | 131 | 88 | 104 | 96 |
| 99 | 71 | 129 | 104 | 148 | 95 | 87 | 105 |
| 105 | 68 | 146 | 130 | 135 | 106 | 119 | 115 |
| 104 | 80 | 151 | 142 | 143 | 101 | 140 | 123 |
| 117 | 83 | 147 | 128 | 116 | 104 | 172 | 124 |
| 81 | 79 | 105 | 86 | 99 | 95 | 119 | 95 |
| 89 | 63 | 112 | 70 | 96 | 150 | 92 | 96 |
| 90 | 73 | 73 | 115 | 101 | 134 | 55 | 92 |

table. I give samples of this simplification here in tables 5 and 6 for $\mathrm{SS}>20$. Figure 9 shows the matter graphically.

The final combinations of two sets of six tables each, with phases shifted to harmonize, is given in figure 9 and table 7, both from Eastport data.

Table 6.-Grouping of six tables when $S S>20$. Period of $9-1 / 10$ months. Eastport, Me.

Second half of the records, 1925 to 1956

| ${ }_{\text {Feb. }} 19$. | $\stackrel{1928}{\text { Feb. }}$ | $\stackrel{3937}{\text { Mar. }}$ | 1940 Apr. l | ${ }_{\text {la }}^{194 .}$ | ${ }_{\text {Feb. }} 1947$ | ${ }^{1950}$ Feb. | [1956 | $\begin{gathered} \text { Means } \\ a_{3} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 77 | 76 | 44 | 76 | 48 | 127 | 116 | 106 | 84 |
| 68 | 72 | 66 | 71 | 50 | 116 | 116 | 129 | 86 |
| 64 | 82 | 89 | 63 | 50 | 136 | 87 | 119 | 86 |
| 69 | 96 | 77 | 75 | 68 | 166 | 117 | II3 | 98 |
| 83 | 109 | 63 | 126 | 54 | 198 | 107 | 89 | 104 |
| 91 | 129 | 74 | 117 | 91 | 134 | 162 | 95 | 112 |
| 93 | 129 | 95 | 128 | 93 | 88 | 114 | 88 | 103 |
| 124 | 119 | 102 | 83 | 101 | 50 | 102 | 93 | 97 |
| 122 | 87 | 67 | 84 | 105 | 69 | III | 101 | 95 |


| 1926 Aug. | ${ }_{\text {May }}^{1927}$ | ${ }^{1929}$ Aug. | 1930 May | ${ }_{\text {I }} 1936$ | ${ }_{\text {I }} 1939$ | ${ }^{1945}$ | ${ }^{1946}$ | 1948. | 1949 May | ${ }_{\text {I }}^{1951}$ Aug. | ${ }_{\text {1952 }}^{\text {May }}$ | ${ }_{\text {June }}^{1955}$ | $\underset{\substack{\text { Means } \\ b 2}}{ }$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 63 | 95 | 92 | 55 | 97 | 87 | 100 | 134 | 94 | 90 | 159 | 130 | 60 | 97 |
| 136 | 135 | 93 | 95 | 91 | 78 | 66 | 83 | 64 | 71 | 135 | 12I | 80 | 96 |
| 145 | 185 | 87 | 107 | 77 | 73 | 110 | 68 | 104 | 54 | 139 | 131 | 89 | 105 |
| 150 | 189 | 74 | 100 | 101 | 84 | 157 | 80 | 130 | 104 | 159 | 99 | 87 | 116 |
| 92 | 180 | 76 | 55 | 92 | 103 | 170 | 82 | 138 | II | 200 | 105 | 87 | 15 |
| 90 | 178 | 104 | 41 | 112 | 65 | 130 | 75 | 127 | 153 | 184 | 81 | 65 | 108 |
| 85 | 167 | 103 | 51 | 104 | 61 | 127 | 112 | 127 | 111 | 145 | 95 | 113 | 108 |
| 70 | 119 | 101 | 65 | 91 | 56 | 119 | 119 | 116 | 138 | 107 | 110 | 102 | 101 |
| 70 | 91 | 69 | 76 | 52 | 79 | 166 | 145 | 108 | 113 | 96 | 136 | 126 | 102 |
|  |  |  | 84 |  |  | 154 |  |  |  |  |  |  |  |


