SMITHSONIAN MISCELLANEOUS COLLECTIONS
VOLUME 62, NUMBER 4

Hodgkins Fund

REPORTS ON WIND TUNNEL EXPERIMENTS IN AERODYNAMICS

(With Five Plates)

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(Publication 2368)

CITY OF WASHINGTON
PUBLISHED BY THE SMITHSONIAN INSTITUTION
JANUARY 15, 1916

The Lord Baltimore Press
BALTIMORE, MD., U. S. A.

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REPORTS ON WIND TUNNEL EXPERIMENTS IN AERODYNAMICS

I. THE WIND TUNNEL OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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An aeroplane or airship in flight has six degrees of freedom, three of translation and three of rotation, and any study of its behavior must be based on the determination of three forces—vertical, transverse, and longitudinal—as well as couples about the three axes in space. Full scale experiments to investigate the aerodynamical characteristics of a proposed design naturally become mechanically difficult to arrange. The experimental work is much simplified if tests be made on small models as in naval architecture, and a further simplification is made by holding the model stationary in an artificial current of air instead of towing the model at high speed through still air to simulate actual flying conditions.

The use of a wind tunnel depends on the assumption that it is immaterial whether the model be moved through still air or held stationary in a current of air of the same velocity. The principle of relative velocity is fundamental, and the experimental discrepancies between the results of tests conducted by the two methods may be ascribed on the one hand to the effect of the moving carriage on the flow of air about the model and to the effect of gusty air, and on the other hand to unsteadiness of flow in some wind tunnels.

The wind tunnel method requires primarily a current of air which is steady in velocity both in time and across a section of the tunnel. The production of a steady flow of air at high velocity is a delicate problem, and can only be obtained by a long process of experimentation. A study was made of the principal aerodynamical laboratories of Europe from which these conclusions were reached: (1) That the wind tunnel method permits a leisurely study of the forces and couples produced by the wind on a model; (2) that the staff of the National Physical Laboratory, Teddington, England, have developed a wind tunnel of remarkable steadiness of flow and an aerodynamical balance well adapted to measure with precision the forces and couples on a model in any position; and (3) that the results of model tests made at the above laboratory are applicable to full scale aircraft.

Consequently it was decided to reproduce in Boston the four-foot wind tunnel of the National Physical Laboratory, together with the aerodynamical balance and instruments for velocity measurement. Dr. R. T. Glazebrook, F. R. S., director of the National Physical Laboratory, most generously presented us with detail plans of the complete installation, including the patterns from which the aerodynamical balance was made. Due to this encouragement and assistance we have been able to set up an aerodynamical laboratory with confidence in obtaining a steady flow of air of known velocity. The time saved us by Dr. Glazebrook, which must have been spent in original development, is difficult to estimate.

The staff of the National Physical Laboratory have developed several forms of wind tunnel in the past few years. In 1912-13 Mr. Bairstow and his assistants conducted an elaborate investigation into the steadiness of wind channels as affected by the design both of the channel and the building by which it is enclosed.[1] The conclusions reached may be summarized as follows:

(1) The suction side of a fan is fairly free from turbulence.

(2) A fan made by a low pitch four-bladed propeller gives a steadier flow than the ordinary propeller fan used in ventilation, and a much steadier flow than fans of the Sirocco or centrifugal type.[2]

(3) A wind tunnel should be completely housed to avoid effect of outside wind gusts.

(4) Air from the propeller should be discharged into a large perforated box or diffuser to damp out the turbulent wake and return the air at low velocity to the room.

(5) The room through which air is returned from the diffuser to the suction end of the tunnel should be at least 20 times the sectional area of the tunnel.

(6) The room should be clear of large objects.

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[1] Technical Report of the Advisory Committee for Aeronautics, London, 1912-13. Report No. 67.

[2] It is of interest to note that Mr. Eiffel has used a helicoidal blower in his new wind tunnel.

The wind tunnel of the Institute of Technology was built in accordance with the English plans, with the exception of several changes of an engineering nature introduced with a view to a more economical use of power and an increase of the maximum wind speed from 34 to 40 miles per hour.

Upon completion of the tunnel an investigation of the steadiness of flow and the precision of measurements was made in which it appeared that the equipment had lost none of its excellence in its reproduction in the United States.

