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LOW-TEMPERATURE RADIATION

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WATER-VAPOR TRANSPARENCY TO LOW-TEMPERATURE RADIATION

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The main purpose of this research was to determine the transparency of water vapor, under atmospheric conditions, to radiation such as the warm earth sends toward space. Upon the absorptive property of water vapor rests in part the virtue of the atmosphere as a conservator of the heat which the earth receives from the sun. Radiation from the sun reaches the earth's surface diminished by a certain portion scattered toward space and certain other portions absorbed in the gases and vapors of the atmosphere. The return of the energy of this radiation back to space is an indirect process. The warmed earth is cooled partly by convection currents playing over its surface and partly by direct and indirect radiation through the constituents of its atmosphere. Of these the principal hindrances to free radiation are aqueous vapor and carbonic acid gas.

The radiation from the sun, at an apparent temperature of 6,000 to 7,000° K.,¹ passes through the atmosphere with comparatively little true absorption. Nearly all of the radiation of a body of this temperature lies at wave-lengths shorter than 2μ . At sea-level on a clear day when the sun is in the zenith only about 6 to 8 per cent is absorbed from the direct solár beam within the great infra-red bands ρ , σ , ψ , ϕ and Ω in its passage to the surface of the earth.² The amount scattered from the direct solar beam by the dust and molecules of the air amounts to considerably more but after subsequent reflections in considerable part reaches the earth.³

The radiation from a body of the temperature of the earth, which may be taken as about 287° K., is of wave-lengths nearly all greater than 2μ and is hindered by quite a different series of absorption bands in its passage outward through the air. These absorption losses are caused principally by the water vapor and carbonic acid gas present in the atmosphere. This present research will treat

¹ In what follows the symbol K. denotes absolute temperature in centigrade degrees.

² *Astrophysical Journal*, 42, p. 406, 1915.

³ See *Annals of the Astrophysical Observatory of the Smithsonian Institution*, Vol. 3.

chiefly of the transparency of water vapor at wave-lengths greater than $2\ \mu$.

Figure 1 shows graphically the relative positions and intensities in the normal spectrum of the energy radiating from two bodies of

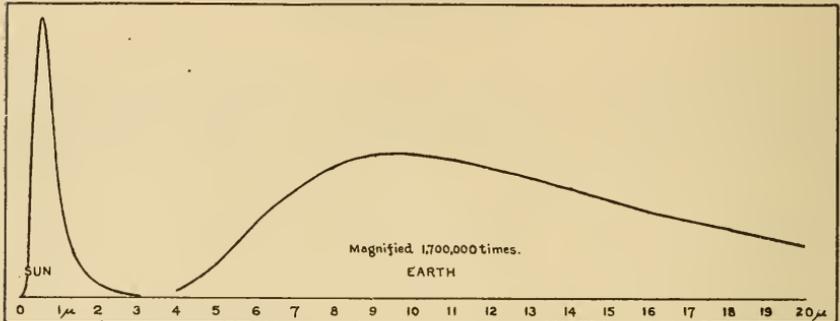


FIG. 1.—Computed black-body energy curves for two bodies of equal size, one at $6,000^{\circ}$ K. (sun), the other at 287° K. (earth).

equal size, one at the temperature of the sun, the other at that of the earth.

HISTORY OF THE PRESENT RESEARCH

At first thought it might seem easy to obtain the main object of this research by observing in the laboratory the total radiation of a body at the temperature of the earth, after that radiation had passed through various amounts of atmospheric aqueous vapor. However, the complication then introduced by the radiation from the vapor itself renders the analysis and interpretation of the results difficult if not ambiguous. So recourse is necessary to the use of the radiation from a source at a high enough temperature to make negligible the radiation from the vapor itself and its low temperature surroundings. But then a new trouble arises for the total radiations of bodies at different temperatures are of different qualities and differently affected by the absorption powers of the aqueous vapor. In order to apply the results to the case of the earth's radiation, it is therefore necessary to know the absorptions from wave-length to wave-length and the comparative distribution of energy in the spectrum of the laboratory source of radiation and of the earth. This requires the introduction of the spectroscope with its attending difficulties.

Preparations for observing quantitatively the transmissibility of radiation by aqueous vapor in the spectrum region of the earth's emission were begun in 1908. The radiation which it was proposed

to observe in the region between 2μ and 20μ is wholly invisible and very feeble even in the emission from the hottest terrestrial sources so that extremely sensitive measuring apparatus was necessary. Few substances are transparent to it. Glass as a rule is non-transparent for wave-lengths greater than 2μ , quartz for those greater than 3μ , and fluorite for those greater than 10μ . A plate of rock salt 1 cm. thick absorbs 50 per cent at 17μ and one of silvite 50 per cent at about 21μ . As silvite was not available it was necessary to use rock salt of which the Observatory, through the courtesy of the Russian Government, possesses a number of large and beautiful prisms.

Preparations had to be made for observing a great range of intensities. The radiation from a Nernst lamp, such as was used in this research as a source of energy at a temperature of about $2,200^\circ \text{K.}$, is at 20μ only $1/100,000$ as intense as it is at 2μ . The difficulties inherent in finding proper means for observing accurately over such a range of intensities will be discussed when the means are described.

No mirror perfectly reflects an incident beam in one direction. An appreciable portion of the beam is scattered in all directions as may be easily noted by looking from any direction at a silvered mirror upon which a beam of sunlight falls. This scattered light becomes more and more troublesome as that part of the spectrum is approached where the intensity is only $1/10,000$ that in the brightest part. Field light, as it is usually called, finally amounts to the whole of the observed energy. Means had to be provided either for eliminating or measuring it.

In order that the air, the transparency of which was to be measured at normal atmospheric conditions as to temperature and pressure, should contain sufficient water vapor, it was necessary to make most of the observations during the hotter summer months. Air of 50 per cent relative humidity at a summer temperature of 40°C. contains 12 times as much water vapor as of the same relative humidity and a winter temperature of 0°C. Even then, with the length of path possible in these laboratory experiments, the air column contained only as much vapor as may be found in the atmosphere during the winter months along the zenith path of the sun's radiation to the earth. On a summer day there may be 10 times as much vapor as this in the sun's path.

In 1908 preliminary experiments were made at intervals between the regular work of the Observatory. During 1909 and 1911, in the

lack of sufficient sensitiveness of the apparatus for measuring the feeble radiation of the longer wave-lengths, certain measures were made connecting the amounts of absorption in the bands of the upper infra-red, 0.7μ to 2.0μ , with the quantity of water vapor producing them.¹ These bands are those affecting the incoming radiation from the sun.

In the early part of the summer of 1913 attempts were made to use a vacuum bolometer which Dr. Abbot had meanwhile prepared. The air-pump, then in the possession of the Observatory, was not suitable and the summer became so far spent that the experiments were finally carried on without a vacuum. When the results were reduced the next winter a serious discrepancy was found, the cause of which was so obscure, that it escaped detection for some time. It necessitated the repetition of the experiments in the summer of 1914. This source of error lay in the circumstance that in the form of lamp then used for the radiation source, the bolometer was exposed to a source of somewhat different effective temperature when the radiation passed through the water-vapor tube than when it passed through the spectroscope alone. The comparison of the observed energies in the two cases was to serve as a measure of the absorptions due to the water vapor and thus the change in quality of the rays just explained caused error. This error was avoided by the use of proper diaphragms and the construction of a more suitable and far more effective form of lamp. Some doubt as to the matter of field light required further observations during the summer of 1916. Because of complications resulting from the absorption of the radiation by carbonic-acid gas, further observations were made during some very cold days of the winter of 1916-17. On such days the losses due to

¹The results of these observations were published in a series of articles in the *Astrophysical Journal* discussing the transmission of radiation through moist and dry air and water vapor between the wave-lengths 0.35 and 2.0μ . The first (l. c. 35, p. 149, 1912) gave the laboratory calibration, with known amounts of water vapor, of the intensity of energy in the bottom of certain absorption bands, the depths of which could be accurately measured bolometrically. The second (37, p. 359, 1913) gave applications of the first paper to the spectroscopic determination of the water vapor above Mount Wilson and a comparison of these values with determinations by Hann's formula. The third (38, p. 392, 1913) treated of the non-selective scattering of dry air and water vapor for the spectrum region 0.35 and 2.0μ . The fourth (40, p. 435, 1914) was concerned with the application of the dry-air transmission coefficients to the determination of Avogadro's constant, the number of molecules in a gram-molecule of any gas. The fifth (42, p. 394, 1915) gave the corresponding selective absorptions in the spectrum region 0.35 to 2.00μ .

aqueous vapor would be at a minimum, whereas the carbonic acid gas would be practically as effective an absorber as in the summer time. Finally, because of both the unexpectedly small absorption of radiation at the longer observed wave-lengths and the too-small quantity of aqueous vapor possible in the laboratory experiments, observations were made in April, 1917, of the atmospheric absorption of the radiation from the sun itself.

The necessity of the right-working of so many unruly processes, the galvanometer, the bolometer, the source of radiation, a transformer working far beyond its capacity in furnishing current for the lamp, the need of sunny weather, not too damp for the use of rock salt nor too windy for the galvanometer or bolometer, and without too many of the ever-prevalent summer cumuli—all these requirements tended to make the securing of good observations a tedious process. Often, too, all the difficulties had to be attended to single-handed.

Before proceeding the writer wishes to express his gratitude to Dr. C. G. Abbot, the Director of this Observatory, who not only suggested the research but at all stages was ever ready to help with suggestions and criticisms in the many perplexing problems.

WORK OF OTHER INVESTIGATORS

Although less than 1 per cent of the sun's incoming energy lies at a wave-length greater than $2\ \mu$, nevertheless the intensity of the sun's heat is so great that measurable energy would be expected at greater wave-lengths unless lost in passage through the constituents of the earth's atmosphere, or possibly in the gaseous envelopes about the sun itself. Langley,¹ in his solar and lunar spectrobolometric researches, found indications of energy between 2.0 and $2.5\ \mu$, 2.8 and $4.0\ \mu$, and 4.5 and $5.3\ \mu$. Beyond the maximum at $4.6\ \mu$, Langley states, "lies the longest break of all, stretching from 5.0 to $7.7\ \mu$." Between 8.4 and $9.1\ \mu$ and at $10.7\ \mu$ there is practically complete transmission as indicated by observations to an air mass of 3.76 , and the observed decrease in transparency for greater air-masses is probably due to the smoke in the atmosphere.

¹The Solar and Lunar Spectrum, Memoirs National Academy of Sciences, IV, p. 159-170, 1888. Note: The longer wave-lengths given by Langley are in error and have been corrected by means of his recorded deviations and a deviation-wave-length curve computed from the dispersion data for rock salt given in Vol. I of the Annals of this Observatory.

Paschen¹ investigated the transparency of carbon dioxide and water vapor. He used a column of steam 7 cm. long at atmospheric pressure. Between the wave-lengths 1 and 9μ he showed definitely the absorption due to each absorbent. He also gives four curves showing the energy curves to about 9μ of a blackened platinum strip at 450° C. observed through an empty fluorite cell and with the cell containing films of liquid water, 0.01 to 0.02 mm., at least 0.03 mm., and 0.08 mm. thick. The principal liquid water bands of this region lie at 2.9, 4.7, 6.1μ .

Rubens and Nichols² found but a small absorption due to water vapor for the energy from a terrestrial source selectively reflected from fluorite at 24.4μ . Energy of this wave-length could not have been found in the solar spectrum by the earlier observers owing to its absorption in their prisms of rock salt or fluorite. Rubens and Aschkinass³ pursued the experiments using selectively reflected sun-light of the same wave-length, 24.4μ , with purely negative results, no deflections were obtained. They then proceeded to the examination of the transmission of energy from a laboratory source through 40 cm. of steam at atmospheric pressure. The results of Rubens and Aschkinass and of Paschen will be given later in more detail in connection with the results of the present research. Their results were all obtained with steam at 100° C. and 76 cm. pressure whereas the results of the present contribution were obtained at atmospheric conditions both as to temperature and to total and partial pressures.

Rubens later, using the residual radiations selectively reflected from various crystals, found that water vapor has strong absorption between 45 and 120μ , this being especially intense at 50μ , 66μ , and 79μ . Relatively high transparency occurs at 47μ , 54μ , 62μ , 75μ , 91μ , and at 115μ .⁴ In an earlier research with Wartenberg⁵ he found that a column of steam 40 cm. long and at atmospheric pressure transmits radiation as follows: 39.6 per cent at 23μ , 0.7 per cent at 52μ , 19.6 per cent at 110μ , and 49.2 per cent at 314μ . These results are not necessarily at variance with the earlier statement that with selectively reflected sun-light purely negative results were obtained at 24.4μ by Rubens and Aschkinass. For some other

¹ Annalen der Physik und Chemie, 51, p. 1, 1894; 52, p. 209, 1894.

² Idem., 60, p. 418, 1897.

³ Idem., 64, p. 548, 1898.

⁴ Berichte Kgl. Preuss. Akademie der Wissenschaften, 1913, p. 513.

⁵ Verh. Deutsche Physikalische Gesellschaft, 13, p. 797, 1911.

atmospheric absorber than aqueous vapor may have removed the energy of this wave-length from the solar beam.

TEMPERATURE AND PRESSURE EFFECTS

The effect of pressure on the absorption of radiation by gases has been investigated principally by Miss Eva von Bahr.¹ In general the absorption of the same quantity of gas is increased by increasing its pressure. The absorptive power exerted by a gas present in another gas does not depend merely upon its partial pressure but upon the total pressure of the mixed gases as well. For instance, let the absorption of a given quantity of a gas in a tube be measured; if the gas be then allowed to expand, in such a manner that the weight of gas in the path remains the same, the absorption would in general decrease. If a non-absorbing gas be then admitted until the total pressure is restored, the original value of absorption will be regained. This absorbing power of a gas in general increases with increasing pressure somewhat rapidly at first, then more slowly, and the maximum constant power of absorption for a gas is not reached until under pressures ranging from 50 cm. for CS₂ to 400 cm. for CH₄ and CO₂.

Unfortunately the variation of the transmission with the pressure has been determined for only one of the absorption bands now under consideration. Miss von Bahr gives for the water-vapor band at 2.7 μ the following values for the absorption of a constant amount of vapor under the varying pressures:

105 mm.	4.6 per cent.	405 mm.	8.5 per cent.
235 "	7.2 "	570 "	10.6 "
370 "	8.6 "	755 "	12.0 "

The increase in pressure was produced by introducing dry air which exercised practically no absorption at this place in the spectrum. Miss von Bahr states that the "change of its absorption as dependent upon the total pressure is in general (for the same gas) the same in the different bands."²

The present research is carried out on vapor at 76 cm. total pressure whereas part of the vapor in the atmosphere is at a considerably

¹ Annalen der Physik, 29, p. 780, 1909; 33, p. 585, 1910; Verhandlungen der Deutsche Physikalische Gesellschaft, 15, p. 673, 1913.

² Die Änderung der Absorption auf Grund des Gesamtdruckes ist in allgemeinen die Gleiche in verschiedenen Banden.

reduced total pressure. It may give a fair estimate of the error committed in using these laboratory values for the transmission of the atmospheric water-vapor in the path of the sun's or earth's radiation, to use the observations of Miss von Bahr just tabulated. Using the distribution of aqueous vapor at different altitudes as given by Humphreys,¹ a vertical column of air, which would give a transmission of 88 per cent with a total pressure uniform throughout at 76 cm., would give, with a distribution of pressures such as actually exists in the atmosphere, 90 per cent in summer, 89 in winter according to the measures of Miss von Bahr. With the distribution of vapor above Mount Wilson, the transmission comes out 90 per cent for both summer and winter. That is, it would take a slightly greater amount of vapor to produce an absorption noted in the spectrum of a celestial body than would be expected from the laboratory data. If the observations are made at the surface of the earth, the difference would be 1 or 2 per cent and about 3 per cent if made at Mount Wilson.

Of the effect of temperature on the amount of absorption of gases comparatively little is known. Miss von Bahr² considers that its effect is certainly different from that of pressure. She interprets her results to mean that an increase of temperature causes a considerable damping by the absorbing particles whereas an increase in pressure up to a certain limit merely influences things by increasing the number of absorbing particles. A comparison of the results obtained with steam with those obtained under atmospheric conditions as regard to temperature indicates that less absorption would be expected at the lower temperature.

APPARATUS AND METHOD OF OBSERVING

The arrangement of the optical apparatus is shown schematically in figure 2. Radiation from the source N, composed of Nernst lamps, passed 42.5 meters through the tube T, containing the water vapor, to the mirror M_1 , 51 cm. in diameter, thence, collimated, 16 meters to the flat mirror M_2 , 76 cm. in diameter, thence back to M_1 , and then to focus on the slit of the spectroscope at S. Before entering the spectroscope the beam could be returned over the path through the water vapor by means of two flat mirrors, F_1 and F_2 , close beside the slit. The first arrangement gave a path through the water vapor of about 117 meters, to which must be added the

¹ Bulletin of the Mount Weather Observatory, 4, p. 121, 1911.

² Annalen der Physik, 38, p. 206, 1912.

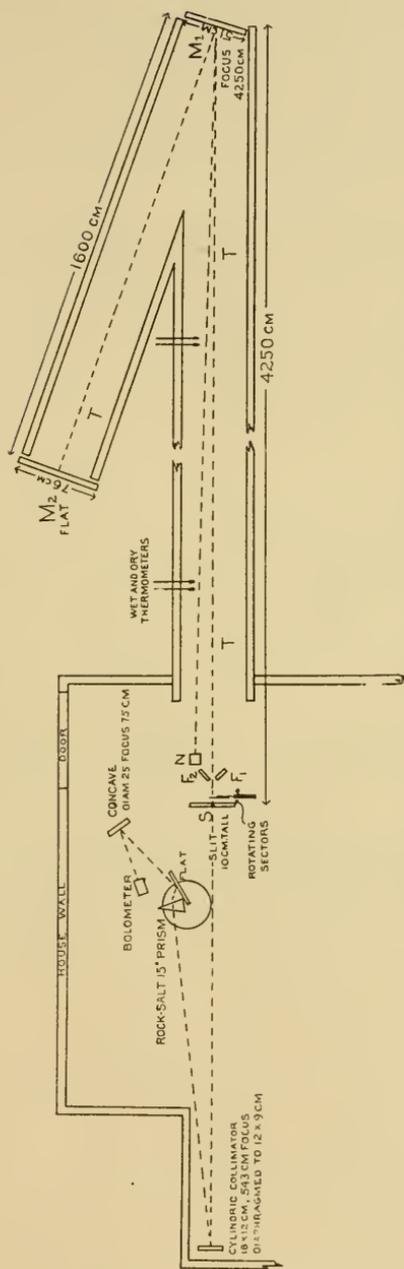


FIG. 2.—Plan of water-vapor tube and spectro-bolometric apparatus.

path through the spectroscope making a total path of 128.5 meters (422 feet). The second arrangement totaled 245.5 meters (806 feet). The source *N* could be turned towards the slit and its radiation passed directly into the spectroscope. Its path through water vapor then amounted to only 11.5 meters (38 feet).

The differences of the observed intensities of the spectrum energy after passing over the various paths served to measure the energy absorbed in the water vapor.

In consequence of the long path of about 43 meters to the collimator M_1 , but a small part of the energy sent out by the lamp was utilized. A shorter focus collimator would have been no better except with a point source of radiation; for as the source was 10 cm. tall, the extreme divergence of the bundles of parallel rays from the

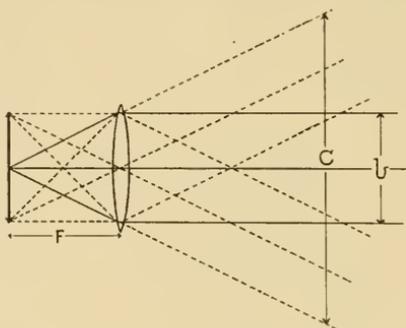


FIG. 3.— F , principal focus of lens; b , collimated rays from point source; C , collimated rays from linear source.

two ends of the source would have overbalanced the gain in incident energy on the collimator from the lamp (see fig. 3). Unfortunately there was a serious vibration of the image of the lamp formed on the slit due to the great magnification of the small tremors of the mirrors. These two causes, the small angular aperture of the mirror and tremor of the image, went far to limit the accuracy obtainable in the observations.