| 1925 <br> Nov | lig28 | ${ }_{\text {lec }}^{1934}$ Dec. | 1935 Sept. | ${ }_{\text {dec. }}^{1937}$ Dec. | ${ }_{\text {l }}^{1938} \mathrm{c}$ ct. | ${ }_{\text {l }}$ | 1947 Nov. | Novo | ${ }_{\substack{\text { Means } \\ c 3}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 116 | 72 | 125 | 71 | 56 | 110 | 79 | 74 | 181 | 98 |
| 91 | 76 | III | 89 | 56 | 73 | 84 | 120 | 194 | 99 |
| 99 | 84 | 102 | 66 | 59 | 65 | 61 | 98 | 165 | 89 |
| 112 | 93 | 63 | 85 | 70 | 74 | 59 | 101 | 114 | 86 |
| 96 | 93 | 54 | 60 | 76 | 92 | 87 | 88 | 139 | 87 |
| 82 | 112 | 62 | 102 | 104 | 125 | 8 I | 141 | 150 | 107 |
| 81 | 95 | 68 | 103 | 131 | 109 | 86 | 146 | 144 | 107 |
| 73 | 80 | 81 | 119 | 131 | 93 | 70 | 176 | I32 | 106 |
| 71 | 63 | 89 | 117 | 146 | 94 | 81 | 121 | 149 | 103 |
|  |  |  |  | 112 |  |  |  |  |  |

The meaning of the symbols on figure 9 is as follows:
ok, no shift.
$\uparrow$, shift backward.
$\downarrow$, shift forward.
Subscripts, number of months shifted.

Table 7.-Phase adjustment. The 6-1/15-month period
Division $=$ Time before and after 1900.
Category $=$ Records when Wolf sunspot numbers $\gtrless 20$.
Phase shifts indicated: ok, $\uparrow_{N}, \downarrow^{N}$, drawn dotted below.
Basis of forecast, over 1 ,ooo montbly records smoothed by 3 -month running means. Forecasts employ 27 periods all exact submultiples of 273 months.
Phases shift with changing atmospheric conditions, but periods remain, and are of the exact lengths found solar variation. It requires 220 tables electronically computed to make a forecast for one station.

| Category 2 Assembly |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{1} \uparrow_{1}$ | $b_{10 k}$ | $c_{1} \uparrow_{3}$ | $a_{20 k}$ | $b_{20 k}$ | $c_{20 k}$ | Mean | $\Delta$ |
| 105 | 95 | 108 | 100 | 96 | 95 | 100 | +1 |
| 102 | 95 | 106 | 93 | 97 | 88 | 97 | -2 |
| 100 | 94 | 95 | 92 | 93 | 80 | 92 | -7 |
| 106 | 92 | 104 | 97 | 94 | 88 | 97 | -1 |
| 105 | 103 | 98 | 106 | 100 | 91 | 101 | +2 |
| 103 | 114 | 108 | 105 | 101 | 99 | 105 | +6 |
|  |  |  |  |  |  | $6 \longdiv { 5 9 3 }$ |  |


| $a_{1} \uparrow_{3}$ | $b_{1} \downarrow_{1}$ | ciok | $a_{20 \mathrm{k}}$ | $b_{2} \uparrow_{2}$ | crok | Mean |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 120 | 95 | 100 | 105 | 96 | 117 | 107 |
| 108 | 86 | 100 | 91 | 94 | 110 | 98 |
| 110 | 86 | 100 | 90 | 92 | 100 | 95 |
| 92 | 97 | 102 | 87 | 84 | 88 | 91 |
| 105 | 91 | 104 | 91 | 90 | 93 | 93 |
| 110 | 98 | 103 | 102 | 94 | 97 | 101 |
|  |  |  |  |  |  | $\begin{array}{r} 6 \longdiv { 5 8 6 } \\ 98 \end{array}$ |



Fig. 9.-Phase shifts in sixfold grouping of periods.

Times.-The growth of population, destruction of forests, multiplying of oil engines, automobiles, and airplanes alter the properties of the atmosphere and thereby shift phases of periods. Hence, as stated above, I divide the thousand months of records into first and second halves and compute the phases and amplitudes within the two parts separately.

Table 7a.-The sixfold groupings.* The 9-I/IO-month period. Eastport, Me.
Values in percentages of normal precipitation
A. WOLF sunspot numbers below 20

| $a_{1} \uparrow_{4}$ | $b_{1}$ ok | $c_{1} \uparrow_{4}$ | as ok | $b_{2} \downarrow 3$ | $c_{2}$ ok | $\Sigma$ | $\Sigma \div$ | $\Delta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IOI | 83 | 94 | 90 | 63 | 106 | 537 | 89 | -9 |
| 88 | 88 | 91 | IOI | 66 | 107 | 547 | 90 | -8 |
| 88 | 79 | 98 | 107 | 73 | 107 | 552 | 92 | -6 |
| 75 | 82 | 8I | 108 | 78 | 100 | 509 | 88 | -10 |
| 97 | 97 | 93 | 110 | 80 | 106 | 583 | 97 | -I |
| 104 | 95 | III | I 15 | 9 I | 110 | 626 | 104 | 6 |
| 106 | III | II2 | 105 | 109 | 122 | 685 | 114 | 16 |
| 106 | 107 | 109 | II8 | 98 | 116 | 654 | 109 | II |
| 101 | II4 | 97 | 107 | 76 | 109 | 60.4 | IOI | 2 |
|  |  |  |  |  |  | Mea | 98 |  |