As will be shown below, the current is steady both in time and across a cross-section within about 1 per cent in velocity. Measurements of velocity by means of the calibrated Pitot tube presented by the National Physical Laboratory are precise to one-half of 1 per cent. Force and couple measurements on the balance are precise to one-half of 1 per cent for ordinary magnitudes. Calculated coefficients which involve several measurements of force, moment, velocity, angle, area, and distance, as well as one or more assumptions, can be considered as precise to within 2 per cent. It is believed that it is not practicable to increase the precision of the observations to such an extent that the possible cumulative error shall be materially less than the above.

Description of Wind Tunnel

A shed 20 by 25 by 66 feet houses the wind tunnel proper, 16 square feet in section, and some 53 feet in length (pl. 1). Air is drawn through an entrance nozzle and through the square tunnel by a four-bladed propeller, driven by a 10 H. P. motor. Models under test are mounted in the center of the square trunk on the vertical arm of the balance to be described later.

The air entering the mouth passes through a honeycomb made up of a nest of 3-inch metal conduit pipes 2 feet 6 inches in length. This honeycomb has an important effect in straightening the flow and preventing swirl.

Passing through the square trunk and past the model, the air is drawn past a star-shaped longitudinal baffle into an expanding cone. In this the plans of the National Physical Laboratory were departed from by expanding in a length of 11 feet to a cylinder of 7 feet diameter. This cone expands to 6 feet in the English tunnel. M. Eiffel affirms that the working of a fan is much improved by expanding the suction pipe in such a manner as to reduce the velocity and so raise the static pressure of the air. Since the fan must discharge into the room, the pressure difference that the fan must maintain is thus reduced. Also with a larger fan the velocity of discharge is reduced, and the turbulence of the wake kept down.

The propeller works in a sheet metal cylinder 7 feet in diameter, and discharges into the large perforated diffuser. The panels of the latter are gratings and may be interchanged fore and aft. The gratings are made of 1½-inch stock with holes 1½ by 1½ inches. Each hole is then a square nozzle one diameter long. The end of the diffuser is formed by a blank wall. The race from the propeller is stopped by this wall and the air forced out through the holes of the diffuser. Its velocity is then turned through 90 degrees. The area of the diffuser holes is several times the sectional area of the tunnel, and the holes are so distributed that the outflow of air is fairly uniform and of low velocity (pl. 2, fig. 1).

A four-bladed black walnut propeller (pl. 2, fig. 2) was designed on the Drzwiecki system and has proved very satisfactory. In order to keep down turbulence a very low pitch with broad blades had to be used. To gain efficiency such blades must be made thin. It then became of considerable difficulty to insure proper strength for 900 R. P. M. as well as freedom from oscillation.

The blade sections were considered as model aeroplane wings and their effect integrated graphically over the blade. The blade was given an angle of incidence of 3 degrees to the relative wind at every point for 600 R. P. M. and 25 miles per hour. The pitch is thus variable radially.

To prevent torsional oscillations, the blade sections were arranged so that the centers of pressure all lie on a straight line, drawn radially on the face of the blade. This artifice seems to have prevented the howling at high speeds commonly found with thin blades. The propeller has a clearance of ½ inch in the metal cylinder.

The propeller is driven by a "silent" chain from a 10 H. P. interpole motor beneath it. The propeller and motor are mounted on a bracket fixed to a concrete block and are independent of the alignment of the tunnel. Vibration of the motor and propeller cannot be transmitted to the tunnel as there is no connection.

The English plans for power contemplate a steady, direct current voltage. Such is not available here. A 15 H. P. induction motor is connected to the mains of the Cambridge Electric Light Company. This motor then turns at a speed proportional to the frequency of the supply current for a given load. Fluctuations of voltage are without sensible effect, and the frequency may be taken as practically constant.