The energy, after entering the slit of the spectroscope, passed to a cylindrical collimator (straight elements vertical) of 543 cm. focus, diaphragmed down to 12 by 9 cm.; thence it passed to a rock-salt prism with Wadsworth minimum-deviation attachment, then to the concave mirror of 75 cm. focus which formed the spectrum on the bolometer. All the mirrors were silvered on glass.

THE WATER VAPOR

The water vapor was contained in a large double-walled galvanized-iron tube, open at its ends. Suitable compartments being provided in the double wall, and numerous holes leading from these

into the inner tube, a 22 horse-power rotary blower served to thoroughly stir the air and aqueous vapor. Alternate compartments were supplied with pressure and exhaust. This stirring was necessary in order to prevent stratification of the air and the consequent blurring and drift of the image of the lamp. It was not desirable to run the blower during an actual spectrum record because of the tremors communicated to the mirrors and to the galvanometer.

The amount of water vapor was determined by wet and dry thermometers at the spectroscope, at the mirror shelters and at several places in the tube. These were read while the air in the tube was stirred by the blower. Check determinations were made several times by Mr. L. B. Aldrich, who absorbed and weighed in tubes of calcium chloride and phosphorus pentoxide the water vapor from known volumes of air taken from the tube. The following table gives the water per cubic meter as measured by the two methods:

By wet and dry thermometers	Absorbed by $\text{CaCl}_2 + \text{P}_2\text{O}_5$
3.25 grams per cu. m.	3.29 grams per cu. m.
3.82 " " " "	3.85 " " " "
7.96 " " " "	8.76 " " " "

The amount of vapor in the tube could be increased by the introduction of a jet of steam into the blower. But when the vapor pressure of the water vapor within the tube differed much from that without an equalization took place so rapidly as to render this procedure useless.

The following table shows a determination of the amount of water vapor in the large tube just preceding, (a), and after, (b), an observation on the transmissibility of radiation through the vapor in the tube.

AUGUST 11, 1914. BAROMETER 76.5 cm.

Dry thermometer....	35°7	37°9	39°7	36°3	36°7	40°7	43°0	42°3
Wet thermometer...	26°9	27°5	27°9	27°0	26°9	28°4	29°0	28°3
g. per m ³	20.4	20.4	20.2	20.4	19.9	20.5	19.3	19.7
Mean g. per m ³	(a) 20.4				(b) 19.9			

$$\begin{aligned} \text{Amount of vapor in path through tube} &= 117 \times 20 \times .0001 \\ &= 0.234 \text{ cm. ppt. H}_2\text{O.}^1 \end{aligned}$$

¹ For brevity the amount of absorbing vapor will be stated as so much precipitable water, "ppt. H₂O," meaning the depth of liquid water, which if evaporated into a column of the same cross section, would produce the absorbing layer of vapor. This should not be construed as meaning that the liquid water produces the same amount of absorption as the corresponding vapor.

SOURCE OF RADIATION

The radiation used in the following experiments was finally furnished by a lamp of 44 Nernst-lamp double-glowers or 88 filaments. It would have been impracticable to use any inclosed radiator because of the absorption in the inclosing material of the very radiations required.

Within a rectangular frame of soapstone (a), figure 4, was mounted the series of filaments (b). To the rear of the filaments the soapstone was cut away so that the radiation observed from between the filaments was that from the walls of the room at nearly the temperature of the rest of the spectroscope and not radiation from the hot material of the frame of the lamp.¹ The glowers were clamped between copper blocks on each side. In the under copper blocks were ruled grooves into which the platinum ends of each glower were carefully placed. Over these ends were laid strips of platinum foil, then the upper blocks were clamped tightly down upon the ends of the glowers. The foil, as well as several horizontal deep-saw cuts in the somewhat stiff upper blocks, was necessary to insure sufficient contact on the filaments to hold them in place and to furnish them with electrical current.

The Nernst glowers do not conduct an electrical current at ordinary temperatures. They were heated by a blast lamp until they would conduct. Then when once conducting they would become hotter and hotter, conducting better and better as their temperature rose, until they would have melted if they had not been provided with so-called "ballast" series resistances. These iron-wire ballasts were so chosen that their increase in resistance with the temperature-nearly compensated the decrease in resistance in the filament due to rising temperature. The ballasts were located in a separate box. The lamps were pushed to their utmost output and they frequently burned out. Although rated at nearly an ampere each the whole 44 in the second lamp would stand but a little over 15 amperes on an open-circuit voltage of 110 volts. The black-body curve best fitting their energy curve indicated an effective temperature of about 2,250° K.

¹ This provision was important. In the earlier lamp the filaments were inclosed in a hollow in the soapstone; but the soapstone, although doubtless helping to keep the lamp at a steady temperature, was not near enough to the temperature of the filaments to produce approximately equal radiation intensity as a part of a black-body source and too near this temperature to have its radiation negligible, whence the error earlier mentioned. The room temperature was so near that of the bolometer that its radiation seen through the interstices of the filaments was negligible.

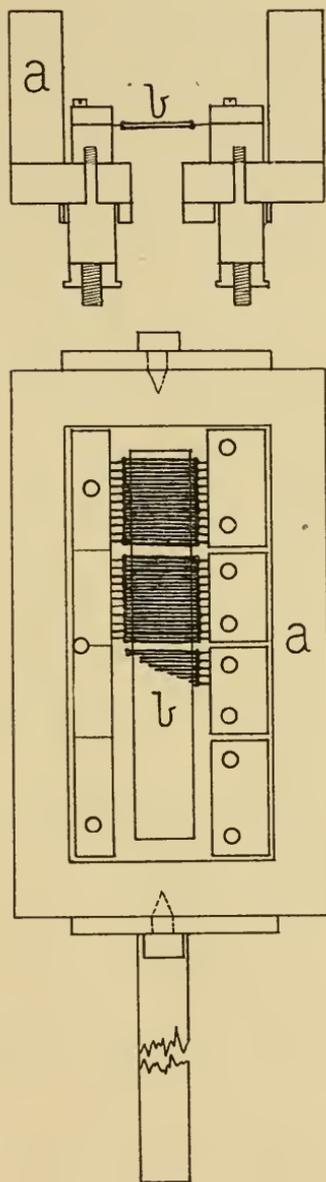


FIG. 4.—Lamp of 88 Nernst filaments. $\frac{1}{2}$ size.

NOTE.—Each pair of filaments was placed so that one filament was slightly to the rear, enabling them to be assembled in closer order.

THE SLIT

The slit was 10 cm. tall with jaws opening bilaterally. Because of the great range of intensity to be observed, from one to a hundred-thousand fold, from one end of the spectrum to the other, some means had to be provided for altering and knowing accurately the amount of alteration in the amount of energy flowing through the slit. It was thought best not to depend upon the readings of the slit micrometer screw for accurately measuring the widths of slits used.

In the earlier part of the research relating to the upper portion of the spectrum, 2μ to 10μ , when the lamp used had its filaments vertical, a lateral change of the slit opening would have been bad because of the irregular lateral changes in the intensity in the lamp. Accordingly a constant slit width was used just wide enough, or less, to be covered by the lamp image, and a series of horizontal grids or diaphragms was used to lessen the intensity of the spectrum. This scheme had the disadvantage of giving a low purity of spectrum corresponding to the great slit width necessary in the least intense part of the spectrum. It had the advantage of allowing the more accurate determination of the energy reducing value of the diaphragm. For the change of deflection when the diaphragm was inserted was not complicated by changes in the purity of the spectrum such as would attend change of slit width. Besides this, owing to having a wide slit, the trouble from diffraction presently to be described did not take place.

For the observations reaching to the greater wave-lengths, a lamp with horizontal filaments was used, the intensity of which was therefore uneven in the vertical direction. The largest slit width for a run was set by closing the slit until the jaws just touched a cylindrical template (4.99 mm., 5.82 mm., or 10.69 mm. in diameter). In the more intense regions of the spectrum the slit was reduced by turning up in succession in front of it two accurately cut slits the widths of which had been measured on a comparator (2.36 mm. and 0.67 mm.). Four rotating sectors could also be used for reducing the energy, which according to their mechanically measured apertures, reduced in the ratios, 0.333, 0.1054, 0.0501 and 0.00512 respectively.

All this provision for making reductions of intensity with accuracy proved somewhat illusory. It has been found here and by others¹ that rotating sectors do not cut down the energy proportionally to their mechanically measured apertures as would be expected. For instance the 0.333 sector cut down the energy by 0.344 as determined

¹ Coblentz, Bulletin Bureau of Standards, 4, p. 455, 1907.

by deflections of the galvanometer before and after the insertion of the sector. It is to be noted that the values of intensity transmitted by rotating sectors which have been observed here in several instances and by Coblenz are always greater than the theoretical value.

Further, not even the vertical slits of fixed widths proved satisfactory and widely differing reduction values were obtained in using them. The cause was obscure until Dr. Abbot suggested that the wave-length of the energy might be sufficiently great to cause large spreading of the beam by diffraction¹ even with the wide slits in use (*e. g.* 2.36 mm.). The fact that this trouble occurred only when the energy passed through the water-vapor tube and an image of the lamp was formed by the mirror M_1 on the slit seems to support the explanation. For in this case the light just filled the diaphragmed collimator of the spectroscope, and diffraction would, as soon as the wave-length was great enough, spread the light beyond both sides of the diaphragm when the narrower slit was up and not when the wider slit was in use. The increasing spread of the light with greater and greater wave-lengths would result in greater and greater reducing ratios for the diaphragm as determined by the deflections for greater and greater wave-lengths. When, however, the energy from the lamp flowed directly through the slit into the spectroscope it so over-filled the collimator that doubtless what was diffracted off from the mirror on one side was diffracted on, so to speak, from the energy already off on the other side. Unfortunately the observations were reduced, because of the press of other work, over a year after they were made, so that there was no opportunity for full investigation of the supposed diffraction phenomena.

THE BOLOMETER AND GALVANOMETER

For the first part of the research extending to 10μ a bolometer strip 0.1 mm. wide by 12 mm. tall was used. Its resistance was about 4 ohms. No plate of any material whatever closed the front of the bolometer case. For the latter part of the work a bolometer strip 0.5 mm. wide by 12 mm. tall was used. The current of 0.04 ampere through the bolometer strip was furnished by two storage cells in parallel. This current was increased to 0.08 and finally to 0.13 ampere for the 1916-17 work. The bolometer strip was placed in an air-tight case, the front being closed with a 1 cm. thick rock-

¹ Annals of the Astrophysical Observatory of the Smithsonian Institution, 1, p. 79, 1900; Lick Observatory Bulletins, 3, 42, 1904.

salt plate fastened on air-tight with white wax. The transmissibility of radiation through this plate will be found in table I under the section relating to the prisms.

The galvanometer is the same as that described on page 32 of Vol. 2 of the Annals of this Observatory. The proportionality of its deflections to the current was tested each year and found to be satisfactory. A time of single swing of about 3 seconds was generally used. The sensitiveness of the apparatus was such that a change of

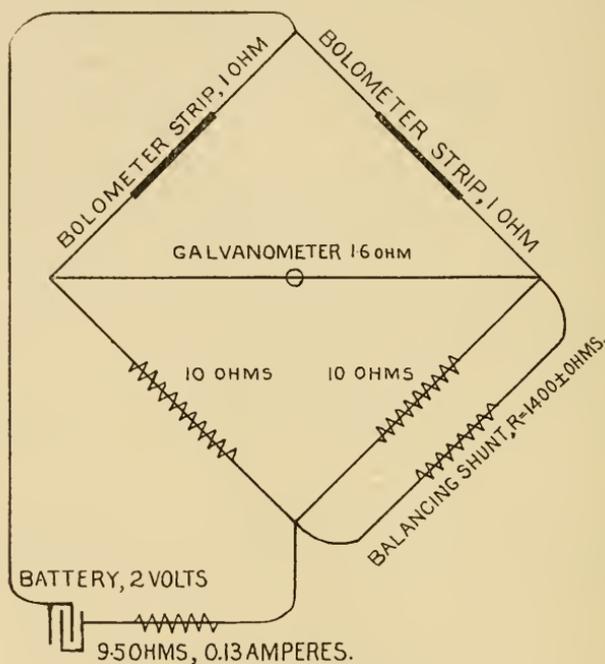


FIG. 5.—Bolometer circuits.

resistance in R (see fig. 5) of one ohm caused a deflection of nearly 3 cm.

The galvanometer deflections were recorded on a photographic plate moved by the same clock which moved the spectrum over the bolometer. The following were the various speeds used for the plate and spectrum:

- 1908, 1 m. of time = 4 cm. of plate = 20' of spectrum
- 1914, 1 m. of time = 1 cm. of plate = 8' of spectrum
- 1916, 1 m. of time = 1 cm. of plate = 4' of spectrum

ROCK-SALT PRISMS

For the work down to about 10μ two rock-salt 60° prisms were available, one with faces 18 cm. tall by 15.5 cm., the other 19 by 12.7 cm. The former was cut from an especially clear crystal and was generally used. For the work extending to greater wave-lengths two rock-salt prisms of about 15° were used with faces 13.4 tall and 8.3 wide, 0.4 cm. thick at the refracting edge, 2.7 cm. thick at the base. Further the front of the bolometer case was closed by a rock-salt plate 1 cm. thick. Table I gives in line (1) the coefficient of transmission for rock salt (a , in the formula e^{-ad} where d is the thickness in cm.), (2) the mean transmission for the smaller prism, using the mean thickness, 1.5 cm., (3) the transmission for the rock-salt plate 1 cm. thick, (4) the transmission for the two combined, (5) approximate values for the amount of light reflected from the 4 rock-salt surfaces and (6) the complete transmission factor.

TABLE I.—TRANSMISSIBILITY OF ENERGY THROUGH ROCK-SALT PRISM AND PLATE

Wave-length	5μ	10μ	12μ	13	14μ	15μ	16μ	17μ	18μ	19μ	20μ
a^1005	.007	.0243	.0715	.167	.414	.662	1.29	2.34	5.1
Prism.....	1.00	.993	.990	.964	.898	.779	.537	.370	.144	.030	.000
Plate.....	1.00	.995	.993	.976	.931	.846	.661	.516	.275	.096	.000
Prism + plate.	1.00	.988	.983	.941	.836	.659	.355	.191	.040	.003	.006
Reflected.....	.161	.149	.142	.137	.133	.129	.124	.118	.115	.107	.090
Transmitted..	.839	.841	.843	.812	.726	.574	.311	.168	.035	.003	.009

To protect the prisms from hygroscopic fogging of the refracting surfaces, they were covered with a thin film of asphalt varnish. A very dilute solution of asphalt in benzol was made, the freshly polished surfaces were dipped in this and the excess allowed to rapidly flow off. An even, very thin, nearly colorless film could thus be obtained. Its absorption for the long-wave radiation used was found to be inappreciable. Despite these protecting films the prisms fogged and were often repolished before each day's observation, and sometimes twice a day. A tube of phosphorus pentoxide (P_2O_5) kept close to the little-exposed and well-varnished rock-salt plate which closed the bolometer case entirely prevented fogging.

The following table gives the data relative to the dispersion of the various prisms used. The wave-lengths, λ , are given in microns or millionths of a meter, μ . The deviations, $\Delta\theta$, are in minutes of arc

¹ Rubens and Trowbridge, Wiedemanns Annalen, 60, p. 737, 1897.

(\prime) measured from the sharp maximum in the spectrum of the lamp at 1.838μ . The true deviations, θ , equal $39^\circ 33.7' + \Delta\theta$ for the 60° rock-salt prism and $8^\circ 9.9' + \Delta\theta$ for the small rock-salt prism. The plate distances, d , are in cm. measured from the position of the maximum at 1.838μ as zero. $d\theta/d\lambda$, given on an arbitrary scale, is for use in transforming the energy curves from the prismatic to the normal scale. $dd/d\alpha$, is the change in deviation, d , in cm. for a change in the angle of the prism of one degree. The deviations were computed from the indices of refraction given by Paschen.³

TABLE 2.—DISPERSION DATA
60° ROCK-SALT PRISM

λ in μ	1	2	3	4	5	6	7	8	9	10
$\Delta\theta$ in \prime	-27.0	+2.5	15.0	28.0	44.0	62.0	84.0	108.0	135.0	170.0
d in cm.....	-5.4	+0.5	3.0	5.5	8.7	12.3	16.7	21.6	27.0	33.9
$d\theta/d\lambda$	260	1175	1410	1199	970	829	721	631	552	491

15° 18.4' ROCK-SALT PRISM

λ in μ	1	2	3	4	5	6	7	8	9	10
$\Delta\theta$ in \prime	-5.2	+0.4	2.8	5.0	7.7	11.0	14.8	19.2	24.5	30.2
d in cm.....	-1.3	0.1	0.7	1.2	1.9	2.8	3.7	4.8	6.1	7.6
$d\theta/d\lambda$	(334)	(1600)	(1804)	1550	1330	1140	965	833	728	648
$dd/d\alpha$	0.09	0.01	0.05	0.09	0.13	0.19	0.26	0.34	0.43	0.53

λ in μ	11	12	13	14	15	16	17	18	19	20
$\Delta\theta$ in \prime	36.8	44.2	52.3	61.2	70.8	81.5	93.1	106.4	120.0	134.4
d in cm.....	9.2	11.0	13.1	15.3	17.7	20.4	23.3	26.6	30.0	33.6
$d\theta/d\lambda$	580	524	475	431	393	360	330	304	281	262
$dd/d\alpha$	0.64	0.77	0.92	1.07	1.24	1.42	1.63	1.86	2.10	2.35

ADJUSTMENTS OF APPARATUS

Each day's adjustments were as follows: A single Nernst glower was lit and placed in the center of the position to be occupied by the larger lamp in front of the slit. Mirror M_1 (see fig. 2) was then so adjusted that the range over which the filament was visible in it, as looked at from just in front of M_2 , was centrally situated with regard to M_2 . Next M_2 was adjusted so that the return beam centrally filled M_1 . This generally caused the image of the lamp to

³ Annalen der Physik, 26, p. 120, 1908. The following were computed from the formula I on p. 130 l. c.:

λ	1.838 μ	14 μ	15 μ	16 μ	17 μ	18 μ	19 μ	20 μ	21 μ
Log n	0.1839026	0.1649292	0.1618148	0.1583802	0.1545992	0.1504420	0.1458745	0.1408578	0.1341792

fall near the slit. Finer adjustments brought the image so that its light passed centrally through the slit. The spectroscope collimator and diaphragm were then adjusted so that the beam was central both on them and on the prism. The last condition was best ascertained by looking towards the collimator through the prism and adjusting so that the filament was visible over the whole extent of the prism. The bolometer was adjusted to look centrally at the projection of the beam on the image-forming mirror and the latter adjusted to bring the image of the spectrum vertically on the bolometer.

Then, when using the 60° prism, sun-light was thrown on the spectroscope slit and the prism¹ adjusted in minimum deviation by slightly turning it back and forth, leaving it in the position where the A line, as seen in the bolometer eye-piece, was at its minimum deviation. With the 15° prism the purity of the spectrum was insufficient to show any solar lines. A Bunsen flame, colored with sodium, was used to illuminate the slit instead of sun-light and the adjustments were made by means of the D lines. The spectrometer circle was then turned to bring the A or D line central on the bolometer strip and the position was read. Energy curves (or bolographs) were started from the spectrometer positions thus determined.

After the apparatus had been thus adjusted the blower was started and readings were made of the wet and dry thermometers in the water-vapor tube before beginning observations.

OBSERVATIONS .

It was originally proposed to observe alternately the energy in the spectrum of the Nernst lamps when shining through the water vapor in the large tube and immediately thereafter when shining through the spectroscope alone. It was found, however, that the curves obtained through the water vapor in the spectroscope remained so constant over the greater part of their wave-length range that in the 1916 observations they were often omitted. The change in aspect of the curves attending the above described change in path of the rays gave a measure of the absorption produced by the water vapor and carbon dioxide in the tube.

¹ Before setting up the prism it was best to give it a new coat of asphalt and slightly warm it out in the sun-light to prevent the fogging of its surfaces by the deposition of moisture from the air.

TRANSPARENCY FROM 1.2 TO 9.0 μ

In the upper portion of figure 6 will be seen a curve drawn from a bologram showing the distribution of energy in the spectrum of the lamp of Nernst glowers after the passage of its radiation through the air, carbon dioxide and aqueous vapor in the path through the spectroscop alone.