B. Wolf sunspot numbers exceed 20

| $a_{1}$ ok | $b_{1} \downarrow 3$ | $c_{1} \downarrow \mathrm{I}$ | $a_{2}$ ok | $b_{2}$ ok | $c_{2} \uparrow 2$ | $\Sigma$ | $\Sigma \div 6$ | $\Delta$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 87 | 92 | 92 | 84 | 97 | 89 | 541 | 90 | -10 |
| 88 | 98 | 98 | 86 | 96 | 86 | 552 | 92 | -8 |
| 98 | 97 | 96 | 86 | 105 | 87 | 569 | 95 | -5 |
| 108 | 105 | 105 | 98 | 116 | 107 | 639 | 106 | 6 |
| 108 | 105 | 115 | 104 | 115 | 107 | 654 | 109 | 9 |
| 108 | 102 | 123 | 112 | 108 | 106 | 659 | 110 | 10 |
| 94 | 100 | 124 | 103 | 108 | 100 | 629 | 105 | 5 |
| 90 | 92 | 95 | 97 | 101 | 98 | 573 | 96 | -4 |
| 79 | 94 | 96 | 95 | 102 | 99 | 565 | 94 | -6 |

[^4]Not only so, but considerable differences of amplitude between the two halves are sometimes found. As forecasts are for present and future time, weights, as $2 / \mathrm{I}, 3 / \mathrm{r}$, or $4 / \mathrm{I}$, are given to favor the second half when considerable differences in amplitude of periods between the two halves appear. It matters not whether the later amplitudes are the less or the greater, the larger weight is ascribed to amplitudes
of the second half. If a backcast were to be made to long ago, the weights would of course be reversed.

At some chosen date all periods must be in the same phase and preferably in zero phase. I chose $1957-0$ as this zero date. To insure that any particular period will be in zero phase with 1957-0 it is necessary to compute ahead from the start at about the year 1870 . This may be done as follows. Take the period 8-3/II months for example.

From 1870 to 1957, 87 years, there are 1,044 months. About 126 periods of $8-3 /$ I I months would cover this interval. But a date must be chosen which is an exact integral multiple of $8-3 / \mathrm{Ir}$. The nearest is that which gives I2I periods in the interim. Multiplying, we find that 121 periods require 1,001 months, or 83 years 5 months. Sub-

Table 8.-Repeated 8-3/II months and round numbers

| $\mathbf{I}$ | 8.2737 | 8 | 7 | 57.9089 | 8 |
| :--- | ---: | :--- | ---: | :--- | :--- |
| 2 | 16.5454 | 9 | 8 | 66.1816 | 8 |
| 3 | 24.8 I 8 I | 8 | 9 | 74.4543 | 8 |
| 4 | 33.0908 | 8 | 10 | 82.7270 | 9 |
| 5 | 4 I .3635 | 8 | 11 | 90.9997 | 8 |
| 6 | 49.6362 | 9 |  |  |  |

tracting these figures from 1957-0 we find 1873-7. Thus a suitable starting point is August 1873. But it was assumed that the record begins about $1870-0$. If so, 43 months would be lost. One therefore counts backward from 1873-7 five periods, and therefore begins with March 1870.
We now come to considering periods ending in fractions of a month. We may make tables of accumulation for them. Again using the period 8-3/II months, table 8 results.
For most of the periods of inexact months, tables to 91 months suffice. But for such as $12-9 / 22,13-13 / 20,24-9 / \mathrm{II}$ and $27-3 / 10$ the tables must be carried on to 273 months.

## RESULTS OF FORECASTS

Having treated of most of the features of the method, the remainder of this paper will disclose the results of these forecasts of precipitation. As I have stated, I discovered discrepancies sometimes as great as io percent between the published monthly normals and new normals obtained by separating years when Wolf sunspot numbers are respectively above and below 20. As my new normals may
be of value to other investigators of periodicity I first give in table 9 the two sets of normals for the 32 cities I have investigated.

The cities are in alphabetical order. The months in the first column apply for all cities. Precipitation is given in inches. Columns $A$ and B give monthly normals for times when Wolf sunspot numbers are respectively less and more than 20.

Departures; observation minus forecast 1950-1958.-There are 20 cities showing (1950-1958) departures in level of 4 percent or more from the values given in table 9. This is to be expected. One could not suppose the mean precipitation, 1950-1958, would be identical with the average precipitation, I870-1958. Table ga gives all the cities where such differences of 4 percent or more occurred.

When I come to give tables and maps of forecasts, 1959-1967, I shall not use table 9 a to correct the maps, but shall quote the results as they are determined from table 9. Persons interested may apply the values of $\Delta$, table 9a, as corrections in level to the forecasts, using them in reverse of the signs given in table 9a.