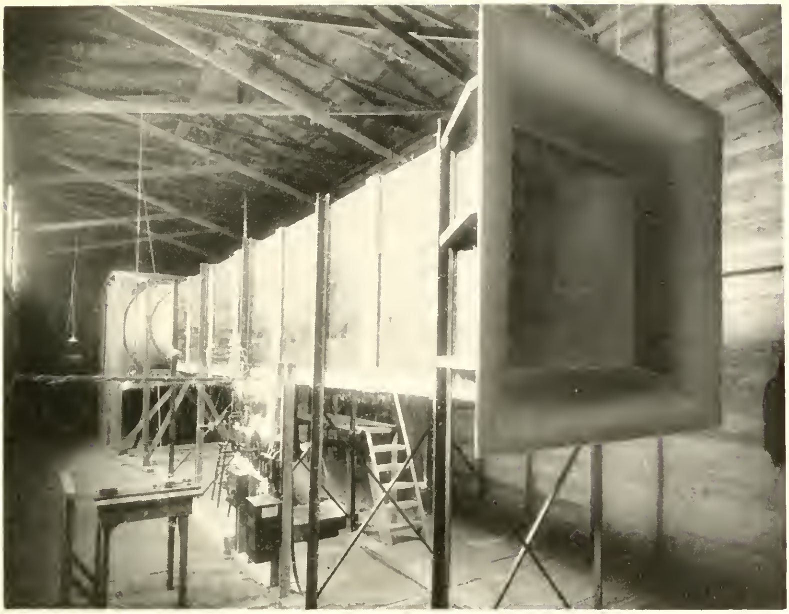


Fig. 1. SUCTION END OF WIND TUNNEL

Fig. 2. ENTRANCE NOZZLE, SHOWING END OF HONEYCOMB

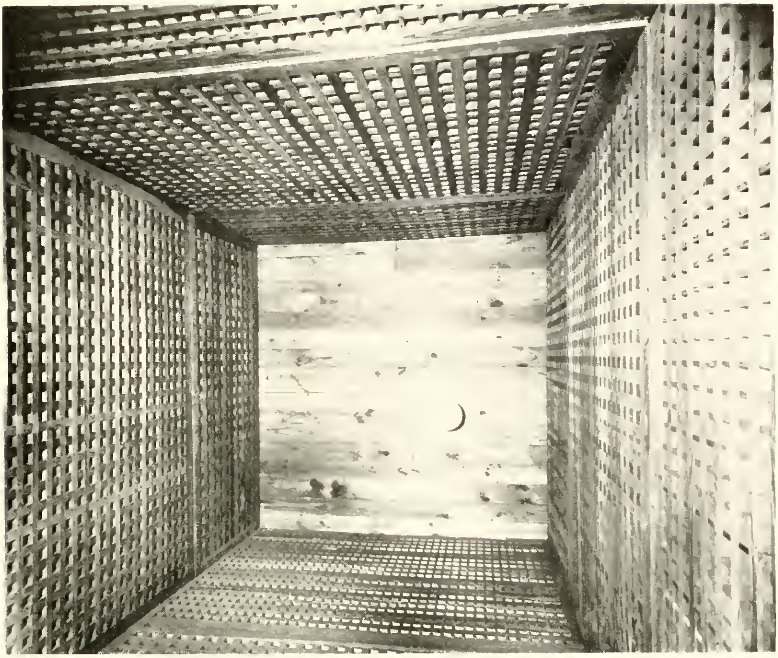


Fig. 1. INTERIOR OF DIFFUSOR, LOOKING FROM PROPELLER

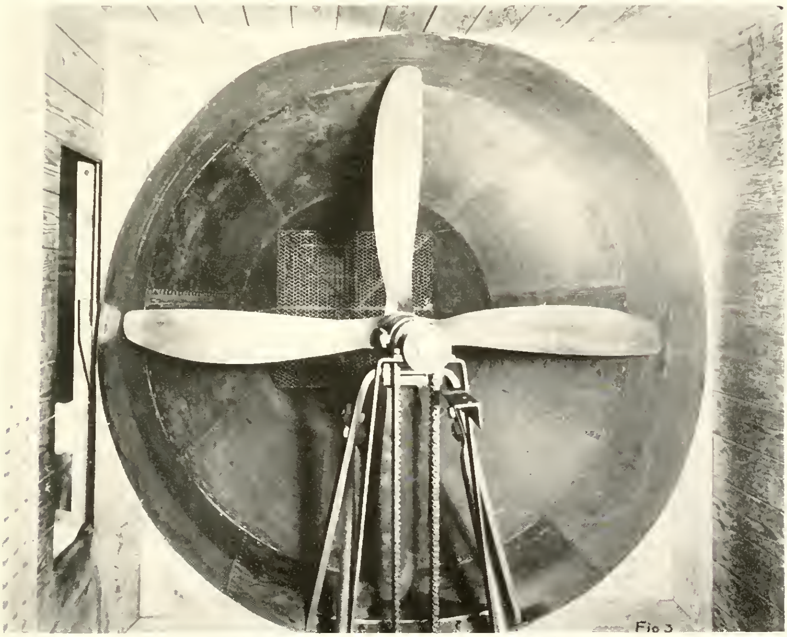


Fig. 2. PROPELLER OF WIND TUNNEL, LOOKING UP STREAM

The induction motor is directly connected to a 12 H. P. direct current generator, which is turned at constant speed and which generates, therefore, a constant direct current voltage for given load.