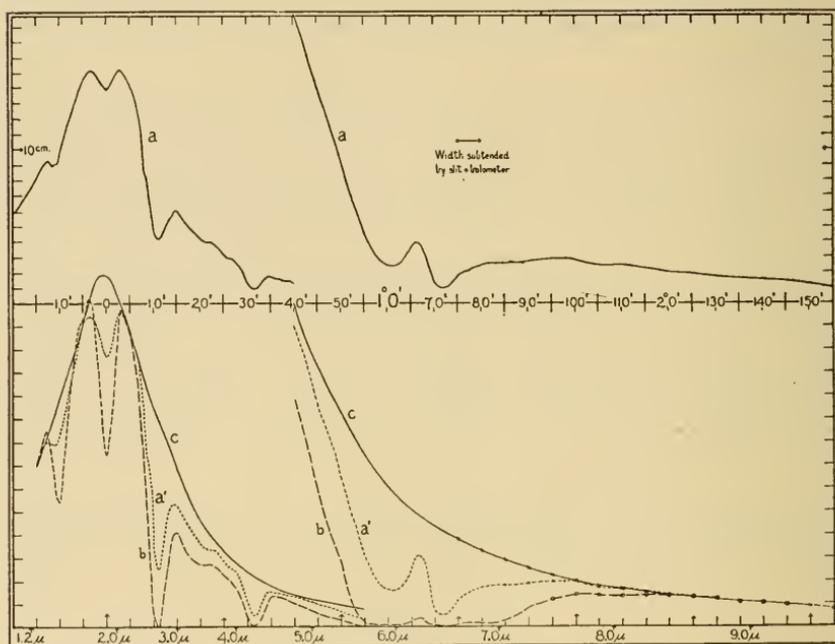


FIG. 6.—*a*, Bologram Nernst lamp, 60° rock-salt spectrum; *a'*, ditto corrected for slit and bolometer widths; ppt. H₂O, 0.01 cm.; *b*, ditto, ppt. H₂O, 0.1 cm.; *c*, black-body curve, 2,200° K.

To see whether any of the energy at the longer wave-lengths was due to energy scattered from the shorter wave-lengths, several bolograms were made with a quartz plate $\frac{1}{2}$ cm. thick inserted between the lamp and the slit. No appreciable deflection was observed beyond 4 μ , where quartz becomes opaque. Hence there is no appreciable energy at the deviations corresponding to wave-lengths greater than 4 μ contributed by radiation of shorter wave-lengths transmissible by quartz.

CORRECTION FOR SLIT AND BOLOMETER WIDTHS

When a spectrum is formed with a slit of finite width (the slits here in use were often necessarily wide), the energy at any point in the spectrum includes an appreciable range of wave-lengths depending upon the angular width of the slit. This apparent so-called impurity of the spectrum is further increased by the finite width of the bolometer. Indeed it is easily seen that the range of deviations observed by the bolometer at any point of the prismatic spectrum is equal to $z(a+b)$ where a and b are the slit and bolometer widths expressed in angular values subtended in the spectrum. In appendix I will be found the derivation of a formula for partially correcting for this impurity of the spectrum. This formula was applied in every case where the resulting correction would be of importance. Such a formula can only partially correct for errors in the readings of the maxima and minima in a spectrum. It of course cannot reproduce from a nearly continuous record, such as is shown in the upper part of figure 6, a purely line absorption spectrum such as is produced by water vapor. Such a formula is best applicable to a continuous spectrum such as would be given, for example, by a black body.

Having corrected the curve in the upper part of figure 6 for the widths of slit and bolometer, it becomes as shown at a' in the lower part of the figure. Here the ordinates of the longer wave-length portion (5 to 9 μ) are magnified 10 times relatively to those of the shorter wave-length section. Curve b , on a uniform scale with a' , records the energy passing through the air, carbon dioxide and water vapor in the 117 meters additional path of the large tube. Curve c on a similar scale is the black-body curve corresponding to a temperature of 2,200° K.

In order to obtain the amount of energy absorbed by the aqueous vapor it would be highly desirable to observe the energy first through the 117 meter path in vapor, then through the same path free of vapor. This was practically impossible. As a substitute for the latter condition, the energy was recorded with the lamp turned so as to observe its energy when passing through the spectroscope alone. With such a process the deflections at any wave-length could not be directly compared but had to be first reduced to the same scale. This was done by making the two sets of curves coincide near deviations $-5'$ and $+5'$ in spectrum regions where the many experiments of this observatory indicate no appreciable absorption by atmos-

pheric gases and vapors. Curve a' of figure 6 is an energy curve so made and it will be noted that even with the comparatively short path through the spectroscop (11.5 meters) some of the absorption bands are still quite pronounced. A black-body curve tangent to the curve a' and corresponding to a temperature of $2,200^{\circ}$ K. was thought to be the best means of representing what might be expected as the form of the energy curve were there no absorption bands present. This is curve c which is tangent to curve a' at deviations $\pm 5'$, at $100'$ to $140'$ (8 to $9\ \mu$) and nearly so at $40'$ ($5\ \mu$).

In figure 7 are plotted the results obtained by dividing by values read from curve c those obtained from curves similar to a' and b . The solid points with the dashed lines connecting the mean values, belong to the class a' , and show the absorption produced by the water vapor and carbon dioxide in the path through the spectroscop alone. The aqueous vapor corresponds to 0.008 cm. ppt. H_2O , the carbon dioxide, to about 7.4 grams¹ in a column of the length of the path (11.5 meters) and a meter square in section. The curve plotted with open circles, squares and diamonds with the solid connecting line corresponds to the class b . This curve shows the absorption due to the medium contained in the path of 117 m. through the great tube. The aqueous vapor corresponds to 0.082 cm. ppt. H_2O and the carbon dioxide to 83 grams in a column of the length of the path and of a square meter in section. Probably the *increase in absorption* indicated by the increase in the area of the absorption bands of the two observed curves is more accurate than the absolute values of absorption for these involve the assumption that the black-body curve of $2,200^{\circ}$ K. would represent the distribution of energy if there was no absorption.

There is also reproduced in the dotted curve extending from 5 to about $7.7\ \mu$ the work of Rubens and Aschkinass (l. c.) taken with a column of steam 30 cm. long at 100° C. and corresponding to 0.0042 cm. ppt. H_2O .

In the region shown here there are two important bands with maximum absorption according to Paschen at wave-lengths 2.73 and $4.63\ \mu$ respectively due to carbon dioxide. Unfortunately it was not feasible to remove this gas from either the spectroscop or the large

¹ The total CO_2 , similarly measured, vertically from the surface of the earth outwards is about 3,000 grams.

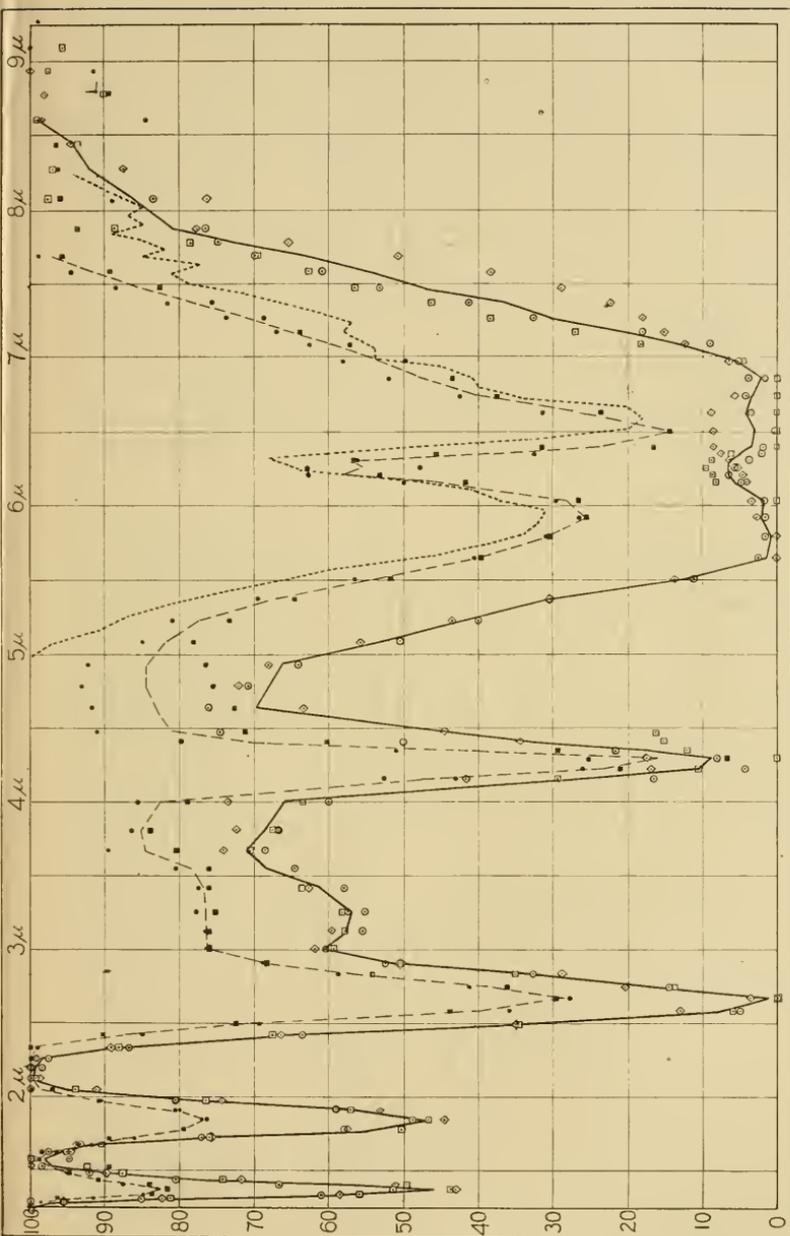


FIG. 7.—Percentage absorptions obtained with 60° rock-salt spectrum.

Lower curve, 0.082 cm. path. H₂O, 83 grams CO₂ in m² path. } Atmospheric conditions.
 Middle curve, 0.008 cm. path. H₂O, 7 grams CO₂ in m² path. } Steam, 100° C.
 Upper curve, 0.004 cm. path. H₂O, 0 grams CO₂ in m² path. } 76 cm. total pressure (Paschen).
 Abscissae are wave-lengths, ordinates, percentage transmissions.

tube. Paschen¹ gives the following data relative to the amount of absorption in these bands:

TABLE 3.—CARBON DIOXIDE BAND 2.34 TO 2.92 μ ; MAXIMUM AT 2.73 μ

Equivalent path in cm. CO ₂ , 76 cm. pressure	0.08*	0.3	7	33
Grams CO ₂ , column 1 m ²	1.6	5.9	140	650
Absorption per cent.	11	30	43

CARBON DIOXIDE BAND 4.26 TO 5.12 μ ; MAXIMUM AT 4.63 μ

Equivalent path in cm. CO ₂ , 76 cm. pressure	0.08*	0.3	7	33
Grams CO ₂ , column 1 m ²	1.6	5.9	140	650
Absorption per cent.	9	55	88	90

* Paschen in obtaining this figure assumes with Kayser (Wied. Ann. 42, p. 310, 1891) that the CO₂ in his laboratory had a partial pressure of about 1/1000 atmosphere. In computing the values given in the present work the partial pressure given by Hann of 0.23 mm. or 3/10,000 of an atmosphere has been used, for the experiments were made under out-of-door conditions.

Remembering that there were respectively about 7 and 80 grams of CO₂ in the meter square absorbing column producing the two curves of figure 7, Paschen's results indicate that probably something like 20 per cent of the absorption in the water-vapor band at 2.65 μ and 60 to 70 per cent in that at 4.3 may be due to carbon dioxide. For the carbon dioxide band at 4.63 μ Paschen considers that 650 grams is enough to produce practically complete absorption, the 10 per cent left in the table probably being accounted for by the impurity of his spectrum. Accordingly 3,000 grams CO₂ in a vertical atmospheric column 1 meter in cross section would suffice to completely extinguish the solar energy in the band whose center appears at about 4.6 μ . It appears from Paschen's work that the absorption for 0.004 cm. ppt. H₂O in the form of steam at 100° C. and 76 cm. pressure, is about equal to that of 0.008 cm. in the form of vapor at somewhat less than 15° C. and 8 mm. partial water-vapor pressure.

The region covered by figure 7 has been further subdivided and the absorption measured is indicated in the following table. The next to the last line summarizes the data of the absorption for the whole region given in parts in the five lines just preceding. The last line summarizes the data of absorption for the whole region from 1.3 to 8.0 μ . The last value, which gives the absorption percentage for the radiation as a whole, depends greatly on the distribution of energy in the spectrum of the beam to be absorbed. In the fourth column the

¹ Annalen der Physik und Chemie, 51, p. 51, 1894.

distribution of energy in the spectrum here observed with the Nernst lamp at 2,200° K. is indicated, and this may be regarded as a measure of the "weights" of the different parts in producing the absorption value here given. If instead of a Nernst glower the sun were observed, the upper part of the table would have even greater weight, while with the earth as a radiator the lower part should have a greater weight. The relative weights in the earth's case are given in the last column and the absorption value in the footnote to the table.

TABLE 4.—ABSORPTION BY WATER VAPOR, 1.3 TO 8.0 μ

Band	Range of wave-length μ	Relative energy 2200° K. Black radiator	Percentage absorbed		Relative energy 287° K. Black radiator
			Precipitable water in cm.		
			0.008	0.082	
Ψ	1.3 to 1.75	2300	6.1	18	0
Ω	1.75 " 2.2	2150	13.6	29	0
X	2.2 " 3.2	2400	23.6	41	1
—	3.2 " 4.0	1050	21.7	37	38
Y	4.0 " 4.9	640	32.5	50	418
Z	4.9 " 5.4	210	18	42	440
"	5.4 " 5.9	150	47	85	545
"	5.9 " 6.4	120	64	97	915
"	6.4 " 7.0	110	68	97	1340
"	7.0 " 8.0	120	25	62	2570
Z	4.9 " 8.0	710	40*	73*	5810
All	1.3 " 8.0	9250	19*	34*	6267

* These figures give the percentage of radiation from the Nernst lamp (2,200°-300° K.) absorbed in the band Z (4.9 to 8.0 μ) and in the whole region 1.3 to 8.0 μ . The corresponding figures obtained by means of columns 4, 5, and 6 for a distribution of energy of a black-body radiating at the temperature of the earth (287° K.) to space (0° K.) would be 43 and 42 per cent for 0.008 cm. ppt. H₂O and 76 and 74 per cent for 0.082 cm. ppt. H₂O.

ABSORPTION DOWN TO 16 μ

With the 60°-prism spectroscopie used in the work just detailed, the energy even at 9 μ had become almost too feeble for accurate measurement. From 10 μ to 16 μ recourse was necessary to the smaller dispersion and consequently greater intensity in the spectrum formed by a 15° rock-salt prism. The reduction of the corresponding observations brought many more difficulties as the detailed discussion coming will show.

Turning to figure 8, there will be seen a reproduction of a record of the deflections of the galvanometer needle which measures the amount of energy absorbed by the blackened bolometer strip, as there passes over it, under varied circumstances, the 15° -prismatic rock-salt spectrum of the lamp of 88 Nernst filaments or glowers. This record was made September 19, 1916, with 0.012 cm. ppt. H_2O in the path between the lamp and the bolometer.

The record consists of three sets of curves: first a lower, dotted, single-branched record indicating the zero of the galvanometer needle when a shutter at a temperature of about $300^\circ K.$ was inserted between the lamp and the slit; second, an upper, five-branched curve,

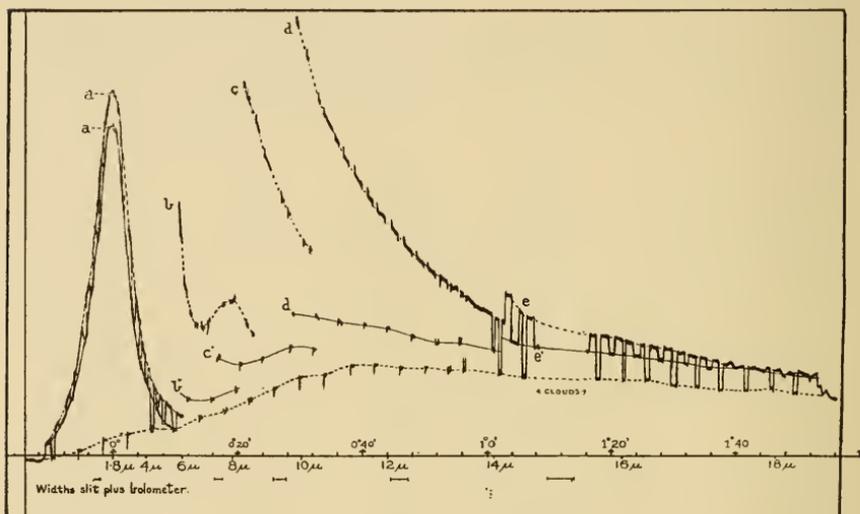


FIG. 8.—Bologram 15° rock-salt prismatic spectrum of Nernst lamp through aqueous vapor and carbon dioxide in the spectroscopic path alone. 7 grams CO_2 in m^2 path, 0.012 cm. ppt. H_2O .

abcde, the observed energy spectrum of the lamp. The branch *a* shows the rapid rise of energy to a maximum at 1.838μ and, for the present record, was made with a slit 0.67 mm. wide, the inflowing energy being further reduced $1/20$ th by a rotating sector. There is a rapid fall of energy on the right-hand or long wave-length side, and the sector was removed to allow sufficient energy to flow through the slit to produce the branch *b*; then for the branch *c* the slit was increased to 2.36 mm., for the branch *d* to 5.82 mm., and finally for the branch *e* to 10.69 mm.¹

¹ These slit-widths were obtained either by the use of brass slits of fixed widths inserted in front of the regular spectroscopic slit, or by closing the latter until its jaws just touched a measured cylindrical template.

The points connected to form the third trace $a'b'c'd'e'$ were obtained by the insertion of a plane-parallel quartz plate one-half a centimeter thick, between the energy source and the slit. Otherwise the conditions were for curve $a'b'c'd'e'$ the same as for the curve $abcde$. The significance of this third curve will be presently considered.

Taking into account the widths of slit and bolometer these curves were first corrected to represent as well as possible "pure" spectrum conditions as described under the work relating to the shorter wavelengths. See also Appendix I.

The next step was the reduction of all the branches of the two upper curves to the same scale. Provision was made to accomplish this by arranging the observations so that in the last three branches ordinates were obtained at the same abscissae for two successive slit conditions. However, when the ratio of ordinates under the wide slit to those under the narrow slit was taken, this ratio was found to progressively increase as the wave-length increased. The cause of this change, as stated in the description of the slit, remained obscure for some time until it occurred to Dr. Abbot that it was probably due to the spreading of the beam by diffraction when the narrower slit was used to produce the lower branch of the curve. As already stated and explained (l. c.) it occurred only when the beam passed through the long tube containing the water vapor and not when employing the path in the spectroscope alone. In the former case, the beam just filled the collimator mirror and diffraction caused it to spread off the mirror. In the latter case the beam considerably more than filled the mirror and the diffraction produced no harmful result.

This disturbance, therefore, rendered difficult the determination of diaphragm values for the reducing of the various parts of the curve to the same scale. With the spectroscope alone no trouble was found, for the slit ratios as measured mechanically and by means of the deflections agreed closely and the energy curve over the whole range of wave-lengths could be reduced to a uniform scale. It was thought best on the whole to use the same slit ratios for the tube work that were used with the spectroscope alone. To avoid error in the tube work as far as possible the overlapping parts of curves made with the smaller slit were rejected. It might, however, be feared that some error from diffraction affects the tube work even with wide slits. This matter will be treated later, and such errors will be shown to be probably negligible.

It was next necessary to reduce the curves "with" and "without" the water vapor to a common scale. It had been intended to do this by comparing the heights of the branches of region *a* and making these heights in the ratio which the higher dispersion work already described on this region indicated. This method had to be abandoned not only because of the difficulty of reducing the successive portions of the curve to the same scale as *a*, but also because of the great sensitiveness of the branch *a* to change in the amount of vapor. For the branch *a*, extending from say about 1 to 4 μ , includes several important water vapor absorption bands, namely, ϕ , ψ , Ω , and X, besides several smaller bands. These bands, because of the impurity of the spectrum, are not indicated in the smooth contour of this maximum. Their existence is felt, however, by its rapidly decreasing height with increasing amount of absorbing vapor as may be seen in figure 8. These separate bands may be seen in the result with the 60° prism shown in figure 7. Only the great band between 5 and 8 μ is noticeable in the impure spectrum recorded with the 15° prism.