Sunspot effect on normals.-Lest readers think the differences between mean precipitation values attending high and low sunspot frequency are merely due to the sparsity of evidence, considering the irregularity of precipitation, I call attention to the numbers of months entering into the mean values of table 9 . For nearly all of the stations approximately a thousand months participated. That indicates about 600 for high sunspot frequency, about 400 for the low. Dividing by 12, there were about 50 values per monthly mean for sunspots exceeding 20 Wolf sunspot numbers, and about 33 per month for the low sunspot frequencies.

Referring to table 9 , the yearly sums show seven cities where sunspot frequency makes no more than I percent difference in the totals. For seven other cities low sunspot activity brings more precipitation, with an average difference of 5 percent. For the remaining 18 cities precipitation averages $5-\mathrm{I} / 2$ percent higher at high sunspot frequency. While the discovery and elimination of these differences by computing new normals was of importance in my forecasting, seasonal differences made the elimination of the sunspot effect imperative. Thus at Salisbury, N. C., precipitation averages 17 percent higher with low Wolf numbers, January-April ; 9 percent lower, May-August; and I I percent higher, September-December, for Wolf numbers below 20 than for those above 20.

Credibility of forecasts.-It is difficult to compress within the limits of a paper, aimed to be available at moderate price to all who desire
Table 9.-Normal monthly precipitation after 1870 through 1957 in inches







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it, the results and comments representing this project. Even with $3^{2}$ stations, the United States is so vast in area and so varied in contrasting conditions that with the fullest use of my results no adequate country-wide coverage of the expected precipitation to 1967 can be made. As stated above, confidence in the forecasts must depend largely on the fidelity with which the first half of the forecast, 19501958, inclusive, fits the observed record.

Table 10 presents in parallel columns for all 32 stations the monthly percentage departures of forecasts and observed records, 1950-1958, from the normals given in table 9.

That readers may see from a graphical standpoint to what degree the forecasts represent the events, I present figure 10 . It gives the march of forecasts and events from 1950 to 1958 for Cincinnati, one of the best, and Denver, a less favorable station.

Table 9a.-Percentage departures (O-F) 1950-1958, from table 9

| $\begin{aligned} & \text { City } \\ & \% \Delta \Delta \end{aligned}$ | Abilene -12 | Augusta <br> $-17$ | Bismarck -6 | Charleston $-11$ | $\begin{gathered} \text { Cincinnati } \\ +4 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| City | Detroit | Eastport | Helena | Independence | Little Rock |
| \% $\Delta$ | -4 | +23 | -II | -17 | +4 |
| City | Natural Bridge | Peoria | Sacramento | Salisbury | Salt Lake |
| \% $\Delta$ | -7 | -6 | -4 | -5 | -7 |
| City | San Bernardino | Santa Fe | St. Louis | St. Paul | Thomasville |
| \% $\Delta$ | +10 | -17 | -8 | -II | -9 |

Figure io shows for a more favorable and a less favorable station a graphic view of data taken from table 10.

A glance at figure to shows for both cities an obvious similarity of the features of the forecasts and of the events for the majority of months covered. There are, to be sure, differences in amplitude of features observed and forecasted. In many cases the forecast, built on average conditions of about 1,000 previous months, hits the features found in the observed record from 1950 to 1958 on the exact months. But in the better station, as well as in the worse, there occur relative displacements of features common to both forecast and event. These displacements are rarely as great as 5 months for any station, but may extend through durations sometimes as great as several years before returning to agreement.

Displacements of features.-Several years ago I published the account of a forecast for 104 years of St. Louis precipitation, including a comparison with the observed records. I quote from my discussion*

[^5]

Fig. 10.-Comparison of forecasts and events, 1950-1958. Upper curves, Denver, Colo.; lower curves, Cincinnati, Ohio. Forecasts, dotted lines; events, full curves.



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| 70 | 67 |
| 59 | 65 |
| 37 | $79+42$ |
| 38 | $86+50$ |
| 52 | $84+3$ |
| 64 | $102+38$ |
| III | $87-24$ |
| 159 | 166 |
| 177 | $133-$ |
| 196 | $124-72$ |
| 165 | 56-10 |
| 161 | 67 - |










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Table 10.-contimued




























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Table io.-continued






















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of discrepancies from pages 2-3 of my paper cited in footnote $\mathrm{I}, \mathrm{d}$, above:
8. Of 100 years of St. Louis precipitation forecasted, 70 seem fairly satisfactory and yield high correlation coefficients with the events. The failure of the other 30 is reasonably explained.
9. As shown by Dr. W. J. Humphreys in his "Physics of the Air," figure 227, great volcanic eruptions, which throw high columns of vapor and dust, profoundly modify weather. He cites the first four cases in the following list [here my table II], and I add several more.