By change of the generator field rheostat and motor field rheostat the propeller speed can be regulated to hold any wind velocity from 4 to 40 miles per hour. The control is very sensitive. Left to itself, the speed of the wind in the tunnel will vary by 2 per cent in 2 or 3 minutes. This variation is so slow that by manipulation of the rheostats the flow can be kept constant within $\frac{1}{2}$ per cent. The cause of the surging of the air is not understood, but is probably due to hunting of the governor of the prime mover in the Cambridge power house causing changes in frequency too small to be apparent. The gustiness of outdoor winds seems to have no effect, although the building is not air-tight.

Aerodynamical Balance

The aerodynamical balance (pl. 3) was constructed by the Cambridge Scientific Instrument Company, England, to the plans and patterns of the National Physical Laboratory. The balance is described in detail by Mr. L. Bairstow in the Technical Report of the Advisory Committee for Aeronautics, London, 1912-13. For details of operation and the precision of measurements reference may be made to the original article.

In general, the balance consists of three arms mutually at right angles representing the axes of coordinates in space about and along which couples and forces are to be measured. The model is mounted on the upper end of the vertical arm which projects through an oil seal in the bottom of the tunnel.

The entire balance rests on a steel point, bearing in a steel cone. The point is supported on a cast-iron standard secured to a concrete pillar, which in turn rests on a large concrete slab. The balance is then quite free from vibration of the floor, building, or tunnel.

The balance is normally free to rock about its pivot in any direction. When wind blows against the model, the components of the force exerted are measured by determining what weights must be hung on the two horizontal arms to hold the model in position. Likewise the balance is free to rotate about a vertical axis through the pivot. The moment producing this rotation is balanced by a calibrated wire with graduated torsion head.

Force in the vertical axis is measured by means of a fourth arm. The model for this measurement is mounted on a vertical rod which slides freely on rollers inside the main vertical arm of the balance. The lower end of this rod rests on one end of a horizontal arm having a knife edge and sliding weight.

For special work on moments, the interior vertical rod is replaced by another having a small bell crank device on its head which converts a moment about the center of the model into a vertical force to be measured as above (pl. 4).

In this way provision is made for the precise measurement of the three forces and the three couples which the wind may impress on any model held in any unsymmetrical position to the wind.

The balance is fitted with suitable oil dash pots to damp oscillations, and devices for limiting the degrees of freedom to simplify tests in which only one or two quantities are to be measured. The balance can be adjusted to tilt for 1/10,000 pound force on the model. In general, the precision of measurements is not so good as the sensitivity, and in the end is limited by the steadiness of the wind and the skill of the observer.

The weights and dimensions of the balance were verified by the National Physical Laboratory, where also the torsion wires were calibrated.

For ordinary forces, weighings may be considered correct to 0.5 per cent. Naturally for very small forces, such as the rolling moment caused by a small angle of yaw, the measurements cannot be so precise.