The portion of the energy curve in which the greatest interest centers is that part where the wave-lengths are greater than 10 μ , or in other words, of greater wave-length than the part treated in the work with the 60° prism. Fortunately this portion was obtained generally without change of slit over its whole range. After correcting for "field light," as will be presently described, all curves were put on the same scale as follows: The assumption was made that there is no absorption by water vapor between the wave-lengths 9 and 10 μ .

The evidence for the validity of the assumption of no absorption of energy between 9 and 10 μ by aqueous vapor is cumulative and as follows: The work with the 60° rock-salt prism which formed the first part of this research indicated no absorption there. Rubens and Aschkinass found certainly less than 5 per cent absorption with *steam*.¹ Further support comes from the work of Langley,² who found at 10.7 μ , from a comparison of high and low sun (air-mass = 3.76) observations, complete transmission of the solar energy of this wave-length. The decreased transmission which he found for greater air-masses was without doubt due to the smoky atmosphere of Allegheny (Pittsburgh, Pennsylvania, U. S. A.). Langley's results are confirmed by similar work on the sun done here and pre-

¹ *Annalen der Physik und Chemie*, 60, p. 418, 1897.

² *Memoirs National Academy of Sciences* IV, p. 159, 1888.

sently to be described. Additional confirmation is given by the agreement in form from 9 to 13.5μ of curves taken here with a range of from 0.003 to 0.028 cm. ppt. H_2O . All the energy curves taken through the spectroscope alone (which involved a range of between 0.003 and 0.012 cm. ppt. H_2O) were reduced and carefully compared. Upon reducing to the same scale between 9 and 10μ they showed a remarkable agreement even over the whole range from 9 to 13.5μ indicating no change in transmission of energy within this range of wave-lengths when the ppt. H_2O increased from 0.003 to 0.012 cm. A yet later set of observations taken on a cold winter day through the long tube with the total ppt. H_2O equal to 0.028 cm. also agreed over this range. It would be very improbable that the percentage absorption would be uniform over the range from 9 to 13.5μ unless the absorption over this whole range is zero. Any departure from such a uniform absorption would have tended to cause the curves to diverge at the longer wave-lengths.

In accordance with the well-grounded assumption just discussed all curves were made to agree in intensity between the wave-lengths 9 and 10μ .

DIFFRACTION EFFECT

It has been stated that with the energy curves made with the beam passing through the spectroscope alone no diffraction effects were noted in determining the diaphragm or slit ratios. It was also stated that the curve just referred to taken with the minimum amount of aqueous vapor (0.028 cm. ppt. H_2O) in the path through the great tube coincided in form from 9 to 13.5μ with those taken through the spectroscope alone. This justifies the inference that the width of the slit (10.69 mm.) used from 9 to 13.5μ in the "tube" observations was sufficient to avoid diffraction losses. Beyond this wave-length the energy in the tube curves is practically all depleted by the carbon-dioxide band central at 14.75μ but which produces an appreciable effect, according to Rubens, all the way from 12 to 16.5μ . Its effect is most important from 14 to 15.5μ .

STRAY LIGHT

Before proceeding with the discussion of the transparency of aqueous vapor attention must be directed to the most troublesome source of error of all. The intensity of energy observed at any point in a spectrum will be due in part to the true energy of the wave-length considered and in part to energy scattered there from other regions of the spectrum. Naturally a portion of energy proper

to the wave-length under observation is lost by scattering into other parts of the spectrum. Even with the slit closed the field may not appear dark because of stray light in the room. General field light like this is not very objectionable, for it is present both when observing the zero of energy with the shutter interposed in front of the slit, and when measuring the intensity in the spectrum, and is eliminated.

In a well-designed spectroscope of moderate dispersion, producing a spectrum the intensity in which ranges 100- or even 1,000-fold, scattered light may cause no trouble. But when, as in the present research, the whole spectrum from the violet to 20μ is contained in a dispersion of only about $1\frac{1}{2}$ degrees and the intensities range 100,000-fold, this disturbance became exceedingly troublesome. In the solar spectrum at 10μ , for instance, the field-intensity due to scattered energy may be over 100 times the true energy belonging there. The safest expedient under such circumstances is to use a sifting train¹ or auxiliary spectroscope forming a spectrum on the slit of the main spectroscope and so adjusted as to allow only that part of the spectrum desired to pass through the main slit. The scattered light of other wave-lengths is then deviated in the main spectroscope to its proper place in the spectrum formed, and is negligible.

A very large proportion of the time and labor consumed in the research was devoted to the elimination of errors from this stray light, but it would seriously break the continuity of the exposition to explain it in full here. Accordingly the subject has been relegated to Appendix II, but the main principles of the method employed to correct for stray light follow.

Since nearly all of the radiation of a Nernst glower is of less wave-length than 4μ , it follows that nearly all of the stray light produced by scattering is transmissible by quartz. Hence if the interposition of a quartz plate at a point beyond 4μ , where the ordinate of the energy curve is y_1 , reduces the ordinate to y_2 , the true ordinate proper to the ray which should be found there does not exceed $y_1 - y_2$. But since a quartz plate reflects approximately 15 per cent of the rays at wave-lengths found in the Nernst glower spectrum above 4μ , it is clear that the ordinate y_2 , which is due wholly to stray light transmissible by quartz, would have reached $1.18y_2$ if the quartz had produced no reflection. Hence the true radiation could not have exceeded $y_1 - 1.18y_2$. Although rays of less wave-length than 4μ ,

¹ Memoirs National Academy of Sciences, IV, p. 159, 1888.

since they constitute nearly all of the energy of a Nernst glower, and still more its energy after water-vapor absorption, must furnish most of the stray light, still it cannot be assumed that the stray light of longer wave-lengths than 4μ is negligible. Accordingly to the ordinate $1.18y_2$ there must be made another addition, which we may call Σy , representing the stray light of all wave-lengths non-transmissible by quartz. By a special research the quantity Σy has been determined for each place in the spectrum, as will be explained. The corrected ordinate now becomes $y_1 - 1.18y_2 - \Sigma y$. Finally each ray is, on the other hand, depleted by sending stray light to all parts of the spectrum. But as will be shown the depletion is so nearly the same proportion of the intensity, namely, about 3 per cent, for all the wave-lengths here considered that to correct for it is merely to multiply all ordinates by the same multiplier, which may be omitted. The increase in scattered energy, Σy , over 1.18 times that observed by the insertion of the quartz plate, ranges from 2 to 11 per cent according to the angular deviation. In addition its amount varies over a range of about 2 per cent for the changes in the amount of absorbing vapor occurring in this research.

All the observations taken during 1916 and 1917 were corrected for stray light by the process detailed in Appendix II. The entire absence of appreciable stray light in the part of the research made with the 60° prism unfortunately led to a false feeling of security in the first work with the 15° prism, and in much of the earlier work with it no field-light experiments were made. Accordingly the following indirect way was resorted to for its elimination: The 1916-17 work showed that with the carbon dioxide present in the long tube no energy at wave-lengths longer than 15μ should reach the bolometer. Accordingly all of the considerable deflection observed there could be considered as stray light. The stray light for shorter wave-lengths was assumed to bear the same ratio to this that the ordinates at the corresponding deviations of the stray light curve of figure 18 do to each other. This involves building up an ever increasing correction from a somewhat small observed value. However, fortunately the larger in absolute amount it becomes, the less its importance, for it becomes a smaller fraction of the total observed deflection. The validity of this procedure is perhaps shown by the depths thus obtained in the great band at 7μ where the somewhat large deflection observed should be nearly all stray light.

Before considering further troubles we will pass to a consideration of the results as shown in figures 9 and 10. The form of the observations was already made familiar through figure 8.

DESCRIPTION OF FIGURE 9

Running across the figure will be noted a heavy, black, smooth curve just above which are indicated the wave-lengths. This is a black-body spectrum curve of a source at $2,200^{\circ}$ K. radiating to one at 300° K., as computed from Planck's formula with C_1 equal to 0.23 and C_2 equal to 1.445 for λ in cm. for a 15° rock-salt spectrum. The curve is in three sections, the ordinates of the second and

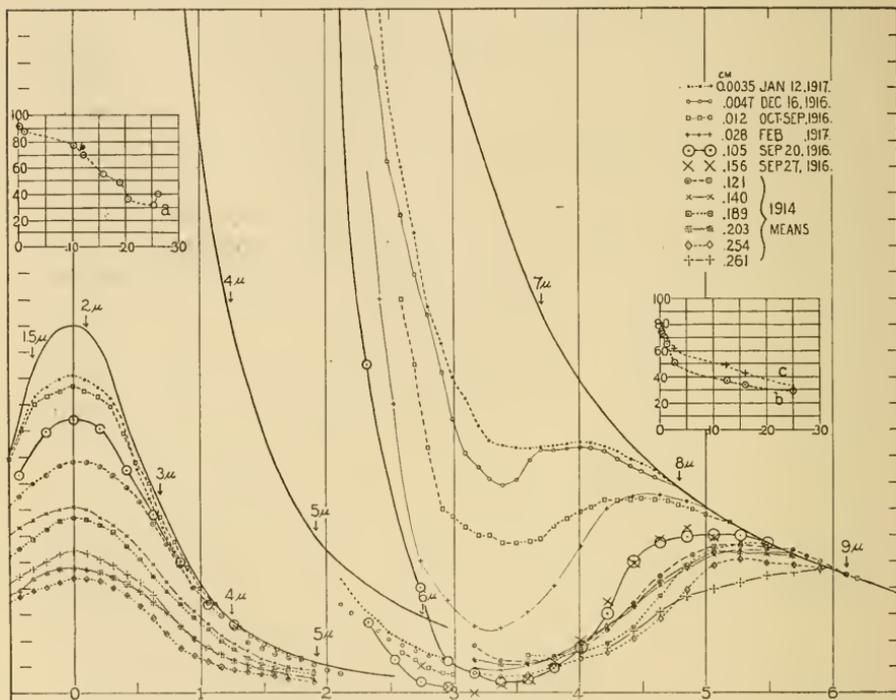


FIG. 9.—Energy curves showing the absorptions produced by water vapor in the 15° rock-salt prismatic spectrum of the Nernst lamp. Ordinates are intensities of energy. Abscissae are proportional to prismatic deviations from 1.8μ . In curves *a*, *b*, *c*, the ordinates are percentage transmissions. Curve *a* gives the transmission percentages for the complex beam including wave-lengths between 1.25μ and 3μ in the lamp spectrum; curve *b*, ditto for between 5μ and 9μ ; curve *c*, ditto but for black body at 287° K.

third section being multiplied relative to the first by 5 and 50 respectively.¹ The plate distances proportional to prismatic devia-

¹ The energy curve is formed for a body radiating from $2,200^{\circ}$ K. to one at 300° K. because the deflection observed with the lamp may be considered as due to the radiation from a body at $2,200^{\circ}$ K. radiating to the bolometer and the deflection due to the insertion of the screen at 300° K. as the radiation

tions, taking as zero the deviation at 1.838μ , are indicated by the numbers at the bottom of the plot.

Underneath these three branches of the black-body curve computed for a temperature corresponding to the "black-body" temperature ($2,200^\circ \text{K.}$) of the Nernst lamp will be found curves representing the energy spectrum of the lamp through increasing amounts of aqueous vapor. These amounts, as indicated in the table on the plot, range from 0.0035 to 0.012 cm. ppt. H_2O when observed through the spectroscope alone; and from 0.028 to .261 when observed through the tube. Each curve is the mean of several sets of observations.

For the curves with 0.0035 to 0.012 cm. ppt. H_2O there were 7 grams carbon dioxide, and in all the others, except the one with 0.254 cm. ppt. H_2O , there were 83 grams carbon dioxide in a 1 m. sq. path; for that one there were 160 grams. For a discussion of its influence on the absorption in the region of figure 9 see the work with the 60° prism.

In the small figure in the upper left-hand corner will be seen a curve (marked *a*) which shows the percentage change in area of the maximum between 1.25 and 3μ plotted against the ppt. H_2O in cm. as abscissae. In the other small figure the similar function is plotted for the region between 5 and 9μ . Curve *b* was read from the large plot and is for a distribution of energy for a body like the Nernst lamp at $2,200^\circ \text{K.}$ Curve *c* was computed for a distribution of energy for a body at a temperature of about 287°K. , which is about the mean temperature of the earth. Table 6 gives more in detail the data from which curves *b* and *c* were drawn. It will be noted in the main curves of figure 9 that between 6 and 7μ the absorption is practically complete for ppt. H_2O of 1 mm. or more.

from a body at 300°K. to the bolometer. Hence the observed intensity of the lamp is really the difference of the deflections in the two cases and should therefore be compared with the black-body curve as drawn. The ratios of the radiation from a body at 300°K. to that from one at $2,200^\circ \text{K.}$, both radiating to absolute zero, are shown in the following table:

TABLE 5.—RATIO OF RADIATION FROM A BODY AT 300°K. TO THAT OF A BODY AT $2,200^\circ \text{K.}$

Wave-length.	2μ	6μ	8μ	10μ	12μ	14μ	16μ	18μ	20μ
Ratio.....	1×10^{-8}	.00067	.0031	.0075	.013	.020	.026	.033	.038

To a wave-length of about 7μ the radiation of the shutter is negligible. At 6μ the observed radiation of the lamp needs to be increased 1/10 of 1 per cent and at 20μ by 4 per cent in order to represent what it would be if radiating to absolute zero.

TABLE 6.—WATER-VAPOR ABSORPTION 5 TO 9 μ

Range of wave-length	Relative energy black-body spectrum.		Percentage absorption						
	2,200° K. Nernst lamp	287° K. Earth.	Precipitable water in cm.						
			.0035	.0047	.012	.028	.125	.16	.25
5-6 μ	1690	142	18	22	25	.43	.55	.59	.65
6-7 μ	947	242	48	54	69	.85	.95	.95	.95
7-8 μ	537	315	15	19	34	.42	.66	.76	.83
8-9 μ	388	360	00	00	02	02	08	13	35
5-9 μ	3562	1059
5-9 μ	Lamp	24	27	35	40	63	66	71
5-9 μ	Earth	18	21	30	38	51	57	67

DESCRIPTION OF FIGURE 10

Figure 10 is a continuation for wave-lengths longer than 9 μ of the curves shown in figure 9. The scale of abscissae is much more condensed. The ordinates of the two right-hand branches are magnified 10-fold relative to the first or left-hand branch. The extreme right-hand curves represent only the work of 1916-17. As there are so many observations it was thought best to shift the abscissae of the 1916-17 work five deviation units to the right to avoid confusion.

As in figure 9, so in figure 10, above the first two branches is given the computed black-body curve corresponding to the effective temperature of the lamp. This will be presently considered. Just below it is another heavy line. It represents the mean of observations of the last three years mainly through the spectroscope alone and within a range of water vapor from 0.003 to 0.028 cm. ppt. H₂O. Within this small range of ppt. H₂O no systematic differences were found.

Observations on a cold day were desired for three purposes: First, to permit the comparison of the curve just described with one through the tube with an amount of vapor of the same order. Generally the tube contained more than 10-fold this amount of water vapor. Second, from a comparison with the mean curve just described to form some estimation of the effect of carbon dioxide. It was supposed that on a very cold day the absorption due to the water vapor in the tube would be so nearly the same as with the observations through the spectroscope alone that the differences would be negligible and thus the increased effect of the carbon dioxide in the tube would be unmasked. Its effect, since its amount in the tube is probably quite constant from day to day, could be then eliminated from the aqueous vapor effects. Third, if coincidence of the two curves taken

with different slit widths occurred, it would serve to assure the absence of errors from diffraction in the tube observations.

Such a cold day was finally obtained in February, 1917, with only 0.028 cm. ppt. H₂O in the optical path. An excellent set of observations was obtained and the mean result of the day, so far as it diverged from the curve of the spectroscope alone, is plotted with simple crosses under the extreme right curve (1916-17 observations). It shows: First, that an increase in water vapor from 0.003 to

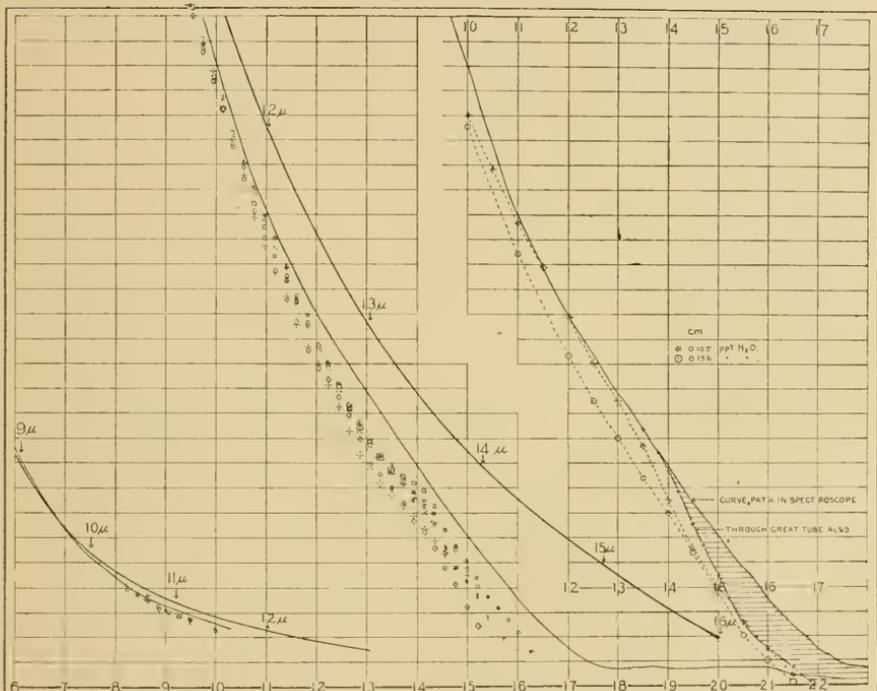


FIG. 10.—Energy curves showing the absorptions produced by water vapor in the 15° rock-salt spectrum of the Nernst lamp. Ordinates are intensities of energy. Abscissae are proportional to prismatic deviations from 1.8 μ . Unless otherwise stated, dates are as indicated in Figure 9.

0.03 cm. (10-fold) produces no appreciable change in transparency of the region from 9 to 13.4 μ . Second, that beyond from 13.4 μ the additional carbon dioxide contained in the tube becomes very effective, and produces practically complete absorption at about 14.6 μ . The cross-hatched portion shows the proportion of energy cut off by absorption. This may be assumed to be the added effect due to the carbon dioxide in the tube over that present in the spectroscope.

It has already been noted in the section on "Diffraction Effects" that the coincidence (up to the region of the carbon dioxide band) of

the curves taken with and without the tube confirmed the view that diffraction with the wide slits used in this region causes little error in the tube work. In table 7 will be found the percentages of absorption at various wave-lengths greater than 9μ as indicated from all the experiments. The values are grouped with regard to the quantity of ppt. H_2O traversed by the beam, but the results of the years 1914 and 1916-17 are kept separate. The lowest line of table 7 comes from observations of Rubens and Aschkinass in which they used an absorbing tube containing 75 cm. of steam at $100^\circ C$. This contained .045 grams ppt. H_2O .

TABLE 7.—WATER-VAPOR ABSORPTION 9 TO 14μ

cm. ppt. H_2O	9.0μ	10.0μ	11.0μ	11.5μ	12.0μ	12.5μ	13.0μ	13.5μ
Up to 0.03.....	0	0	0	0	0	0	0	0
.10.....	0	0	0	..	2	0	3	15
.15 (1914).....	0	0	..	2	3	8	14	15
.16 (1916-17).....	0	0	8	11	15	21
.25 (1914).....	0	3	8	13	24	26
.045 rubens, steam....	5	6	6	8	12	18	13	25

NOTE: 0 indicates no effect detected.

Although Rubens and Aschkinass' work shown in figure 11 was done in a purer spectrum it will be noted from the lines marked *s* in the plot, that a wide range of angular deviation was subtended in the spectrum by the widths of the bolometer and the slit. Not even in their work, and still less in the present research, could the line detail which doubtless exists in the water-vapor bands be expected to show. Referring to the table it may be seen that, as in the observation at smaller wave-lengths made with the 60° prism, the same amount of water absorbing (A) in the form of steam at 76 cm. pressure and (B) at the comparatively low pressure of the order of 1 cm. which prevails under atmospheric conditions, exercises much greater absorption in the form of steam.