Table II.-Great atmospheric disturbing causes

| Approximate dates | Volcanic eruptions |
| :---: | :---: |
| 1856 | . Cotopaxi and others. |
| 1883-1890 | . .Krakatoa and others. |
| 1901-1904 | .. Pele, Santa Maria, Colima, and others. |
| 1912 | Katmai. |
| 1924 and I9 | Many great eruptions. |
| 1930 ... | Great eruptions. |
| 1947 | Niuafoo Island. |

10. Of 30 unsatisfactory years, in 100 years of synthesis of St. Louis precipitation, these lie in groups as follows: 1854 first half; 1856 to $1860 ; 1887$ to 1889; 1900; 1901 ; 1905 to 1907 ; 1912 last half; 1913 first half ; 1915 to 1917 ; 1920 ; 1923 to 1926; 1930; 1940 to 1950 . It will be seen that many of these unsatisfactory intervals fall either soon after tremendous volcanic eruptions occurred or there was tremendous use of explosives in war or explosions of atomic bombs. As has been pointed out, atmospheric changes alter the lags in the weather effects of all solar impulses, and of course unequal periods have unequal lags. These unusual atmospheric disturbances may very well have mixed up the timing of terrestrial responses to the 23 periods so as to cause the events to differ from the predictions.

At some future time it may be possible to connect theoretically the displacements found in my forecasts with causes producing atmospheric alterations of importance in weather. As yet I have been unable to name with certainty causes operative to produce these occasional displacements. For the practical inquiries of farmers, however, it is of importance to estimate the magnitude of forecasting error rather than the cause attending such discrepancies.

As a step toward that, I cite the case of Spokane, Wash., figure II. A computation made in 1957 derived a "correlation coefficient" of $59 \pm 5$ percent over the interval March 1950 through October 1956 between forecast and event in Spokane precipitation. In simple language this means that my forecast represented the observed precipitation 59 percent perfectly for almost 7 years.

Fig. iI.-Comparison of forecasts and events, 1950-1958. Upper curves, Spokane, Wash.; lower curves, Washington, D. C. Forecasts, full curves ; events, dotted curves.

When records through 1958 became available two considerable discrepancies between forecast and event were noted. In the months January and March 1950 heavy precipitation (over twice the normal even in 3-month running means) raised the February observed curve far above the predicted curve. Both curves, as has been said, are smoothed by using 3 -month running means in all computations, hence the February effect. Not until April did the two curves come close together. Yet there was a difference of only 6 percent of normal precipitation between the averages of their heights, January-April, 1950.

Beginning October 1956, and extending through August 1957, there was a shift of 5 months, leaving the predicted curve in the rear, and exposing opposed high and low values of the prediction and event. When the two curves were averaged over this interval of II months, the predicted curve was in6 percent of the normal and the observed curve 96 percent of the normal.
To sum up: At Spokane, in the 9 -year interval, my forecast gives for over 7 years a correlation with observations of 59 percent. Two intervals of marked discrepancy occurred. The first, of 4 months, culminating with February 1950, was obviously caused by extraordinary precipitation in two almost adjacent months. It produced a difference of only 6 percent between the averages over these 4 months. The second discrepancy, extending II months, was of unknown cause. It involved a 5 -month shift of phases and produced 20 percent difference between forecast and event in average precipitation over those II months.

Having set forth those discrepancies I remark that this is in the infancy of my method of forecasting, before any help has come to me from theoretical meteorologists. It may be that some of them will discover the causes of occasional displacements of features between forecast and event. If so, it may reduce error of forecasts greatly. Then, too, my method assumes that the average behavior of periods in weather in a thousand months that are past will be followed in the months to come. It perforce neglects changed conditions which may arise from unpredictable storms, volcanoes, or even from man's interposition, as from forest destruction, invention of new powerful devices, wars and the like. Even a minor atmospheric change may alter the time of a feature in precipitation by a month. All these factors tend to lower the coefficient of correlation.

The 273-month period in weather features.-It will have occurred to some readers that if one were backcasting from April 1927 or from July 1904 he would employ the same tabular data that I have used
in forecasting from January 1950. Hence one might infer that the precipitation following these earlier dates should parallel that following January 1950.

There is indeed a partial similarity, as I pointed out many years ago, between the march of weather at successive intervals of 273 months. But the correspondence is very imperfect. This appears in figure 2, where the precipitation at Nashville following July 1904 is compared to that following January 1950. However, I call attention to the close agreement of the two curves for the last three years of the comparison. I have computed for several cities, including Nashville, the coefficients of correlation of the observed precipitation following April 1927 and July 1904, and compared with the forecast made to follow January 1950. These coefficients have fallen between i8 and 22 percent, while, as stated in my foreword, the correlation following January 1950 ranges from 52 to 59 percent.