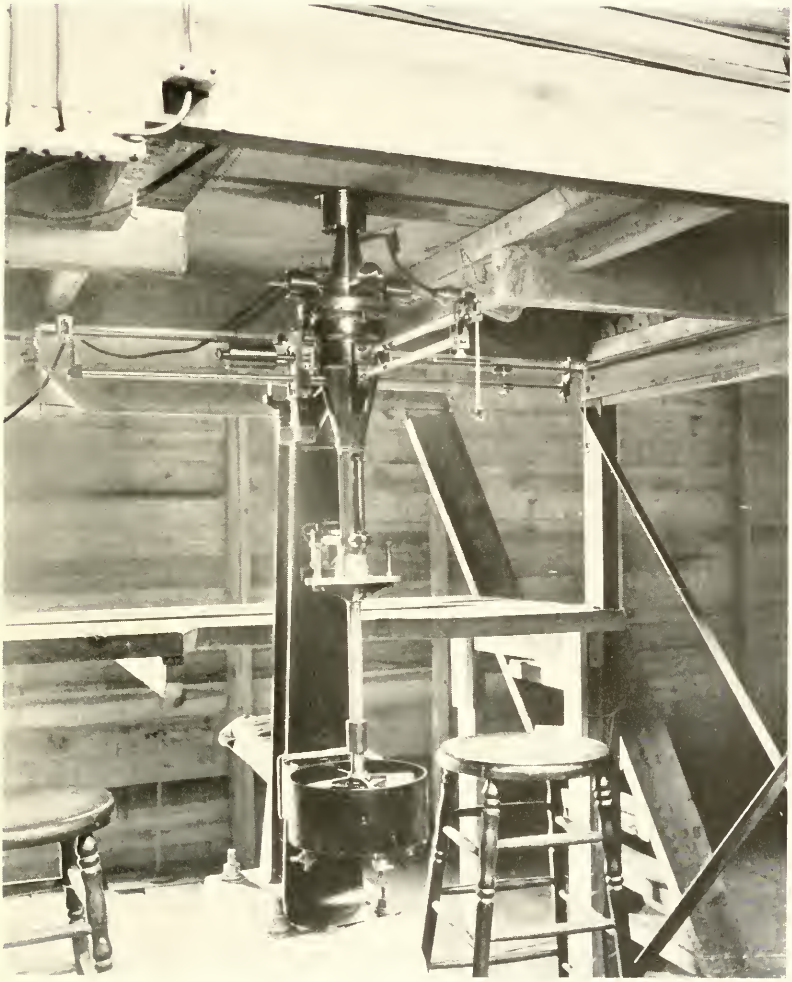
Alignment of Tunnel

The axis of the wind tunnel was desired to be horizontal from the honeycomb to the baffle plates in front of the propeller. To accomplish this an engineer's level was mounted on a platform, built on the floor of the house, opposite the mouth of the tunnel, and sighted on the intersection of diagonal threads placed at 6-foot intervals. By this means the distance of the center line of the tunnel above or below the horizontal line could be estimated to one-eighth of 1 inch.

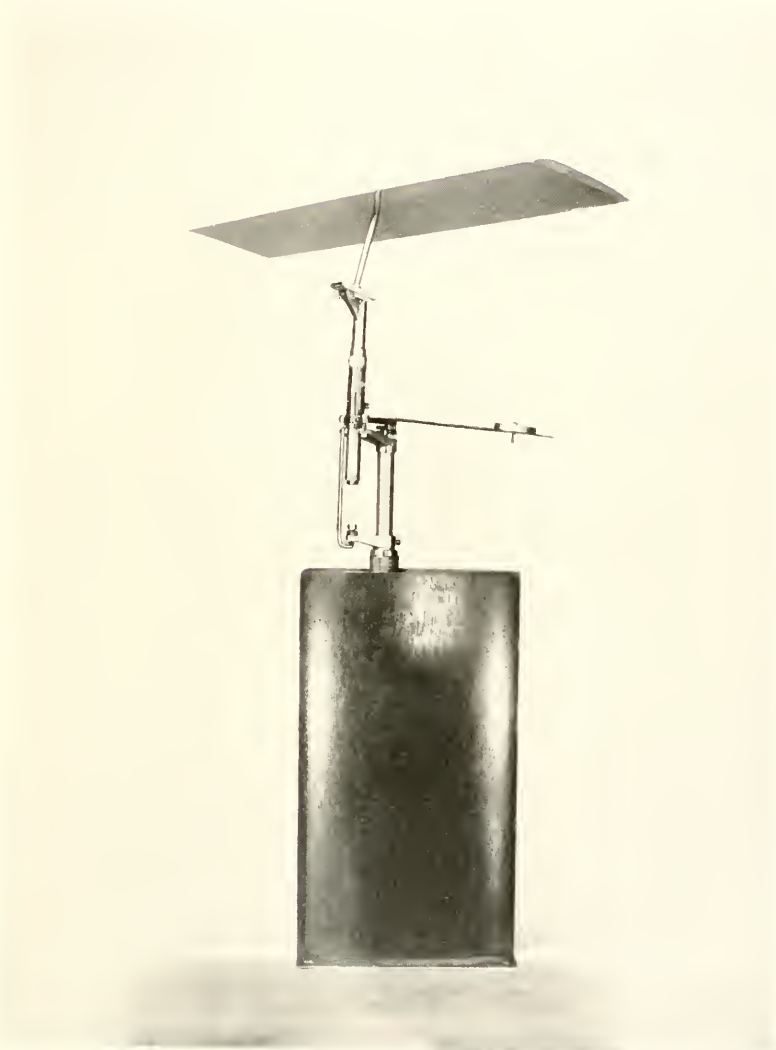
The tunnel beling low in the center, it was raised by wedges until the reference marks coincided with the horizontal. This was attained to within one-eighth of 1 inch in 6 feet of tunnel length. The tunnel may, therefore, be said to have its axis horizontal to within one-tenth degree.