COMPARISON OF NERNST LAMP WITH BLACK-BODY SPECTRUM

Down to wave-length 10μ the curve computed from Planck's formula for black-body radiation from a source at $2,200^\circ K$. radiating to a black body at $300^\circ K$. shows no remarkable departure from the Nernst lamp spectrum, when due allowance is made for the absorption bands. From that point on the greater wave-lengths the observed radiation of the lamp systematically falls below that of the black-body curve. The black body curve as plotted in figure 10

was made to agree with the other curve between 9 and 10 μ and allowance has been made in it for the absorption and reflection losses from the rock-salt prism and plate, as computed in table I.

TABLE 8.—COMPARISON OF NERNST LAMP AND BLACK-BODY SPECTRUM

Wave-length μ	11	12	13	14	15	26	52	108
Percentage drop of lamp below black-body radiation	9	15	20	41	86
Excess of transmission of lamp black over that at 10 μ	4	8	11	14	17	47	61	62

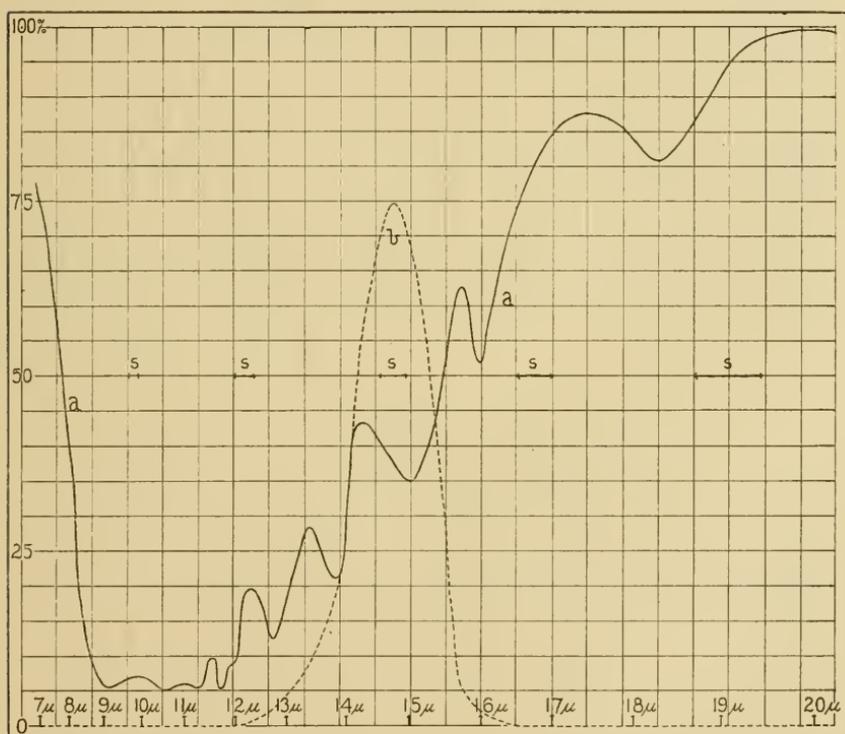


FIG. 11.—Ordinates are percentage absorptions, wave-lengths are indicated at the bottom of the figure. Curve *a*, 75 cm. steam, 76 cm. pressure, 0.045 ppt. H₂O. Curve *b*, 20 cm. CO₂, 76 cm. pressure. Work of Rubens and Aschkinass, *Annalen der Physik und Chemie*, 64, p. 598, 1898.

In the second line of the table are given values for the percentage excess of transparency of lamp black over that at 10 μ . The values were obtained with candle smoke, for a layer containing 1.8 mg. per cm². The figures are principally interpolated from values given

by Rubens.¹ In view of these figures it seems possible that the divergence of the observed Nernst lamp spectrum curve from the computed black-body curve may be partly due to the increasingly incomplete absorption of the rays by the bolometer strip. A part of the divergence is surely due to carbon-dioxide absorption. Although the experiments do not indicate any absorption by water-vapor less than 0.03 cm. ppt. H₂O, they are not quite conclusive. Owing to the impurity of the spectrum, fine line absorption, like that which Abney photographed in $\rho\sigma\tau$, may occur here undetected, and it may possibly produce its complete effects with very minute quantities of water vapor. If so, a part of the discrepancy may be due to water-vapor absorption. Finally, it is not certain that the two energy curves should coincide, for the Nernst lamp may depart widely from being a perfect radiator.

It will be noted that when observed through the spectroscope alone energy from the lamp is found beyond 16 μ even after allowance is made for the field light. The lamp curve is the mean of many observations and no doubt is felt that there is some kind of energy here. Although there may be in it some field light a return of energy is to be expected in this spectrum region beyond the carbon dioxide found where the work of Rubens indicates increased transparency of water vapor.

ATMOSPHERIC TRANSMISSION OF SOLAR ENERGY BETWEEN 9 AND 14 μ

Even with the maximum amount of water vapor possible in the tube work (0.26 cm. ppt. H₂O) the transparency observed between 9 and 14 μ had appeared to be very great. It was thought worth while to attempt a measure of this transparency between 9 and 14 μ by observing the amount of energy transmitted from the sun through the earth's atmosphere for various air masses. In this way there would be brought about a several-fold increase of the amount of absorbing vapor in the path. Indeed at the maximum this amounted to 3 cm. ppt. H₂O.

Reference has already been made,² in discussing the transparency of this wave-length region to the remarkable early work of Mr. Langley. As far as air-mass 3.76 practically no absorption was found by

¹ Verhandlungen Deutsche Physikalische Gesellschaft, 13, p. 88, 1911. His values are at 2 μ , 0.5 per cent transmission; 4 μ , 8.6; 6 μ , 16.0; 12 μ , 37.6; 26 μ , 76.7; 52 μ , 91.3; 108 μ , 91.5.

² S. P. Langley, The Solar and the Lunar Spectrum. Memoirs National Academy of Sciences, IV, p. 159, 1886.

him near wave-lengths 8.8μ and 10.7μ . The apparent decrease in transmission for greater air-masses he attributed to the smoky atmosphere prevalent at Pittsburgh. The great increase in sensitiveness and accuracy of the present day spectro-bolometric apparatus made it seem worth while to repeat these observations.

The scattered light, which in the spectrum of the Nernst lamp was troublesome, becomes absolutely prohibitive here in a directly formed 15° -prismatic rock-salt solar spectrum. At 10μ with the large galvanometer deflection of 20 cm., practically all this deflection remained, and was therefore field light, upon the insertion of the quartz plate. In the case of the Nernst-lamp spectrum at this wave-length only 16 per cent of the deflection was found to be false. Langley eliminated the stray light by using a sifting train, but this was not feasible in the present case. Dr. Abbot suggested the use of a screen of solid iodine, which Coblenz' work¹ showed to be opaque to the visible radiation and increasingly transparent for the longer wave radiation. Accordingly Dr. Abbot with Mr. Aldrich and the writer prepared two plane parallel rock-salt plates 0.65 and 0.34 cm. thick. These were uniformly heated, flakes of iodine placed on one, the other plate quickly placed on top and the two plates squeezed together. After several attempts a thin, fairly uniform film of iodine was thus obtained which micrometer measures showed to be between 0.005 and 0.007 cm. thick.

TRANSPARENCY OF IODINE

Although the direct measures on the solar spectrum were made first, it is perhaps proper to discuss here the later measures on the transmission of the iodine film. The measurement of this is not so simple as might at first be thought, again because of the scattered light. No separate allowance will be made here for the light reflected from the rock-salt plates used to protect the iodine film. The effect of the insertion of the screen as a whole is wished.

The observations consisted in making an energy curve of the 15° -prismatic rock-salt spectrum of the Nernst lamps as usual, except that every few seconds the iodine screen was inserted, then the iodine screen with the quartz plate, then the quartz plate alone, then all screens removed. The significance of the respective deflections measured were as follows when the plate was inserted at wave-length λ greater than 4μ : λ_x is used to signify the wave-length.

(a) Lamp alone: Energy of λ_x + scattered energy of $\lambda < 4 \mu$, + scattered energy of $\lambda > 4 \mu$.

¹ Physical Review 16, p. 72, 1903.

(b) Iodine screen: Energy of $\lambda_x \times$ transmission I for $\lambda_x +$ scattered energy of $\lambda < 4 \mu$, greatly diminished by opaqueness of I for $\lambda < 4 \mu$, + scattered energy of $\lambda > 4 \mu \times$ absorption for it of I .

(c) Iodine and quartz: Scattered energy of $\lambda < 4 \mu$, greatly diminished by opaqueness of I for $\lambda < 4 \mu$, and diminished by reflection from surfaces of quartz.

(d) Quartz: Scattered energy of $\lambda < 4 \mu$, diminished by reflections from surfaces of quartz.

The point especially to be noted is that the "field-light" is not the same in the two deflections (a) and (b) whose quotient would give the transmission of the iodine screen if there were no field light. Before taking the quotient (b)/(a) it was necessary to subtract

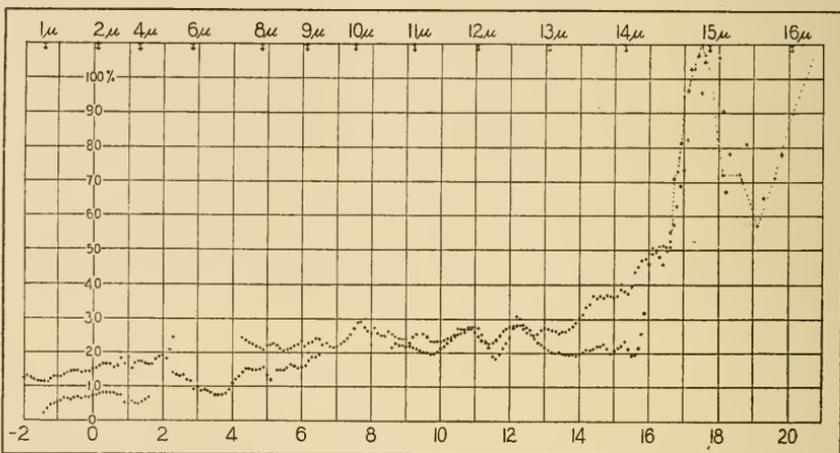


FIG. 12.—Transparency of iodine film about 0.006 cm. thick. Abscissae are proportional to deviation distances on plate from 1.8 μ . The wave-lengths in microns (μ) are indicated at the top of the figure. Ordinates are percentage transmissions.

from (b), (c) + 1.17 (c) + light scattered from other wave-lengths greater than 4 μ and from (a), (d) + 1.17 (d) + light scattered from other wave-lengths greater than 4 μ (see section on field light). Due allowance was computed for all field light as discussed in the Appendix II.

Figure 12 shows graphically the results of the two bolographs. Unfortunately the intense heat from the 44-glower Nernst lamp upon the iodine screen only about 10 cm. away, despite the protection of the rock-salt plates caused an evaporation of the iodine film. This is probably the cause of the discrepancy in height between the two curves in the figure. It should be remembered that the galvanometer

deflections with the iodine screen in, after subtracting the field light, were of course very small for wave-lengths less than $9\ \mu$ partly because of the opaqueness of the iodine itself and partly because of the great water-vapor band preceding $9\ \mu$.

SOLAR OBSERVATIONS

Returning now to the observations on the sun, the following energy curves were made with the assistance of Dr. Abbot and Mr. Aldrich:

Bgms. I and V. Solar spectrum made with apparatus prepared for North Carolina solar-constant work. Bgm. I taken at hour angle

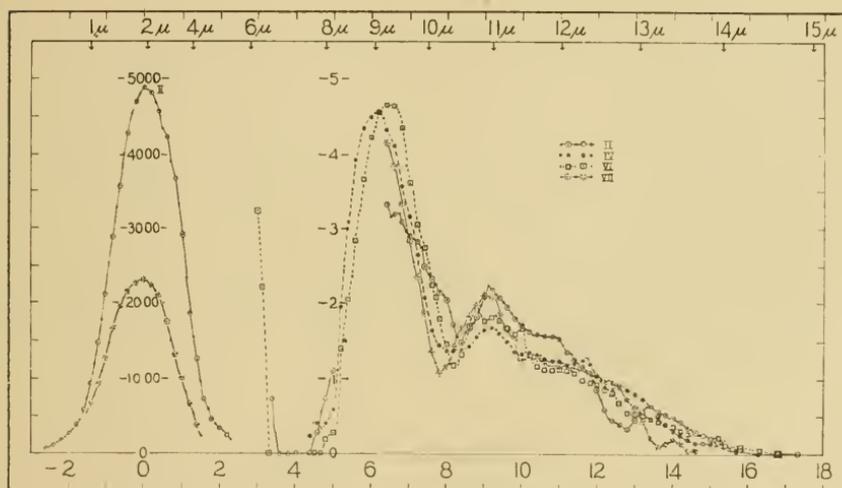


FIG. 13.—Bolograms of solar 15° rock-salt spectrum. Wave-lengths are at the top of the plate.

Bgm. II, air mass 1.5, ppt. H_2O 0.7 cm.

Bgm. IV, air mass 2.1, ppt. H_2O 1.1 cm.

Bgm. VI, air mass 3.0, ppt. H_2O 2.0 cm.

Bgm. VII, air mass 4.1, ppt. H_2O 3.0 cm.

$1^h\ 42^m$ W., air-mass 1.3, for purpose of determining by the depths of the water-vapor bands ρ and ϕ the amount of aqueous vapor in the path of the sun's rays through the atmosphere. These bands have been calibrated (see *Astrophysical Journal* 35, p. 149, 1912) against known amounts of water vapor.

Bgm. II. Solar spectrum made with spectroscope as used in the tube work, but with the iodine screen in front of slit. H. A.

2^h 38^m W., air-mass 1.5. Proper observations were made to eliminate the field light.

Bgm. III	H. A. 3 ^h 27 ^m W.,	$m = 1.8$,	similar to Bgm. II
Bgm. IV	3 55	2.1,	similar to Bgm. II
Bgm. V	4 21	2.4,	similar to Bgm. I
Bgm. VI	4 47	3.0,	similar to Bgm. II
Bgm. VII	5 15	4.1,	similar to Bgm. II

The amounts of water vapor for the various bolograms obtained directly for Bolograms I and V and by interpolation for the others were:

Bgm. I	0.6 cm. ppt. H ₂ O	Bgm. V	1.4 cm. ppt. H ₂ O
Bgm. II	0.7 cm.	Bgm. VI	2.0 cm.
Bgm. III	0.9 cm.	Bgm. VII	3+ cm.
Bgm. IV	1.1 cm.		

The observations on the long-wave-length spectrum of the sun will be found reproduced in figure 13 after due allowance for slit plus bolometer widths and field light.

Again the evidence indicates a great transparency for the aqueous vapor in the atmosphere from the wave-length 9 μ to perhaps 12 or 13 μ . No systematic decrease in the heights in this region of the energy curves with increasing aqueous vapor is found.

The band central at 10 μ , probably due to ozone (Ladenburg and Lehman, *Ann. der Phys.* 21, p. 305, 1906), is the only place within this wave-length region which shows a consistent decrease of energy with increasing air-mass.

SUMMARY OF RESEARCHES ON ATMOSPHERIC ABSORPTION OF RADIATION

A. Water Vapor. Wave-lengths less than 2 μ .

(1) Actual atmospheric conditions; molecular scattering; banded absorption.

B. Water Vapor and Carbon Dioxide. Wave-lengths 1.2 μ to 9 μ .

(1) Laboratory conditions approximating atmospheric conditions as to temperature and pressure. Temperature, 20° to 30° C., total pressure 76 cm., partial pressure of aqueous vapor about 1 cm.

(2) Steam, 100° C., total pressure = partial pressure = 76 cm.

C. Water Vapor and Carbon Dioxide. Wave-lengths 9 to 14 μ .

(1) Temperature and pressure conditions as in B (1).

(2) Temperature and pressure conditions as in B (2).

D. Steam, 76 cm., 100° C. Wave-lengths greater than 20 μ .

E. Liquid Water.

(1) Wave-length less than 2 μ .

(2) Wave-lengths 1 to 8 μ .

(3) Wave-lengths 1 to 18 μ .

F. Atmospheric Absorption of Solar Energy. 9 to 15 μ .

G. Collected Results on Atmospheric Absorption for Long-wave Radiation.

H. Subsidiary Investigations.

(1) Corrections for slit and bolometer widths.

(2) Field energy due to scattering.

(3) Transparency of iodine.

(4) Comparison of Nernst-lamp spectrum with that of black-body.

It is proposed first in this summary to indicate the information available on the absorption in aqueous vapor, not only as obtained in the present research, but also as selected from the work of others.

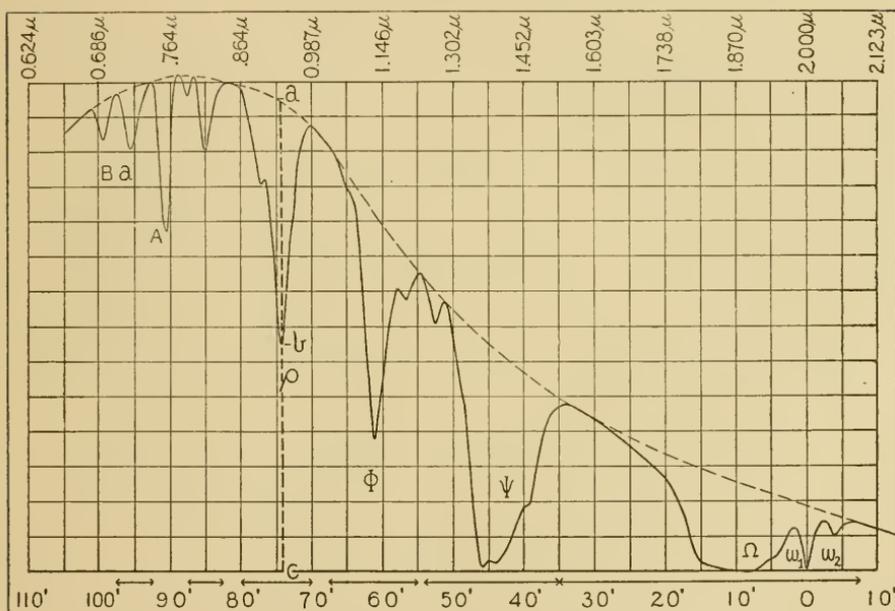


FIG. 14.—Contour solar energy-curve, 60° ultra-violet glass prism. Region of atmospheric absorption bands.

A. ABSORPTION IN WATER VAPOR, WAVE-LENGTHS LESS THAN 2 μ

(1) ATMOSPHERIC CONDITIONS

The region from 0.6 μ to 2.1 μ contains the absorption bands due to atmospheric water vapor and oxygen which chiefly affect the incoming energy from the sun. For wave-lengths less than 0.6 μ , except for a slight absorption due to a large number of small lines in the neighborhood of the D lines, the losses are due principally to molecular scattering. These losses are fully described in the following references:

"Avogadro's Constant and Atmospheric Transparency," *Astrophysical Journal*, 40, p. 435, 1914.

"The Transparency of Aqueous Vapor," *Astrophysical Journal*, 42, p. 394, 1915.

"Atmospheric Transparency for Radiation," *Monthly Weather Review*, 42, p. 2, 1914. (See also *Meteorologische Zeitschrift*, 6, p. 270, 1914.)