This difference is easily explained. Over 40 percent of perfect 100 percent correlation is unpredictable as yet. There are several causes. (a) There is occasional unusual precipitation, as occurred in January and March 1950 at Spokane. (b) There are displacements of features as yet unexplainable. (c) The graphs I have published show large discrepancies in amplitude between forecast and event of obviously identical features. (d) Unpredictable events occur to alter weather from the averages of $\mathrm{I}, 000$ months.
In the march of precipitation from April 1927 and from July 1904, the vicissitudes of the later years up to January 1950 cannot have affected the observed precipitation of the earlier times as they have done that following 1950. As such vicissitudes account for 40 percent and more in coefficients of correlation, the tabulation suited to January i950 can only roughly forecast what follows these earlier dates.

## FORECASTS, 1959 TO 1967

Table 12 gives for 32 stations for the interval 1959-1967 the expected monthly mean percentages of the normal precipitation tabulated in table 9 . The reader will recall that all forecasts are made from 3 -month running means taken from published monthly mean values, and expressed in percentages of the normal values of table 9 .

Expressing these forecasts in a more usable form, table i3 gives average percentages of the normal for the intervals January-April, May-August, September-December, of each year, 1959 to 1967 , inclusive.













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At the end of text are 27 maps of the United States with the 32 cities as listed above, and each accompanied above the circle by a number identical with the appropriate number in the column headings in table 13. Below is the predicted departure from normal. Each group of three maps covers the three intervals per year of four months each named with table 13. Large areas of approximately equal departures from normal precipitation are clearly noticeable on the maps. These area similarities may aid farmers remote from the 32 cities to estimate the precipitation probable in their locations.

## APPLICATIONS

Periods control long-range weather. - I have sought to present to meteorologists evidence of two important propositions. First, that there exists in weather a family of periods, all exact submultiples of 273 months. These periods are hidden from immediate recognition because their phases are shifted according to the state of the atmosphere. When, however, the long monthly records are grouped and reduced with reference to time of the year, sunspot activity, and march of population, the family of periods is clearly disclosed with constant length, and with approximate sine-curve forms.

Second, long-range precipitation is to nearly 60 percent governed by this family of periods. By evaluating the average forms and amplitudes of these periods from thousand-month records, precipitation and temperature may be forecasted for years in advance, with considerable approximation to the event.

Whether these forecasts will appear to interested parties as trustworthy guides to help in planning their future operations must depend on the agreement attained between forecasts and events, 1950-1958. I therefore prepared table 14 which gives for 32 cities the 4 -month forecasts and observations, 1950-1958, and the differences in percentages of normal precipitation, $\Delta$, in the sense observed minus forecasted. Their means are given disregarding signs.

Agricultural requirements.-For agricultural purposes a foreknowledge of seasons rather than of individual months is most desired. Hence I give in table 144 -month mean values computed from table 10. But it is the difference between forecast and event which would be the controlling factor in estimating the value of the forecasts. ${ }^{5}$ The average differences, $\Delta$ (observed minus scale-corrected forecasts) are

[^6]entered at the bottom of the columns of $\Delta$ in table 14. These averages will be needful to the use of table 15 which is to follow.

Assuming that the degree of success attained in the forecasts from I950 through 1958 will be attained from 1959 through 1967, I have prepared table 15 from which the probable sizes and numbers of discrepancies between forecasts and events in 4 -month mean values over the entire interval of 9 years, 1959-1967, may be estimated. Selected from table 14, four groups of cities, 25 in all, are tabulated in table 15. The first group of ir cities have average 4 -month mean discrepancies, 1950-1958, of about 20 percent between forecasts and events.

The second group of six cities have mean 4 -month discrepancies of about 26 percent, the third group of five cities, 30 percent, and the fourth group of three cities, 40 percent. All the percentages relate to normal precipitation given in table 9 , with the scale correations from table ga used in table 14.

The six columns of table I5 give, respectively, the numbers of cases in table 14 when the discrepancies between forecast and event, i9501958, are (a) less than one-fourth, (b) one-fourth to one-half, (c) one-half to one times, (d) one to one and one-half, (e) one and onehalf to two, and (f) over two times the average discrepancy of the group.

If the same degree of success is reached $1959-1967$ as was reached 1950-1958, the interested person of a city in Group I would expect the numbers of discrepancies (O-F) among the 4-month means stated in the mean values at the bottom of the columns of table 15 to occur in the entire interval of 9 years with magnitudes in percent of the normals as stated at the top of the columns of the first group. If he were located at a city of Group 4, the percentages would be twice as large, because the numbers heading Group 4 are twice those heading Group I. But the numbers of cases would be the same.