Alignment of Vertical Axes of Balance

A concrete foundation having been built for the balance, the latter was set in its approximate position. Three wedges were then



AERODYNAMICAL BALANCE



MODEL WING MOUNTED IN WIND TUNNEL ON MOMENT DEVICE OF BALANCE

inserted under the base plate of the balance standard, and the whole balance raised to its proper height. It was now necessary to rectify the vertical axis of the balance.

To bring the axis of the balance more nearly vertical by more sensitive means the following method was employed: The small torsion wire, used in aerodynamical measurements with the balance, was set in place. The lower pivot of the balance was engaged in its bearing, leaving the balance free to rotate about its vertical axis, but constrained from tipping laterally.

The torsion wire was adjusted by means of the micrometer head until the cross-hair on the fixed telescope coincided with that on the mirror attached to the balance proper.

A weight of 0.4 pound was placed on one balance arm. The micrometer head was again turned until the cross-hairs were coincident. By setting up on the holding-down bolts, the balance axis was adjusted until placing a weight on either of the arms required no further rotation of the torsion head to maintain coincidence of the cross-hairs. In such case the axis of the balance is vertical. The final adjustment admits of a possible error of less than 1/400 inch-pound on the torsion wire. The angularity of the balance axis remaining uncorrected may be computed as follows:

Let

F = force hung on arm.

β = angle of balance axis to vertical.

Then, taking moments about the vertical axis

$$F \sin \beta \times 18'' = 0.4 \times \sin \beta \times 18'' = 1/400$$

$$\text{or } \beta = 0^\circ.025.$$

Determination of Wind Direction in the Horizontal Plane

As a first approximation, the wind was assumed parallel to the axis of the tunnel. A vertical flat plate was mounted on the balance arm and carefully set parallel to a line drawn on the floor of the tunnel in the direction of its axis. The plate was inclined 8 degrees to right and left of this position and the transverse force measured on the balance. The observations were repeated for 6 degrees and for a second plate to eliminate errors due to irregularities in the plates. The transverse force on one side was greater than on the other, indicating an error in the assumed wind direction. A new line was drawn on the tunnel floor making an angle of 0.3 degrees with the original line. The observations for transverse force were repeated. It was then found that the average of transverse force readings taken for equal angles to right and left of mid position differed 0.5 per cent from the mean of all readings. The extreme error of one observation, including the error of 0.5 per cent due to lack of sensitivity of the balance and the personal error of the observer, may then be as great as 1 per cent in case the two errors are cumulative.

It is not considered practicable to obtain a closer setting with the methods of alignment employed.

Setting Arms of Balance

Knowing the true direction of the wind, it is necessary to set the horizontal arms of the balance parallel and perpendicular to this direction. To do this the floating part of the balance was rotated by an adjustment provided by the design until the force recorded on the "drift" arm (the resistance) was equal for equal angles of the plate to right and left of the wind direction. The final setting indicates a remaining error of .2 per cent.

After making the slight adjustment required here the error in the transverse force measurement was not found to be increased.

Measurement of Air Velocity

The velocity of flow in the wind tunnel is measured by a pressure tube anemometer commonly called a double Pitot tube.

Our laboratory standard is a double Pitot tube presented by the director of the National Physical Laboratory, England. This tube was compared with the National Physical Laboratory standard which had been calibrated on a whirling arm by F. H. Bramwell.[1] Its constant had been determined to be unity to a precision of 0.1 per cent. Our tube was compared with it by a method allowing a precision of 0.25 per cent. A discrepancy of about 0.25 per cent was found. Its readings may then be taken as correct to this degree of precision. In all cases a uniform rectilinear current is implied.

The Pitot tube, in common with all anemometers, has the disadvantage of obstructing the channel, and where models are to be tested the channel should be kept entirely clear. The expedient of using a side hole in the channel is due to M. Eiffel.[2]

In a channel of uniform section, air is forced to flow practically parallel to the axis of the channel. Hence stream lines are all parallel and across any section, taken normal to the channel axis, there should be no component of velocity at any point. This statement is of course true only for a steady, uniform flow free from turbulence. The

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[1] Technical Report of the Advisory Committee for Aeronautics, London, 1912-13.

[2] La Resistance de l'Air et Aviation, Paris, 1912.

static pressure should be constant across a section, for if pressure differences existed there would be a transverse flow of air created. Tests in our wind tunnel showed constant pressure across a section to a good approximation. Incidentally the constancy of this static pressure across a section is a measure of the uniformity of flow.[1]

A small hole in the side of the tunnel can then be used to measure the static pressure, but the dynamic pressure measured by the impact end of the Pitot tube is

$$p+\frac{\rho v^2}{2g}=p_0, \text{ by Bernoulli's equation,}$$

where

p = pressure at any point in a stream line.
v = velocity at any point in a stream line.
ρ = density at any point in a stream line.
p_0 = pressure where v is zero.