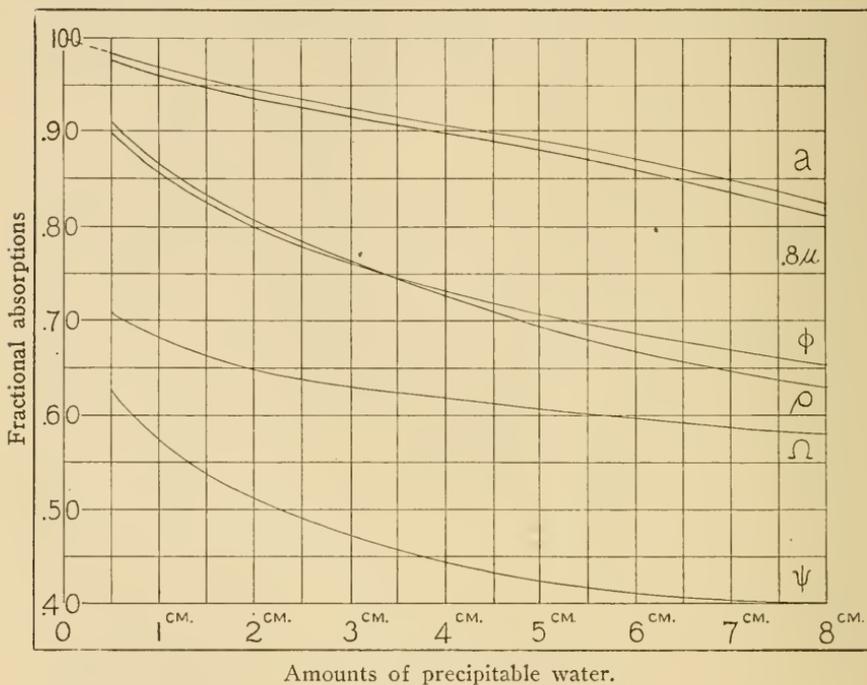


FIG. 15.—Fractional absorption of energy by water-vapor bands.

From the second of the papers just cited is reprinted figure 14 indicating the atmospheric bands between 0.6 and 2.1 μ . Figure 15 shows the absorptions produced by the various bands over the regions indicated by the lines at the bottom of figure 14, and table 9 shows the losses which the incoming solar energy suffers from these absorptions and from scattering in the atmosphere before reaching sea-level. In table 10 will be found the transmission coefficients, $a_{w\lambda}$ for 1 cm. ppt. H_2O and for zenith passage of solar energy to sea-level ($a_{a\lambda}$) for use in the formula, $e_\lambda = e_{0\lambda} \{ a_{a\lambda} a_{w\lambda} \}^m$ for computing the additional losses due to molecular scattering. In this formula w is the ppt. H_2O in cm., m the air mass, in terms of the path vertically through

the atmosphere at sea-level taken as unity. Figure 15 gives means to compute the losses due to absorption in the bands. The scattering losses should be computed first, and then allowance made for the

TABLE 9.—WASHINGTON. ATMOSPHERIC ABSORPTION FOR DRY AIR AND DRY AIR PLUS DUST AND VARIOUS AMOUNTS OF WATER VAPOR

Altitude sea-level; Barometer 76.0 cm.

Incident solar radiation, 1.93 15° C. gram-calories per sq. cm. per minute

Air masses	m=1		m=2		m=3		m=4		m=5		m=7	
	Gram-Cal. lost	Percentage lost	Gram-Cal. lost	Percentage lost.	Gram-Cal. lost.	Percentage lost	Gram-Cal. lost	Percentage lost	Gram-Cal. lost	Percentage lost	Gram-Cal. lost	Percentage lost
Precipitable water vapor												
0.00 cm.												
Air scattered.....	0.18	9.3	0.33	17.1	0.44	22.8	0.53	27.5	0.61	31.6	0.73	37.8
Air absorbed.....	.01	0.5	.01	0.5	.01	0.5	.01	0.5	.01	0.5	.01	0.5
Total lost.....	.19	10.0	.34	18.0	.45	23.0	.54	28.0	.62	32.0	.74	38.0
0.5 cm. February 15												
H ₂ O scattered.....	.08	4.1	.15	7.8	.21	10.9	.26	13.5	.30	15.5	.37	19.2
H ₂ O absorbed.....	.12	6.2	.13	6.7	.14	7.3	.14	7.3	.14	7.3	.14	7.3
Total lost.....	.39	20.0	.62	32.0	.80	42.0	.94	49.0	1.06	55.0	1.25	65.0
1.8 cm. October 4												
H ₂ O scattered.....	.26	13.5	.42	21.8	.53	27.5	.61	31.6	.67	34.7	.74	38.3
H ₂ O absorbed.....	.15	7.8	.16	8.3	.16	8.3	.14	7.3	.13	6.7	.11	5.7
Total.....	.60	31.0	.92	48.0	1.14	59.0	1.29	67.0	1.42	74.0	1.59	82.0
2.4 cm. May 14												
H ₂ O scattered.....	.38	19.7	.56	29.0	.67	34.7	.74	38.3	.78	40.4	.83	43.0
H ₂ O absorbed.....	.16	8.3	.16	8.3	.15	7.8	.14	7.3	.13	6.7	.10	5.2
Total.....	0.73	38.0	1.06	55.0	1.27	66.0	1.42	74.0	1.53	79.0	1.67	86.0

TABLE 10.—60° ULTRA-VIOLET GLASS PRISMATIC SOLAR ENERGY-CURVE: ALSO DRY AIR AND AQUEOUS VAPOR (1 CM. PRECIPITABLE WATER) TRANSMISSION COEFFICIENTS

λ.....	0.342	0.350	0.360	0.371	0.384	0.397	0.413	0.431	0.452	0.475
c _o λ.....	102	130	160	198	227	322	437	518	681	807
a _a λ.....	(0.595)	(0.626)	0.655	0.686	0.713	0.752	0.783	0.808	0.840	0.863
a _w λ.....	0.920	0.926	0.934	0.940	0.945	0.949	0.953	0.957	0.961	0.964
λ.....	0.503	0.535	0.574	0.624	0.686	0.764	0.864	0.987	1.146	1.302
c _o λ.....	907	1044	1197	1334	1416	1435	1431	1306	1025	775
a _a λ.....	0.885	0.898	0.905	0.929	0.950	0.979	0.987	0.992	0.996	0.997
a _w λ.....	0.968	0.972	0.970	0.975	0.981	0.984	0.986	0.987	0.987	0.987
λ.....	1.452	1.603	1.738	1.870	2.000	2.123	2.242	2.348
c _o λ.....	586	435	343	262	187	123	88	74
a _a λ.....	0.998	0.999	0.999	0.999	0.999	0.999	0.999	0.999
a _w λ.....	0.987	0.987	0.987	0.987	0.986	0.985	0.984	0.983

band absorption losses by means of figure 14, remembering that the losses there given pertain to the energy over a range of wave-lengths indicated by the corresponding lines at the bottom of figure 14.

B. ABSORPTION IN WATER VAPOR AND CARBON DIOXIDE FOR WAVE-LENGTHS 1.2 TO 9.0 μ

(1) LABORATORY CONDITIONS APPROACHING ATMOSPHERIC CONDITIONS AS TO TEMPERATURE, TOTAL AND PARTIAL PRESSURES

Observations described in the main body of this report were made in a 60°-prismatic rock-salt spectrum, of the effect of water vapor and carbonic-acid gas to deplete energy from the radiation of a Nernst lamp (at 2,200° K.) in this spectrum region. The absorbents comprised in some experiments 0.008 cm. ppt. H₂O and carbon dioxide of such an amount that there would have been 7.4 grams in a column the length of the path and a meter square in section; and in other experiments 0.082 cm. ppt. H₂O and 83 grams of carbon dioxide. For mental comparison, the total amount of carbon dioxide similarly measured vertically from the surface of the earth outwards is about 3,000 grams.

The appearance of the absorption bands in the energy curve of the lamp is shown in the lower half of figure 6. The computed percentage absorptions are given in figure 7. The percentage absorptions in regions extending over various wave-length ranges are given in table 4. Paschen's results for carbon dioxide are given in table 3. It is through table 4 that the results are most easily applicable. The amount of radiation available before absorption between the wave-lengths indicated in column 2 should be multiplied by the ratio belonging to the proper amount of ppt. H₂O taken from the columns headed "percentage absorbed" and the result will be the amount lost in the corresponding water vapor. Table 4 may be supplemented for greater amounts of vapor by the data in table 6 obtained from the observations with the 15° rock-salt prism.

(2) RESULTS FOR STEAM (1.2 TO 9.0 μ)

The results obtained by Paschen for steam are given in the *Annalen der Physik und Chemie*, 52, p. 209, 1894. In figure 7 are shown the results for the water-vapor absorption band from 5 to 8 μ . With equal quantities of ppt. H₂O, water vapor in the form of steam evidently produces considerably more absorption than when dispersed at small partial pressure as in the atmosphere, even although the same total pressure prevails. Referring to figure 7 the partial pressure of Paschen's steam (at 100° C.) was equal to its total pressure, 76 cm. The partial pressure of water vapor in the present research (at 20 to 30° C.) was of the order of a centimeter, the total pressure 76 cm.

C. ABSORPTION IN WATER VAPOR FOR WAVE-LENGTHS 9.0 μ TO 14 μ
AND BEYOND

(1) LABORATORY CONDITIONS APPROXIMATING ATMOSPHERIC CONDITIONS AS TO
TEMPERATURE, TOTAL AND PARTIAL PRESSURES

In the main body of this report will be found in detail the observations for this region made with a 15° rock-salt prism. The amounts of vapor range from 0.003 cm. to 0.26 cm. ppt. H₂O. Figure 8 shows the appearance of a record of bolographic observation, somewhat complicated by the observations necessary to eliminate field light. The principal results relating to the absorption of water vapor are shown in figure 10 and table 7.

For information on the effect of carbon dioxide in this spectrum region (partially indicated by the shaded region in figure 10) the main body of the paper must be consulted. A path through the atmosphere of the order of 100 meters is sufficient to produce complete absorption in the carbon dioxide band between 14 and 15.5 μ (see fig. 11). Besides the work published by Rubens and Aschkinass, and shown in figure 11, further data will be found in "Über das ultrarote Absorptions-spectrum der Kohlensäure in seiner Abhängigkeit von Druck und Partial-druck," von G. Hertz, Verh. Deutsche Physikalische Gesellschaft, 13, p. 617, 1911.

(2) RESULTS FOR STEAM (7 TO 20 μ)

In figure 11 will be found the results of Rubens and Aschkinass. They show the percentage absorption due to 0.045 cm. ppt. H₂O in the form of steam at 100° C. and 76 cm. pressure. Again by comparison with table 7 it will be noted that the same amount of water in the form of steam gives a greater absorption than when at a smaller partial pressure although the same total pressure.

D. RESULTS FOR STEAM. WAVE-LENGTHS GREATER THAN 20 μ

Isolated results for greater wave-lengths have been obtained by the use of the "Restrahlungen" from various crystals. Rubens and Wartenberg¹ found that a column of steam 40 cm. long 100° C. 76 cm. pressure (0.024 cm. ppt. H₂O), transmits radiation as follows: 39.6 per cent at 23 μ , 0.7 at 52 μ , 19.6 at 110 μ and 49.2 at 314 μ . At the same wave-lengths 20 cm. of carbon dioxide at 76 cm. pressure showed no absorption. Rubens² later found that water vapor in the whole region between 45 and 120 μ causes many bands of great

¹ Verh. Deutsche Physikalische Gesellschaft, 13 p. 797, 1911.

² Berichte Kgl. Preuss. Akademie der Wissenschaften, p. 513, 1913.

absorption. It is especially absorbing at 50μ , 66μ and 79μ and apparently also at 58μ and 103μ . It is relatively more transparent at 47μ , 54μ , 62μ , 75μ , 91μ and 115μ .

E. ABSORPTION BY LIQUID WATER

(1) FOR WAVE-LENGTHS LESS THAN 2μ

It has been thought worth while both for comparison and for its practical value to include the following data for liquid water. As already stated, in giving the absorption of water vapor in terms of the precipitable water, it should not be inferred equivalent to that of the same quantity of liquid water. In general the absorption of an equivalent amount of water is different in the three states, liquid, atmospheric vapor, and steam.

For the shorter wave-lengths data have been obtained by Kreuzler,¹ Ewan,² and Aschkinass³ from which are taken the following values of a in the formula $i = i_0 e^{-ad}$ in which d is in cm., i_0 the original intensity of a ray and i its intensity after transmission. A more complete table will be found in Kayser's Handbuch, Vol. 3.

TABLE II.—ABSORPTION OF RADIATION BY LIQUID WATER
WAVE-LENGTHS LESS THAN 0.5μ

λ in μ	.186	.193	.200	.220	.230	.240	.260	.300	.415	.430	.450	.475	.487	.500
a	.0688	.0165	.0090	.0061	.0057	.0034	.0025	.0015	.00035	.00023	.00020	.00020	.00014	.00020

Between the two absorption bands at 0.115μ (metallic absorption⁴) and at 0.50μ these values of absorption are what would be expected from purely molecular scattering.⁵ When the same water is in the form of atmospheric vapor its absorption is greater. This will be seen by comparing $a_w \lambda$ of table 10, for example at $.5 \mu$, with the a in the above table at the same wave-length remembering $a_w \lambda = e^{-a}$. The vapor value and liquid value in the same units are 0.33 and $.0002$ respectively.

As the true absorption bands are approached the liquid water rapidly becomes a far more effective absorber than the vapor. The following coefficients, a , are given by Nichols⁶ for the transmission of a cell of water 1 cm. thick.

¹ Annalen der Physik, 6, p. 412, 1901.

² Proceedings of the Royal Society, 57, 117, 1894.

³ Annalen der Physik und Chemie, 55, 401, 1895.

⁴ Martens, Annalen der Physik, 6, p. 603, 1901.

⁵ Astrophysical Journal, 38, p. 392, 1913, and unpublished data.

⁶ Physical Review, 1, p. 1, 1894.

TABLE 12.—TRANSMISSION OF LIQUID WATER (1 CM.) .8 TO 2.8 μ

λ in μ	.779	.865	.945	1.19	1.41 to 2.8
Transmission per cent . .	76.2	74.4	58.4	14.4	Too small to measure

W. Schmidt¹ in his "Absorption der Sonnenstrahlung im Wasser" gives computations based upon the data of Aschkinass shown in figure 16. He gives the transmission for various thicknesses of water and states that a layer 1,000 m. thick produces complete absorption for wave-lengths greater than 0.6 μ , 10 m. for those greater than 0.9 μ , 10 cm. for those greater than 1.2 μ , 1 cm. for those greater than 1.5 μ and 1 mm. for those greater than 2.4 μ . It should be remembered that his results are from computation. Because of the

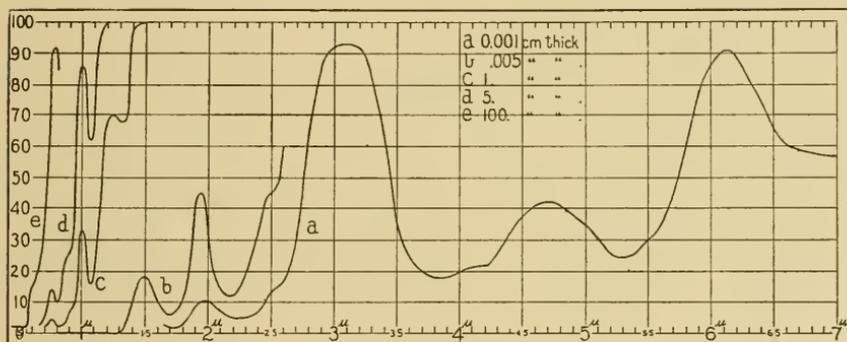


FIG. 16.—Absorption of radiation by liquid water. Aschkinass, *Annalen der Physik und Chemie*, 55, p. 401, 1895. Abscissae are wave-lengths in microns (μ). Ordinates are percentage transmissions.

impurity of the spectrum used to obtain the transmission of vapor, and the banded nature of the absorption, such computations based on Bouguer's formula are doubtful, and inferences obtained from the curves of figure 16 would probably be as satisfactory.

(2) ABSORPTION BY LIQUID WATER, WAVE-LENGTHS 1 μ TO 8 μ

In figure 16 will be found a portion of the data obtained by Aschkinass for layers of water ranging from 0.001 cm. to 100 cm. thick. The curve continues on nearly horizontally from 7 to 8 μ . The data will be found given in much greater detail in the original publication.²

¹ *Sitzungsberichte K. Akad. der Wissenschaften*, 117, 2 A, p. 237, 1908.

² *Annalen der Physik und Chemie*, 55, p. 401, 1895.

(3) ABSORPTION BY LIQUID WATER, WAVE-LENGTHS 1 TO 18 μ

In figure 17 are given data obtained by Rubens and Ladenburg¹ with the use of two bubble films, one (b) 3.09 μ thick and containing 10 per cent glycerine, the other (a) 1.89 μ thick and containing 25 per cent glycerine. The maxima in both curves at 7.0 and 9.5 μ are probably due to glycerine.

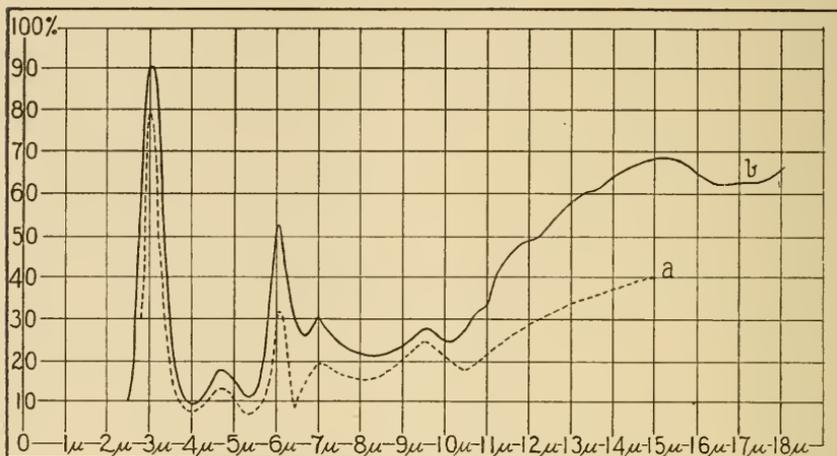


FIG. 17.—Absorption of radiation by liquid water films. Rubens and Ladenburg, *Verh. D. Phys. Gesell.* 11, p. 16, 1909. Abscissae are wave-lengths in microns (μ). Ordinates are percentage absorptions. *a*, film 0.00789 mm. thick; *b*, 0.00309 mm. thick.

The main laboratory results of this research, done to obtain quantitative measures of the transparency of aqueous vapor for radiation, have just been summarized. Below the great water-vapor band 6 to 8.5 μ , which is very strong even with the small amounts of vapor present in the laboratory, the great transparency of the region stretching from 9 towards 14 μ was surprising and absorption effects there were scarcely perceptible. The total depletion of energy by the great carbon dioxide band 14 to 15.5 μ and the decreasing transparency of the rock-salt prism employed prevented results for longer wave-lengths.

F. ATMOSPHERIC TRANSMISSION OF SOLAR ENERGY. 9 TO 15 μ

The unexpected transparency for wave-lengths just longer than 9 μ led to direct observations of solar energy through paths in air which increased with decreasing altitudes of the sun. A range of

¹ *Verh. Deutschen Physikalischen Gesellschaft*, 11, p. 16, 1909.

aqueous vapor from 0.6 cm. to over 3 cm. ppt. H_2O was thus obtained. Except in the band probably due to ozone at about 10.3μ no systematic change appeared in this region of spectrum (see fig. 13) and thus great transparency was indicated. This band at 10.3μ systematically deepens with increasing air-mass.

G. COLLECTED RESULTS ON ATMOSPHERIC ABSORPTION FOR LONG-WAVE RADIATION

Table 13 contains a summary of the results on the absorptive power of atmospheric water vapor and their application to determine the total radiation outwards vertically to space from the earth when the atmosphere contains 0.003, 0.03, 0.3 and 3.0 cm. precipitable water. The effect of carbon dioxide and the band of unknown origin at about 10μ has been included:

TABLE 13.—ATMOSPHERIC ABSORPTION OF EARTH RADIATION

Range of wave-length μ	Energy of black body $287^\circ K.$	Percentage absorption			
		Precipitable water in cm.			
		0.003	0.03	0.3	3.0
3-4	5	10	30	50	75
4-5	50	15	45	70	95
5-6	142	16	43	66	95
6-7	242	45	85	95	100
7-8	315	13	42	85	100
8-9	360	0	2	40	50
9-10	380	0	0	0	15
10-11	370	0	2	5	40
11-12	350	0	0	4	10
12-13	320	0	0	13	20
13-16	810	100	100	100	100
16-20	510	(90)	100	100	100
20-30	900	(70?)	(80?)	(90?)	100
30-40	300	(100?)	(100?)	(100?)	100
40-50	150	(100)	(100)	(100)	(100)
50-60	75	(100)	(100)	(100)	(100)
3-60	5,279	49	57	66	75

In accordance with the values given in the last line of the table the vertical transmissions of the earth's radiation are therefore 51, 43, 34, and 25 per cent, corresponding to 0.003, 0.03, 0.3, and 3 cm. ppt. H_2O . Further, applying these last figures to the transmission of radiation outwards in all directions from a horizontal surface at sea-level, assuming Lambert's cosine law, and 1 cm. ppt. H_2O , it is found that 28 per cent of the earth's radiation under such circumstances passes directly out to space.