Stated numerically, a person residing where the mean departure of forecast from observation, given in table 14 for 4 -month intervals from i950 through 1958, was about 20 percent of normal precipitation, may expect the following numbers and magnitudes of departure from the forecast of 4 -month means during the entire 9 years, 19591967, given in table 14.

| Numbers of departures..... | 4.6 | 4.5 | 6.1 | 6.0 | 2.8 | 3.0 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| Magnitudes in percent..... | 0-5 | 6 -10 | II-20 | $21-30$ | $31-40$ | $>40$ |

If he resided where the mean departure given in table 14 was greater, the numbers of departures as just given would be unchanged, but $\dagger$

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Table 13.-continued







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Four-month mean values






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Table 15.-Expected numbers of discrepancies of forecasts between assigned limits

Numbers expected of 4 -month intervals in 9 years, 1959-1967, when (O-F) has certain values
Group 1. Mean $(O-F)=20$ percent

|  | $<5$ | 6 -10 | 10-20 | 20-30 | 30-40 | $>40$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bismarck | 6 | 3 | 8 | 5 | I | 4 |
| Charleston | 4 | o | ıо | 8 | 1 | 4 |
| Cincinnati | 4 | 4 | 7 | 3 | 4 | 5 |
| Independence | 4 | 5 | 4 | 8 | 2 | 4 |
| Madison | 3 | 5 | 8 | 4 | 4 | 3 |
| Nashville | 2 | 7 | 2 | 8 | 5 | 3 |
| Port Gibson | 8 | 2 | 6 | 5 | 4 | 2 |
| Rochester | 5 | 5 | 5 | 7 | 3 | 2 |
| Spokane | 3 | 6 | 7 | 6 | 2 | 3 |
| St. Louis | 6 | 4 | 3 | 6 | 4 | 4 |
| Washington | 6 | 5 | 8 | 4 | 2 | 2 |

Group 2. Mean $(O-F)=26$ percent

|  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Albany, Oreg. $\ldots \ldots \ldots$ | $<6$ | $7-13$ | $14-26$ | $27-40$ | $41-52$ | $>52$ |
| Augusta $\ldots \ldots \ldots \ldots$ | I | 5 | 7 | 5 | 4 | 2 |
| Denver $\ldots \ldots \ldots \ldots$ | 3 | 2 | 9 | 6 | 1 | 3 |
| Little Rock $\ldots \ldots \ldots \ldots$ | 5 | 6 | 10 | 7 | 3 | 2 |
| Peoria $\ldots \ldots \ldots \ldots$ | 4 | 3 | 6 | 6 | 1 | 5 |
| Salisbury $\ldots \ldots \ldots \ldots$ | 5 | 5 | 3 | 8 | 4 | 3 |

Group 3. Mean $(O-F)=30$ percent

|  | $<7$ | $8-15$ | $16-31$ | $32-46$ | $47-62$ | $>62$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Detroit $\ldots \ldots \ldots \ldots$ | 5 | 5 | 4 | 8 | 2 | 3 |
| Natural Bridge, Ariz..... | 6 | 4 | 4 | 9 | 2 | 2 |
| Salt Lake $\ldots \ldots \ldots \ldots$. | 6 | 2 | 4 | 6 | 4 | 5 |
| Santa Fe $\ldots \ldots \ldots \ldots$. | 5 | 5 | 7 | 4 | 2 | 4 |
| St. Paul $\ldots \ldots \ldots \ldots$. | 3 | 5 | 8 | 6 | 4 | 1 |

Group 4. Mean $(O-F)=40$ percent

|  | < 10 | 11.20 | 21-40 | 4-60 | -8o | $>80$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| El Paso | 3 | 5 | 5 | 3 | 7 | 4 |
| Sacramento | 0 | 4 | 11 | 6 | 3 | 3 |
| San Bernardino | 3 | 2 | 9 | 7 | 2 | 4 |
| Sums of 25 . |  | 106 | 159 | 152 | 74 | 80 |
| Means |  | 4.2 | 6.4 | 6.1 | 3.0 | 3.2 |
| Limits | $<\frac{1}{4}$ | $\frac{1}{4}-\frac{1}{2}$ | $\frac{1}{2}-1$ | I-3/2 | 3/2-2 | $>2$ |

their magnitudes would be greater in proportion as the mean departure of his place bears to 20 percent.

As actual cases, farmers living near Albany, Oreg., or Augusta, Ga., both by table 14 lying in the 26 -percent class of table 15 , may expect, according to table 15 , during the 9 years 1959-1967, the numbers of 4 -month averages found in table 14 to differ as follows from the 27 mean 4 -month departures from normal precipitation they will actually experience: Four cases less than 7 percent; four cases between 7 and I3 percent ; six cases between I4 and 26 percent ; six cases between 26 and 39 percent ; three cases between 39 and 52 percent ; and four cases over 52 percent. Farmers living near one of the cities of the 20 -percent class might expect this same division of the 27 cases for the 4 -month mean departures from normal precipitation, but these departures would be smaller in percentages in the ratio $\frac{20}{26}$. It will be for their judgment to dictate whether it is worth while to procure from the Smithsonian Institution, and make use of this paper, "A Long-range Forecast of U. S. Precipitation."