In our wind tunnel a fan sucks air through the tunnel which is therefore all under suction. The air is discharged by the fan through a strainer into the building at one end, whence it returns at low velocity to the other end to pass again into the tunnel. At a point in the room the pressure transmitted by an impact tube would be

$$p_r+\frac{\rho v'_r{}^2}{2g}=p_0.$$

But the room is 30 times as large as the section of the tunnel, and when a wind of 30 miles is blowing in the tunnel there is only a gentle draft in the room of about 1 mile per hour. Thus the ratio $\frac{v'_r{}^2}{v^2}=\frac{1}{900}$ and the pressure in the room can be taken as

$$p_r=p_0=p+\frac{\rho v^2}{2g}$$

neglecting v_r^2.

[1] The static holes of the National Physical Laboratory Pitot tube were connected to an alcohol gage, and the velocity being kept constant, the tube was moved along the vertical center line of the tunnel. The following readings were taken:

Head in ½ mm. alcohol	Distance from wall
440.2	3″
438.0	6″
439.5	12″
439.8	18″
439.5	24″
441.2	30″
441.0	36″
441.0	42″
441.2	45″

If then we connect a hole in the side of the tunnel with one end of a liquid manometer, and leave the other end open to the room, the gage reading is proportional to the difference in pressure or to

$$p_r - p = p_0 - p = \frac{\rho v^2}{2g}.$$

The reading of the manometer thus is a measure of the velocity.

Due to loss of head from friction in the mouth of the tunnel and in the honeycomb, the relation

$$p_r = p + \frac{\rho v^2}{2g}$$

is not strictly true. An unknown loss in friction would be represented by adding a term to indicate the friction head pressure. Then

$$p_r = p + \frac{\rho v^2}{2g} + p_f.$$

The use of the side plate method ignores the effect of p_f. A comparative test showed an error of 3 per cent when velocity was calculated from side plate readings. It is, therefore, necessary to calibrate the side plate and its manometer against the standard Pitot tube and its manometer.

The side plate used (fig. 1) consists of a thin brass disk about 3 inches in diameter set flush in the wall of the tunnel. The disk is flat and highly polished. Near its center, five holes 0.02 inch in diameter are drilled. These holes are connected with a brass tube soldered to the back of the plate and projecting through the side of the channel. Rubber tubing is used to transmit the static pressure from the small holes to one end of a manometer. As explained above, the other end of the manometer is open to the air in the room.

The pressures transmitted by the side plate have been found to respond very quickly to changes in velocity, and the method is even more sensitive than the Pitot tube. Naturally its precision is no better than that of the Pitot used for its calibration.

The pressure difference transmitted by the side plate is read on an inclined alcohol manometer on the Krell principle. Both the side plate and this alcohol manometer require calibration against a standard. For convenience, the side plate and its manometer were calibrated together against the standard Pitot tube and a Chattock manometer.

The standard National Physical Laboratory Pitot tube was mounted in the center of the tunnel in the place where models are tested. This tube was connected to the Chattock gage. The side plate in the wall opposite the tube was then connected to the alcohol gage. The wind speed was then adjusted to 2, 4, 6, 8, etc., up to 40 miles per hour and both gages read. Some 100 readings were taken. From the Chattock gage readings the true speed was taken from its calculated curve (for standard air). The readings of the alcohol gage were then plotted on true speeds. The curve so made was then a calibration of the side plate and alcohol gage in combination. The Pitot tube and Chattock gage may now be removed, and in future