H. SUBSIDIARY INVESTIGATIONS

(1) CORRECTION FOR SLIT AND BOLOMETER WIDTHS

The different angular widths of the bolometer and the slit necessitated the derivation of a new formula for reducing approximately the observed spectrum intensities to what they would have been with an infinitely narrow slit and bolometer. This derivation with its limitations is given in Appendix I.

(2) FIELD ENERGY DUE TO SCATTERING

The complication arising from the immense amount of scattered light present in a spectrum of such great range of intensities (100,000 fold) and small dispersion required extended study. This discussion covered the energy scattered into any region from all wavelengths as well as that scattered away. It will be found in Appendix II, where a separate summary gives the generalizations.

(3) TRANSPARENCY OF IODINE. FIGURE 12

The need of a suitable absorbing screen to cut out the intense short wave energy, the scattering from which rendered observations otherwise useless in the solar spectrum, led to the preparation of a thin film of iodine, 0.005 to 0.007 cm. thick between two rock salt plates whose combined thickness was about 1 cm. The results of two measures of the transparency of the iodine film are shown in figure 12.

(4) COMPARISON OF NERNST LAMP SPECTRUM WITH THAT OF BLACK BODY

The comparison of the radiation from a black-body radiating at 2,200° K. (apparent black-body temperature of the lamp) to one at 300° K. (the temperature of the shutter used for zero deflection) is shown in figures 6, 9, and 10. The agreement, with due allowance for the absorption bands, seems satisfactory down to about 10 μ . From here on the lamp curve increasingly falls below the black-body curve. It is uncertain that the lamp radiates here as a black-body but the departure may be chiefly due to the decreasing absorption of the incident radiation by the soot-blackened surface of the bolometer.

NOTE

There are two doubtful points upon which light has been thrown since the above was written: 1st, May not even the small amount of aqueous vapor present in the path through the spectroscope exercise nearly complete and considerable absorption in narrow lines

in the region from 9 to 15 μ and yet the additional amounts of vapor produce no further absorption there? 2d, In treating of the earth's radiation outward, complete absorption for atmospheric amounts of vapor has been assumed for wave-lengths greater than 20 μ . Is this certain? The first point could be settled only by observing the radiation through a path containing no water vapor. Dr. Coblentz (Proc. Nat. Acad. Sc. 3, p. 504, 1917) has just published observations on a black body radiating at 800° C. Comparing the radiation after passing through a tube containing 0.00045 cm. ppt. H₂O with that when the tube was exhausted he found the absorption due to this amount of vapor to be about 0.9 per cent. If all the absorption had been produced in the region between 9 and 15 μ , the mean absorption in this region must have been 12 per cent. 12 per cent, then, is the maximum absorption permissible here with this amount of vapor. Assuming the absorption in the known water-vapor bands is proportional to the amount of vapor, within the range from no vapor to the smallest amount used in the present research, the following table is probably representative of the absorptions from wave-length to wave-length in Dr. Coblentz' case.

Wave-lengths (μ)	1.5-2.0	2-3	3-4	4-5	5-6	6-7	7-8	8-9
Percentage absorption..	0.5	1.0	1.0	1.0	2.0	5.0	1.5	0
Relative energy black body, 800°C.....	300	1380	1400	1030	750	550	400	270

Total energy same scale, 6825.

The absorption computed from the above data would be 1.3 per cent, which is probably within experimental error the same as that found by Dr. Coblentz. This confirms the conclusion of great transparency in the region between 9 and 15 μ .

As to the second point, recent nocturnal experiments by Mr. L. B. Aldrich on Mount Wilson with the pyranometer, employing salt and sylvine screens tend to confirm the assumption that the atmosphere transmits no appreciable quantity of radiation from the earth of wave-lengths greater than 20 μ .

APPENDIX I

CORRECTION TO OBSERVED ENERGY CURVE FOR WIDTH OF SLIT AND BOLOMETER

The true monochromatic intensity in a spectrum would occur only with the use of an infinitely narrow slit and then the intensity would be infinitely small. As both the slit and the observing device must

be of finite width, an intensity is observed which corresponds to the sum of the intensities of the energy of the various wave-lengths falling on the observing device, a bolometer in the present case. A correction must be determined for reducing the observed intensity to the pure or monochromatic value. Such a correction for the case where both the slit and the observing apparatus subtend the same angular value in the spectrum has been determined by Runge.¹ His method of deriving the correction is not directly applicable to the present case where the widths may be unequal and the following modification in the derivation was suggested by Dr. Abbot and worked out in cooperation with the writer in 1909. The resulting equation was applied to all the observed energy curves of the present research.

Let x denote the position (deviation) in the spectrum under consideration. At x , then, lies the middle of the slit image of the color whose wave-length may be λ . This slit image will extend from the deviation $x - \frac{a}{2}$ to $x + \frac{a}{2}$ where a is the slit width expressed as its angular value in the spectrum. It is assumed here, with safety, that the spectrum on each side of x is a minimum deviation spectrum, although this is not strictly true. Each wave-length as it falls in succession from the center of the slit upon the center of the bolometer is kept automatically in minimum deviation but the image of the whole spectrum formed at any time at the bolometer is not strictly a minimum deviation spectrum.

The amount of energy, which should appear in the deviation from x to $x + dx$, is not condensed in the interval dx but spread over the interval from $x - \frac{a}{2}$ to $x + \frac{a}{2}$. Let the amount of this energy be adE_x where dE_x is the amount of energy flowing through unit slit in unit time. Of this energy only the portion $dE_x \cdot dx$ falls in the interval between x and $x + dx$. However portions of the slit images belonging to the deviations $x - \frac{a}{2}$ to $x + \frac{a}{2}$ to each side, do fall within the interval dx so that the total amount of energy falling in the interval between x and $x + dx$ is equal to

$$dx \int_{-\frac{a}{2}}^{+\frac{a}{2}} \frac{dE_{x+v}}{dv} dv = \left\{ E \left(x + \frac{a}{2} \right) - E \left(x - \frac{a}{2} \right) \right\} dx.$$

¹ "Ueber die Differentiation Empirischen Functione," Zeitschrift für Mathematik und Physik, 42, 1897; see also Hyde, Astrophysical Journal, 35, p. 237, 1912.

The intensity of the spectrum at x may be defined as the energy over an indefinitely small interval dx divided by dx . That of the impure spectrum at x is therefore

$$\left\{ E \left(x + \frac{a}{2} \right) - E \left(x - \frac{a}{2} \right) \right\}.$$

That of the pure spectrum is $\frac{dE_x}{dx} \cdot a$ as a becomes infinitely small.

However, only the relative intensity from one part of the spectrum to another is required and not necessarily the absolute infinitely small intensity which the abstract pure spectrum would have.

In measures with the bolometer not even the intensities in this impure spectrum are obtained directly but rather the sum of the intensities in it over the region covered by the bolometer. If the middle of the bolometer is at the deviation x and its width is b , then it receives in a unit time the amount of energy

$$F(x) = \int_{-\frac{b}{2}}^{+\frac{b}{2}} \left\{ E \left(x + v + \frac{a}{2} \right) - E \left(x + v - \frac{a}{2} \right) \right\} dv.$$

The observed intensity in the spectrum is proportional to this $F(x)$.

A development of $\frac{dE_x}{dx}$, which is proportional to the intensity in the pure spectrum, in terms of $F(x)$ is desired.

If $f(x)$ is taken to denote $\frac{dE}{dx}$, then by Taylor's theorem

$$E \left(x + v + \frac{a}{2} \right) = E(x) + f(x) \left(v + \frac{a}{2} \right) + \frac{1}{2!} f'(x) \left(v + \frac{a}{2} \right)^2 + \dots$$

so that

$$\int_{-\frac{b}{2}}^{+\frac{b}{2}} E \left(x + v + \frac{a}{2} \right) dv = E(x)b + f(x) \frac{ab}{2!} + f'(x) \frac{3a^2b + b^3}{3!4} + \dots$$

Making a similar development for the second term of the expression for $F(x)$ and subtracting, there results

$$F(x) = 2 \left\{ f(x) \frac{ab}{2!} + f''(x) \frac{a^3b + ab^3}{4!2} + \dots \right\}$$

Let $b = an$, a being the slit width and n the ratio of the width of the bolometer to that of the slit, then

$$F(x) = 2 \left\{ \frac{na^2}{2!} f(x) + \frac{na^4(1+n^2)}{4!2} f''(x) + \frac{na^6(3+10n^2+3n^4)}{6!4^2} f^{iv}(x) + \dots \right\}.$$

Similar developments may be made for $F''(x)$, $F^{iv}(x)$, , whence using the undetermined coefficients C_1, C_2, \dots as multipliers and adding:

$$F(x) + C_1 F''(x) + C_2 F^{iv}(x) + \dots = 2 \left\{ \frac{na^2}{2!} f(x) + \left[\frac{na^4(1+n^2)}{4!2} + \frac{na^2}{2} C_1 \right] f''(x) + \left[\frac{na^6(3+10n^2+3n^4)}{6!4^2} + \frac{na^4(1+n^2)}{4!2} C_1 + \frac{na^2}{2} C_2 \right] f^{iv}(x) + \dots \right\}.$$

Putting $k_1 = n$, $k_2 = n(1+n^2)$, $k_3 = n(3+10n^2+3n^4)$, and equating coefficients, it follows then that

$$a^2 k_1 f(x) = F(x) - \frac{k_2 a^2}{k_1^2} F''(x) + \frac{10k_2^2 - k_2 k_1}{k_1^2} \frac{a^4}{5760} F^{iv}(x) \dots \quad (1)$$

Now letting $2ca$ be the portion of the energy curve covered by the slit plus the bolometer, c being equal to $1+n$ and therefore $2ca$ to $2(a+b)$, then

$$F(x+ca) = F(x) + caF'(x) + \frac{c^2 a^2}{2!} F''(x) + \frac{c^3 a^3}{3!} F^{iii}(x) + \frac{c^4 a^4}{4!} F^{iv}(x) + \dots$$

Forming a similar development for $F(x-ca)$ and adding and transposing, there follows

$$F''(x) = \frac{2}{c^2 a^2} \left\{ \frac{F(x+ca) + F(x-ca)}{2} - F(x) - \frac{c^4 a^4}{4!} F^{iv}(x) + \dots \right\}$$

which may be put in the form

$$F''(x) = \frac{2}{c^2 a^2} \left\{ \phi_1(x) - \frac{c^4 a^4}{4!} F^{iv}(x) + \dots \right\}$$

Substituting in (1) there follows that

$$a^2 k_1 f(x) = F(x) - \frac{k_2}{k_1} \frac{1}{12c^2} \phi_1(x) + \frac{10k_2^2 - k_2 k_1 + 20k_2 k_1 c^2}{5760k_1^2} a^4 F^{iv}(x) + \dots \quad (2)$$

Now

$$\phi_1(x) = \left\{ \frac{F(x+ca) + F(x-ca)}{2} - F(x) \right\},$$

and

$$\phi_1(x+ca) = \phi_1(x) + ca\phi_1'(x) + \frac{c^2 a^2}{2!} \phi_1''(x) + \frac{c^3 a^3}{3!} \phi_1^{iii}(x) + \dots$$

making a similar development for $\phi_1(x-ca)$, adding and transposing

$$\phi_1''(x) = \frac{2}{c^2a^2} \left\{ \frac{\phi_1(x+ca) + \phi_1(x-ca)}{2} - \phi_1(x) \right\} + \dots$$

which let equal, say, $\frac{2}{c^2a^2} \phi_2(x)$; but

$$\phi_1(x) = \frac{c^2a^2}{2} F''(x) + \dots$$

$$\phi_1''(x) = \frac{c^2a^2}{2} F^{iv}(x) + \dots$$

Whence

$$\begin{aligned} F^{iv}(x) &= \frac{2}{c^2a^2} \phi_1''(x), \text{ approximately} \\ &= \frac{4}{c^4d^4} \left\{ \frac{\phi_1(x+ca) + \phi_1(x-ca)}{2} - \phi_1(x) \right\} \\ &= \text{say, } \frac{4}{c^4d^4} \phi_2(x). \end{aligned}$$

Substituting in (2) there results the following development of the energy curve $f(x)$ as, with certain reservations, it might be expected to appear with an infinitely narrow slit and bolometer, in terms of the observed energy curve $F(x)$.

$$\begin{aligned} a^2k_1f(x) &= F(x) - \frac{k_2}{k_1} \frac{1}{12c^2} \phi_1(x) \\ &\quad + \frac{10k_2^2 - k_3k_1 + 20k_1k_2c^2}{1440k_1^2c^4} \phi_2(x) + \dots \\ &= F(x) - \frac{1+n^2}{12c^2} \phi_1(x) + \frac{7+10n^2+7n^4+20c^2(1+n^2)}{1440c^4} \phi_2(x) + \dots \end{aligned}$$

When $n=1$, or the slit and the bolometer are of the same angular width in the spectrum, which is Runge's case, the second half of the above equation reduces to

$$= F(x) - \frac{1}{6} \phi_1(x) + \frac{2}{45} \phi_2(x) + \dots$$

The coefficients for the various cases in the present research were:

TABLE 14.

Slit mm.	a^* mm.	Bolometer mm.	b^* mm.	Coefficients of		$\frac{2ca}{2a} \text{ or}$ $\frac{2}{1+n}$
				ϕ_1	ϕ_2	
0.67	0.05	0.5	0.29	$\frac{1}{4}$	0.086	0.68
2.40	.19	$\frac{1}{6}$.047	0.96
4.99	.40	$\frac{1}{6}$.046	1.38
5.82	.46	$\frac{1}{6}$.049	1.50
10.69	.85	$\frac{1}{3}$.063	2.28

* Equivalent to differences of deviation as expressed in millimeters on the bolographs or energy curves.

An example will best show the method of using the formula. Taking the case with a slit of 0.67 mm. linear width and subtending in the spectrum an angular width equivalent to 0.05 mm. as measured on the plate (equal to a) and a bolometer 0.5 mm. wide and subtending 0.29 mm. (equal to b) on the plate, then the formula becomes

$$a^2k_1f(x) = F(x) - 1/4 \cdot \phi_1(x) + 0.086\phi_2(x)$$

$$= F(x) - \frac{1}{4} \left\{ \frac{F(x+3mm) - F(x-3mm)}{2} - F(x) \right\}$$

$$+ 0.086 \left\{ \frac{\phi_1(x+3mm) - \phi_1(x-3mm)}{2} - \phi_1(x) \right\} \dots$$

TABLE 15.—EXAMPLE OF THE APPLICATION OF THE FORMULA FOR CORRECTING FOR THE FINITE WIDTH OF THE SLIT AND THE BOLOMETER

x	$F(x)$	$\phi_1(x)$	$\phi_2(x)$	$-\frac{1}{4}\phi_1(x)$	$+0.086\phi_2(x)$	$a^2k_1f(x)$
0	1,000
1	1,120
2	1,220
3	1,300	-103	+26
4	1,360	-105	+26
5	1,390	-105	+26
6	1,395	-100	-10	+25	-1	1,419
7	1,390	-105	-3	+26	0	1,416
8	1,350	-100	+5	+25	0	1,375
9	1,290	-118	+66	+29	+6	1,325
10	1,210	-110	+81	+28	+7	1,245
11	1,110	-85	+70	+21	+6	1,137
12	950	-5	-24	+1	-2	949
13	810	+46	-70	-11	-6	793
14	700	+70	-84	-18	-7	675
15	600	+60	-38	-15	-3	582
16	503	+62	-21	-15	-2	486
17	430	+56	-14
18	370	+50	-12
19	320	+36	-9
20	272
21	241
22	210

The first step as followed out in table 15 was to read $F(x)$, the observed energy curve at every mm. Next $\phi_1(x)$ was obtained by subtracting, for example, the value of $F(x)$ at $x=3$ from the mean of the values at $x=3-3$ or 0 and $x=3+3$ or 6 and so on proceeding down the column. $\phi_2(x)$ was obtained from $\phi_1(x)$ exactly as was $\phi_1(x)$ from $F(x)$. The completion of the process is evident from the headings of the columns, the final column giving the corrected energy curve.

LIMITATIONS

It is evidently impossible by any such process as that just developed and described to obtain, in general, from an impure spectrum, a pure spectrum, except in the case of a continuous energy curve such, for instance, as would be expected from an incandescent black body. A line cannot be made to appear which has been completely obliterated by the impurity of the spectrum although one which has been rendered shallow may be deepened, and a maximum which has been lowered may be made to approximate its proper height.

APPENDIX II

STRAY LIGHT AND ITS DETERMINATION

The most intense region in the rock-salt prismatic spectrum of the Nernst glower lies at wave-lengths less than 4μ . From this region, which is transmissible by quartz, most of the field light comes. At 4μ the intensity of the spectrum had already decreased nearly to $1/10$. Interest in the present research centers in wave-lengths greater than 4μ and *not* transmissible by quartz. Energy proper to any place in the spectrum of wave-length greater than 4μ has been distinguished from that scattered into it from the intense short-wave-length spectrum region by observing what portion of the deflection remains when a plate of quartz is inserted between the source of energy and the spectroscopie slit. Since the quartz is opaque to radiation of wave-lengths greater than 4μ , the deflection observed through quartz must be all false and due to energy of wave-length less than 4μ . Without the quartz, the deflection represents this false energy, plus the true longer-wave radiation, plus certain other corrections to be presently considered.

Turning again to figure 8, the significance of the central curve $a'b'c'd'e'$ as a means of determining this field light will be considered. This curve, which is essentially an energy curve of the source observed through a plane parallel quartz plate $1/2$ cm. thick, consists of two parts of quite different significance, namely, region a and region $b'c'd'e'$.

Region a'.—Curve a' indicates approximately the intensity of energy in the lamp spectrum for wave-lengths between 0.6 and 4.0μ for which quartz is nearly transparent. The difference between the areas a and a' (after making certain correction for the absorption in the quartz of the longer waves between 3 and 4μ) measures the amount of energy of this region reflected from the quartz surfaces.

This amount can also be computed by Fresnel's and by Bouguer's formulae for reflection and absorption, respectively, from the known coefficients of refraction (Rubens)¹ and transmission (Merritt).² Several computations made before the idea of inserting the quartz in this branch had occurred, gave this loss as about 19 per cent. Considering the uncertainties due to the great impurity of the spectrum used for computing this loss, this is in sufficient agreement with the mean value of 15 per cent found from subsequent curves like *a* and *a'*.

Region b'c'd'e'.—For the energy of prismatic deviations belonging to the region *b'c'd'e'* quartz is opaque. The energy recorded must therefore belong to wave-lengths for which quartz is transparent. That is to say, the observed energy is scattered from and belongs to region *a*. There must be reflected from the quartz surfaces some of the energy scattered into the second region (namely, an amount corresponding to the difference in energy between the areas *a* and *a'*). Therefore the curve *b'c'd'e'* does not represent all the energy scattered here from *a'* but requires to be increased by the mean observed ratio *a/a'* (or 1.18) in order to represent the total field light from region *a'*.³ But even this is not sufficient to give the total field light in the long-wave parts of the spectrum for it gives only that coming from region *a* and it will be directly seen that the scattered energy of even longer wave-lengths than $4\ \mu$ is appreciable. The somewhat complex determination of the correction for stray light, not transmissible by quartz, will now be considered.

The two regions *a* and *b'c'd'e'* corrected as just described and reduced to a more convenient scale are reproduced in figure 18 as *a* and *b₁'c₁'*, the latter magnified respectively a thousand fold and three thousand fold relative to region *a*. With close approximation the first curve may be considered as the energy curve of the region producing the scattering shown in the second. The energy curve of the region *a*, from which the energy is scattered, seems so symmetrical that it appears probable that the distribution of scattered energy from it, *b₁'c₁'*, would not be materially altered if all its energy should be concentrated at the center of the region. Let it be assumed that the whole energy of region *a* is concentrated in a central strip 4' of spectrum (1 cm. of plate) wide as indicated by the dotted lines in figure 18. Let this central strip be joined to the curve *b₁'c₁'* by

¹ Rubens, *Annalen der Physik und Chemie*, 54, p. 476, 1895.