## COUNTRY-WIDE TRENDS IN PRECIPITATION

The maps of the United States presented below show large areas over which similar forecasts prevail. This should be helpful to interested persons who reside at a distance from the 32 cities for which forecasts were made.

I have been interested to search further to see if similar trends of precipitation sometimes prevail over the whole United States. Table 14 gives the actual departures of 3 -month consecutive means of precipitation as averaged over three 4 -month intervals per year, 19501958. A working table of these results was prepared, giving the 32 departures from normal of the cities employed in each line of a table of 27 lines, 3 lines per year for 9 years. Recording separately plus and minus departures, sums were taken for each line. These plus and minus departure-sums were plotted in figure 12 , lower two curves. Plus sums are given in full lines, minus sums in dotted lines.

The plus and minus departure curves run generally in opposite directions, and in some 4 -month intervals are widely separated. In such cases of wide separation the 4 -month intervals were strongly heavy in precipitation if the high points are on full lines, and strongly drought-prevailing if dotted. With this explanation it is seen that the autumn of 1951 and winter of 1952 were wet periods generally for the whole United States, and similarly from the summer of 1957

through the summer of 1958 . On the other hand from the summer of 1952 through the autumm of 1956 the country was generally dry.

This interpretation of generality over the country is justified by the fact that the high points of figure 12 depend on observations of identity of signs for more than 20 out of 32 cities, in 15 cases. Some peaks are supported by 28 cities out of 32 .

When both curves are near the heavy horizontal line the precipitation of the country as a whole was nearly normal. That is, through 1950 and the first four months of 1951, and for portions of the years 1953, 1954, and 1957 precipitation generally averaged nearly normal. The curves of figure i2 show plainly that the entire country is subject to nearly simultaneous trends of precipitation, depending, as they do, on nearly universal agreement of observations of departures in 32 cities over an interval of 9 years.

With this result established, turn to the two upper curves on figure 12. These are plotted similarly to those below, but are from table 13 which gives the 4 -month mean departures from normal precipitation forecasted 1959-1967.

Reading these upper curves: After the dry winter of 1959 there should follow a short well-watered interval, and an interval of nearly normal precipitation before a rather well-watered period in 1960. Then, following normal precipitation in 1961, should come pretty dry conditions in the winter and early summer of 1962. A long period of normal rainfall follows from the autumn of 1962 through the summer and autumn of 1964. A very wet winter of 1965 follows, and fairly normal precipitation thereafter, except for the dry summer of 1966.

The last preceding paragraph concerns the country as a whole. For details of forecasts for individual stations, the predictions may be found in tables 12 and 13 , and in the 27 maps of the United States.

## MAPS

Twenty-seven maps of the United States follow, with circles showing location of 32 cities. Numbers above the circles refer to the cities given in table 13, which are numbered correspondingly. Numbers below the circles give percentage departures from normal precipitation as forecasted as means for 4-month intervals in table 13, 1959-1967, A, B, and C, for each year. Three maps form one chart. The nine charts are dated from 1959 to 1967.


FORECAST OF U. S. PRECIPITATION-ABbot



NO. 9 FORECAST OF U. S. PRECIPITATION-ABBOT



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[^0]:    ${ }^{1}$ a, Journal of Solar Energy, Sci. and Eng., vol. I, No. I, January 1957; b, ibid., vol. 2, No. I, January 1958; c, Smithsonian Misc. Coll., vol. 122, No. 4, August 1953; d, ibid., vol. 128, No. 3, April 1955; e, ibid., vol. 128, No. 4, June 1955; f, ibid., vol. 134, No. I, September 1956; g, ibid., vol. 138, No. 3, February 1959.

[^1]:    ${ }^{2}$ See in reference, footnote 1 , e, above, figure 3 and table 3.

[^2]:    ${ }^{8}$ See discussion of backcasts at a later page.

[^3]:    ${ }^{4}$ Refer also to the clearing of overriders from the period of $68-1 / 4$ months at St. Louis. Note I, g, figure 3.

[^4]:    * The shifting of phases is indicated by arrows as in figure 9 and table 7. The accompanying subscripts indicate the number of months shifted up or down.

[^5]:    * Text continued on page 44.

[^6]:    ${ }^{5}$ As differences in level of obscrved precipitation, 1950-1958, from the averages of 1,000 months, are disclosed in table ga, I refer to that table for possible corrections of level which might be applied to values for some stations in table 14.

[^7]:    $\dagger$ Text continued on page 67.