² Merritt, *Annalen der Physik und Chemie*, 55, p. 459, 1895.

³ As stated above the mean loss observed is 15 per cent. The reciprocal of the 85 per cent remaining is 1.18.

what seems a plausible interpolation. Then this central strip, with its wing-like appendage $b_1'c_1'$, may be assumed to represent the energy curve of an approximately monochromatic source of radiation, including the portion of its energy scattered through the spectrum region on its long wave-length side. A symmetrical wing should evidently stretch to its short wave-length side also.

It is proposed to use the curve just described to obtain the scattering taking place at each point in the spectrum from wave-lengths

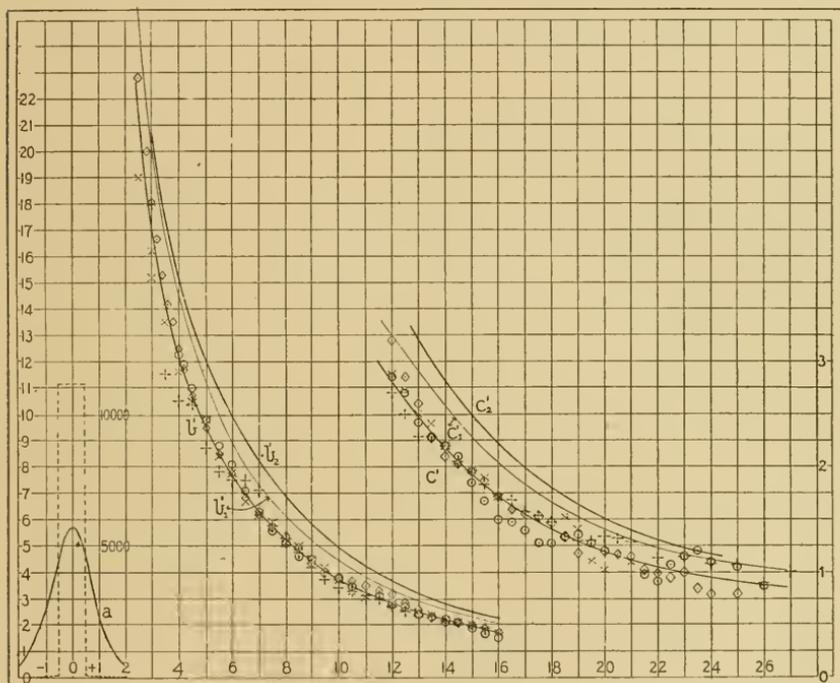


FIG. 18.—Field-light curves.

greater than 4μ and not directly obtained by the insertion of the quartz plate. This involves the assumption that the contour of this curve representing the scattering is the same for all wave-lengths coming into consideration. Gorton¹ has published curves showing, for great angles of incidence and unpolished, comparatively rough surfaces, the variation of the scattering with the wave-length. These show, as might be expected, less scattering for the longer wave-lengths. The smallest angle of incidence used was 54° . The obser-

¹ Physical Review, 7, p. 74, 1916.

vations indicate, as the angle of incidence decreases, a much more uniform reflective power for different wave-lengths, even with such rough surfaces. Accordingly the assumption of a comparatively uniform scattering power at nearly normal incidence (about 3° in the present case) for polished surfaces is not unreasonable.

In order to compute the relative amount of true and scattered radiation at any prismatic deviation an analytical expression for the observed spectrum in terms of these quantities will be built up and then modified to suit actual computations. The first term of this equation should represent, for any deviation, the energy due to the ordinary dispersion of the prism, the second term that lost from the region by scattering and the third that gained by scattering.

Let θ be the deviation taken as zero at 1.8μ . Let $S(\theta)$ represent the function which is the analytical expression of the complete, symmetrical curve of scattering of approximately monochromatic light, the central and right-hand portion of which curve is shown as $a_1'b_1'c_1'$ in figure 18. Let the true energy curve of the lamp be $E(\theta)$, and the observed energy curve $E_0(\theta)$.

The first term of the equation for $E_0(\theta)$ is evidently $E(\theta)$, the energy proper to the deviation.

The second term represents the energy properly belonging to this deviation, but scattered elsewhere. To a close degree of approximation, it is equal to the area of the curve of scattering taken so that its maximum ordinate is equal to the true intensity in the spectrum. With the reservation stated in the second succeeding paragraph, which also applies to the third term, it is therefore equal to $\int_{-\infty}^{+\infty} S(x) dx$ multiplied by the ratio of the ordinate of the true energy curve, $E(\theta)$, at this deviation to that at 1.8μ , $E(\theta_0)$, where θ_0 is taken to signify $\theta=0$.

The third term represents the energy belonging to other deviations but scattered here. To a close degree of approximation it is equal to a sum of successive infinitesimal areas determined by $S(\theta)$, the distance of each area, infinitely narrow in abscissae, from the maximum of $S(\theta)$ being determined by the distance $(\theta+x)$ of the deviation contributing the scattering. The height of the area is so taken that the height of the corresponding maximum, $S(\theta_0)$, shall be equal to the true height of the energy curve at the deviation from which this scattered portion is derived. Analytically this is equal to $S(x) dx$ multiplied by the ratio of $E(\theta+x)$ to $E(\theta_0)$ and integrated over all deviations.

The integrals of the second and third terms will be seen to both contain the term $S(x) \frac{E(\theta)}{E(\theta_0)} dx$ which is not desired but which fortunately occurs in the two integrals with opposite sign. In other words, the function used for the scattered energy includes the energy which should be observed at $\theta=0$ and which is not to be considered as scattered energy. This energy is accounted for in $E(\theta)$.

Accordingly the following expression is obtained:

$$E_0(\theta) = E(\theta) - \frac{E(\theta)}{E(\theta_0)} \int_{-\infty}^{\infty} S(x) dx + \frac{1}{E(\theta_0)} \int_{-\infty}^{\infty} E(\theta+x) S(x) dx,$$

or transposing,

$$E(\theta) = E_0(\theta) + \frac{E(\theta)}{E(\theta_0)} \int_{-\infty}^{\infty} S(x) dx - \frac{1}{E(\theta_0)} \int_{-\infty}^{\infty} E(\theta+x) S(x) dx.$$

All the functions on the right-hand side of the second equation are known except $E(\theta)$. For this may be substituted a first approximation to it obtained by subtracting from the observed lamp deflections the amount of scattered energy indicated by the quartz plate, suitably corrected, as already described, for the reflections from the surface of the plate. The resulting values, which call $E_a(\theta)$ will be found at their greatest to differ by only about 1 per cent where they are of importance in deriving the additional scattered energy. Hence $E_a(\theta)$ will be a sufficiently close approximation for use in the second and third terms.

Table 16 shows the process used to evaluate this expression for the total field light. The numbers in the first column and the first line indicate the prismatic deviations (θ) measured in centimeters on the plate (1 cm. = 4' of deviation in the spectrum, zero of deviation at 1.8 μ). Each other number is a measure of the energy under the curve $S(\theta)$ summed for a difference of deviation extending from -0.5 to +0.5 from the indicated deviation. It represents that portion of energy scattered from the deviation at the top of the column into that indicated in the first column.

$$E_a(\theta)$$

Consider first the numbers surrounded by the rectangles and appearing like a flight of steps. These represent $E_a(\theta)$, the first approximation to the true amount of energy in the spectrum of the lamp for the corresponding deviations indicated in the first line or column. They represent also the terms common to both integrals.

$$\frac{E(\theta)}{E(\theta_0)} \int_{-\infty}^{+\infty} S(x) dx$$

Consider next the numbers in the other vertical columns. These give the energy scattered from the "step" values into the deviations indicated in the first column and are determined by the function S . The maximum in each case falls at the deviation corresponding to the "step" value, the other values falling off similarly above and below to represent the energy scattered towards the violet and the long wave-lengths respectively. These values plotted with the

Deviations																				A	B	B/A													
	$\theta = -4$	-3	-2	-1	0	$+1$	$+2$	3	4	5	6	7	8	9	10	11	12	13	14	15	16		17	18	19	20	Field	Light	Total						
-10	9	20	71	237	450	185	30	5	3	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	472	981				
-9	11	24	85	278	520	181	35	5	4	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1134	1181				
-8	15	30	103	334	612	1209	42	6	4	4	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1345	1344				
-7	20	39	130	405	735	246	48	8	5	4	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1623	1647				
-6	28	54	168	510	1892	295	57	9	6	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2004	2031				
-5	36	75	233	660	1120	358	69	10	7	6	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2551	2581			
-4	11180	96	324	915	1454	450	83	13	8	7	4	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3318	3354			
-3	36	29960	418	1274	12010	583	104	15	9	8	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4425	4464			
-2	28	96	129700	1640	2800	808	134	15	11	10	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5506	5559			
-1	20	75	418	508600	3600	1125	167	24	14	12	6	4	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5425	5474			
0	15	54	324	1274	118000	1450	260	34	18	15	8	4	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3743	3824		
+1	11	39	233	1274	13600	449000	334	48	25	19	10	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5491	5804		
+2	9	30	168	915	2800	1450	04000	61	35	26	12	7	4	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5372	5522		
3	7	24	103	660	12010	1125	334	19000	45	36	18	9	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4240	4381		
4	6	20	130	510	1450	808	260	61	4100	47	24	12	6	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3157	3315		
5	5	16	85	405	1120	583	197	48	45	14530	31	17	8	4	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2401	2541	
6	4	14	71	334	6921	450	134	34	35	47	19780	22	12	6	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1894	2041	
7	4	12	60	278	735	358	104	24	25	36	31	15	7	4	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1581	1702	
8	3	10	52	237	612	245	83	19	18	26	24	12	6	4	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1244	1421	
9	3	9	45	205	520	246	69	15	14	19	18	10	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1047	1216	
10	3	8	38	175	450	209	57	13	11	15	12	12	12	11	7	4	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	940	1041	
11	2	7	34	148	385	181	48	10	9	12	10	9	8	9	4	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	806	893	
12	2	6	30	132	325	155	42	9	8	10	8	7	6	6	7	7	1677	4	3	1	1	0	0	0	0	0	0	0	0	0	0	0	693	749	
13	2	5	27	117	1201	131	35	8	7	8	6	5	5	4	5	5	1374	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	608	672	
14	2	5	23	104	1258	117	30	6	6	7	5	4	4	3	3	4	4	4	1049	2	1	1	0	0	0	0	0	0	0	0	0	0	539	573	
15	2	5	21	93	1228	104	27	5	5	6	5	4	3	3	3	3	3	3	750	2	1	0	0	0	0	0	0	0	0	0	0	0	480	529	
16	1	4	20	83	1244	92	24	5	4	5	4	3	3	2	2	2	2	2	498	1	1	0	0	0	0	0	0	0	0	0	0	0	430	470	
17	1	4	17	76	1183	82	21	4	4	4	3	3	2	2	2	2	2	2	360	1	1	0	0	0	0	0	0	0	0	0	0	0	384	421	
18	1	3	16	70	1167	74	19	4	3	4	3	2	2	2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	350	379	
19	1	3	15	64	1153	67	17	3	3	3	3	2	2	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	320	347	
20	1	3	14	59	1141	62	16	3	2	3	2	2	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	246	320
21	1	3	13	56	1130	57	14	3	2	3	2	2	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	274	295
22	1	3	13	53	1123	52	13	3	2	2	2	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	258	275
23	1	2	12	51	1116	49	12	2	2	2	2	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	243	258
24	1	2	12	49	1112	45	11	2	2	2	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	232	242
25	1	2	11	47	1108	43	11	1	2	2	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	223	232
\sum_{-4}^{+25}					17113																														
$2 \sum_{-4}^{+25}$					34226																														
$2 \sum_{-4}^{+25} \frac{1}{11180}$					0.0305																														

TABLE 16

deviations of the first column would make a curve similar to $S(\theta)$ but with the maximum ordinate equal to $E_a(\theta)$. These values, added from the top to the bottom of a column and including the "step" value, would give the value of the term containing the first integral, or the total amount of energy belonging to the deviation indicated by the "step" value,¹ *i. e.*, this sum indicates the energy observed at

¹ The columns to the left are not carried upward to their full extent.

this deviation plus that scattered to other deviations. The sum, excluding the "step" value and therefore equal to the amount scattered and lost from the corresponding wave-length, is expressed at the foot of the column of zero deviation. It amounts to about 3 per cent of the intensity observed at the deviation to which the energy belongs. This correction of 3 per cent, in accordance with one of the assumptions, is independent of the wave-length and therefore does not alter the shape of the energy curve.

$$\frac{I}{E(\theta_0)} \int_{-\infty}^{+\infty} E(\theta+x)S(x)dx$$

Consider next the horizontal lines. The numbers added in this direction, including the "step" value, give the values of the term containing the second integral. When added, excluding the "step" value, they give the amount of light scattered into the region indicated by the "step" value of the line from the regions whose deviations are indicated at the tops of the columns of the individual terms of the sum. Remembering that the first-integral term does not affect the shape of the desired energy curve since it may be put in the form of a constant factor multiplying the observed energy of each deviation, the values connected with the second integral are those in which interest will at present center.

In the set of sums in the right-hand group of columns, will be found in the first column the sums taken horizontally from the 2d to the 8th columns inclusive. These values give the energy scattered from the wave-lengths between the deviations -4 and $+2$ cm. (*i. e.*, wave-lengths transmissible by quartz) into the corresponding deviations of the horizontal lines. These values therefore should be, and are, proportional to the field energy obtained by the insertion of the quartz plate. The next column gives the sums of the numbers taken horizontally completely across, but omitting the "step" values. They represent the total field energy due to scattering from all wave-lengths. The next column gives the division of the second of these sums by the first, or is the ratio by which the total field light exceeds that contributed from the region for which quartz is transparent. The last column includes the factor 1.18 to correct for the reflection from the quartz surfaces.

It is evident then from inspection of the last two columns that the deflection obtained on the insertion of quartz has to be increased not only by 18 per cent for losses by reflection from the quartz surfaces but also by an added amount of from 2 to 11 per cent in order to obtain the total energy scattered from all spectrum regions into that

under observation. The total field light therefore ranges (last column) from 20 to 30 per cent greater than that observed with the quartz plate. This added amount depends on the shape of the energy curve. It is somewhat less the greater the amount of water vapor because of the considerable absorption by the water vapor of the energy less than 9μ which plays such a disturbing rôle in the production of field energy.

SUMMARY

The following table gives a summary indicating the amounts and increasing importance of the scattered light with increasing angular deviation. This increasing importance with increasing prismatic deviation depends on the more rapid decrease of the energy in the prismatic spectrum than in the scattered band of energy overlapping this spectrum.

TABLE 17.—FIELD LIGHT SUMMARY

1 Wave-length in (μ)	1.8	5.0	5.7	7.0	8.0	10.0	12.0	14.0	17.5
2 Deviation in ($'$)	0	8	10	15	20	30	45	60	100
3 Deviation in (cm.)	0	2.0	2.5	3.8	5.0	7.5	11.2	15.0	25.0
4 Quartz transmits	85,000	504	206	117	85	50	27	17	8
5 1.18 \times ditto	100,000	595	242	141	100	59	30	20	10
6 Total field light		600	244	146	107	66	33	22	11
7 Black-body radiation	100,000	8,000	5,000	1,800	1,000	400	140	65	20
8 Nernst lamp	100,000	8,000	5,000	1,800	1,000	400	140	50	2

Line 1 gives the wave-lengths in millionths of a meter (μ).

Line 2 gives the difference of deviation in minutes of arc ($'$) in a 15° rock-salt prismatic spectrum, the deviation at 1.8μ being taken as zero.

Line 3 gives these differences as cm. measured on the plate $1 \text{ cm.} = 4'$.

Line 4 gives the Nernst-lamp spectrum observed through a quartz plate, $\frac{1}{2}$ cm. thick. The energy indicated belongs to the deviation but not to the wave-lengths indicated as it must be all of wave-lengths transmissible by quartz, namely, less than 4μ .

Line 5 is 1.18 times line 4, allowing for reflection of energy of wave-length less than 4μ from the quartz surfaces. It nearly represents the energy curve of a monochromatic line of intensity 100,000 together with the energy scattered to the long wave-length side. See curve ab_1c_1' of figure 18 for a representation of this energy curve.

Line 6 is line 5 increased by the process developed in this appendix to give the total light scattered from all wave-lengths as it would occur in the spectrum of a Nernst lamp when the intensity of the latter is taken as 100,000 at 1.8μ (see line 8). These values are represented by the curves b_2c_2' of figure 18.

Line 7 computed relative intensities in the black-body spectrum of a source at $2,200^\circ \text{ K.}$ radiating to a similar one at 300° K. , corrected for absorptions

and reflections at the rock-salt prism and plate by table 1. Intensity taken as 100,000 at 1.8μ .

Line 8 corrected intensities in the Nernst lamp spectrum approximating the conditions of line 7 but differing at the greater wave-lengths probably because of the decreasing absorption of energy by the lamp-blackened surface of the bolometer strip. (See discussion relative to comparison of black-body spectrum in section so headed in main body of paper).

It was found that the total energy scattered from any region is only about 3 per cent of that belonging to that region. The angle of incidence of the radiation on the image forming mirror was slightly less than 3° . The intensity of scattered energy falls off very rapidly to each side from the central image (see line 5). If the intensity is 100,000 in the central image, at an angle $10'$ it amounts to only $\frac{1}{4}$ of 1 per cent, at $20'$, $1/10$ of 1 per cent, then falling off more slowly, it amounts to $1/100$ of 1 per cent at $100'$. The total field energy (line 6) at $10'$ amounts to 5 per cent of that of the lamp spectrum upon which it is superposed, at $60'$ it amounts to nearly 50 per cent and at $100'$ to over 500 per cent. As already stated, in a solar spectrum formed with the same apparatus, the stray light at 10μ was more than 100 times as intense as that which belonged there.

The above data were obtained with silver-on-glass mirrors, the surfaces of which were in excellent condition, freshly polished to a hard, compact surface. The following table shows a comparison with results obtained several months later after the mirrors were so badly tarnished as to be unfit for work in the visible spectrum. It will be noted that the scattering had increased relatively more for the greater deviations from the central image.

TABLE 18—INCREASE OF SCATTERING WITH TARNISHING

Deviation ($'$) ...	16	20	24	32	40	48	60	80	100
Wave-length (μ)	4.0	8.2	8.9	10.3	11.4	12.5	13.8	15.8	17.5
Increase ratio...	1.22	1.26	1.27	1.34	1.42	1.40	1.47	1.66	1.70

It would be of interest to know how far this work is applicable to determine stray light in other cases. The above results were a side issue of the more extensive investigation described in this report, and no more time could be spared to investigate this matter. In the work on the solar spectrum, involving resilvering of all the mirrors, a different distribution of energy in the spectrum, in which the upper infra-red much more greatly predominated over the longer wave portions than with the Nernst glower, produced curves of stray light which were the same within the limits of observational errors. Part

of the research involved the use of a different spectroscope using a rock-salt prism of 60° . In this spectroscope at $10\ \mu$ with a deviation of $150'$, (instead of the $30'$ with the 15° prism), practically no field light was observable upon the insertion of the $\frac{1}{2}$ cm. thick quartz plate. Now line 6 of table 16 shows that at $10\ \mu$ the deflection, which (with a rock-salt prism in both cases) should be dependent only on the wave-length, would be $400/100,000$; whereas the field light, if dependent only upon the deviation, would be perhaps $4/100,000$ or only 1 per cent of the true deflection. The deflection in the spectrum was so small at this wave-length (see fig. 6, upper part) that a deflection of 1 per cent of it would not have been detectable.

Two idiosyncrasies of the field light may be noticed: (1) Mention has been made of the error produced in the determination of the diaphragm values supposed to be caused by diffraction with the long wave-lengths and narrow slits. If this explanation were true, it would be expected that the values obtained at the same deviations with the field light would not show this error, since the field light is principally due to the more intense short wave-length energy. This was found to be so. (2) Another cause besides the natural decrease of energy with the greater wave-length tends to make the field light increasingly disturbing with the longer wave-lengths. For the absorption in the rock-salt of the true spectrum energy gets more and more effective as the wave-length increases; whereas the field light, being of short wave-lengths, passes through the prism almost unhindered.