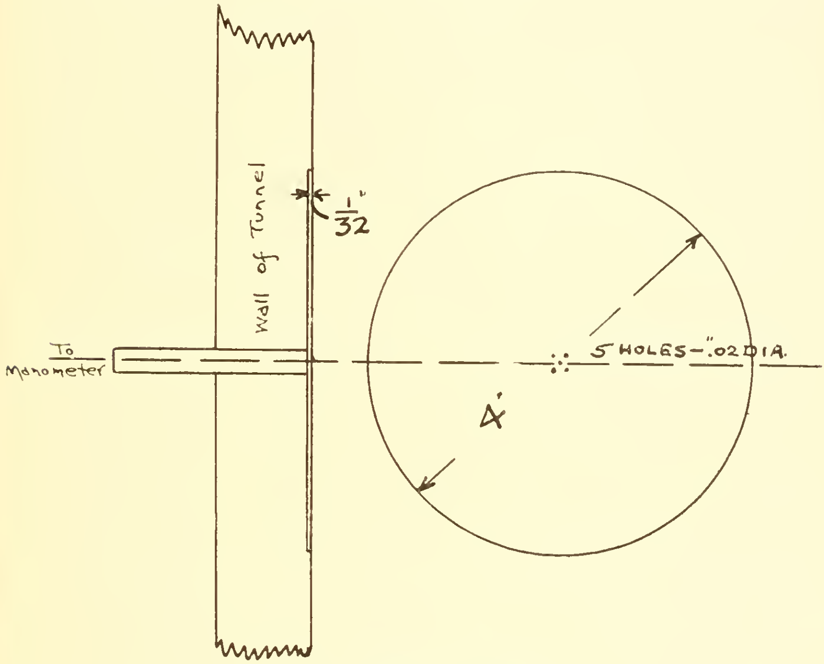


FIG. 1.—Side pressure plate.

model testing the alcohol gage readings may be used to measure the velocity at the center of the tunnel.

It is shown below that the velocity over the section varies about 1 per cent over a 2 foot 6 inch square at the center of the tunnel.

Chattock Micromanometer

The Chattock gage mentioned above has been adopted as our laboratory standard, but is used only for the calibration of other gages which may be preferred on account of ease of reading. The following notes on this gage are introduced here in the hope that someone may have use for a delicate pressure gage. Working drawings will gladly be supplied to anyone contemplating the construction of such a gage.

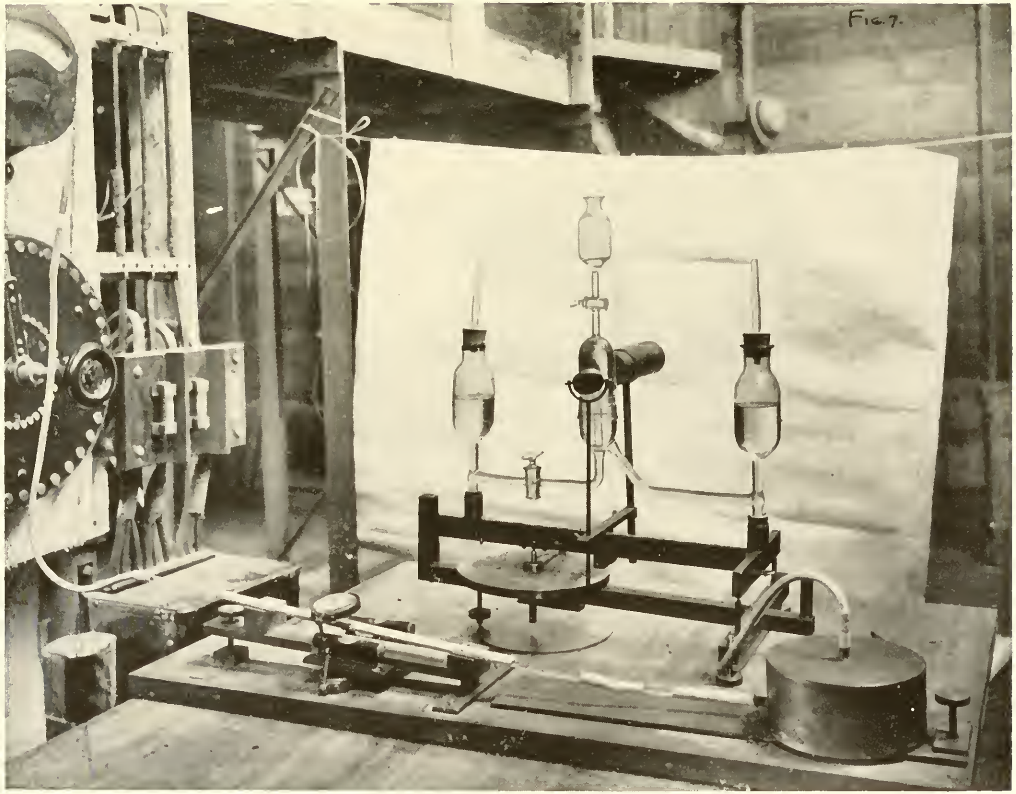
The Chattock micromanometer was devised by Professor A. P. Chattock and Mr. J. D. Fry for the precise measurement of very small pressures. The gage is described by Dr. T. E. Stanton in the *Proceedings of the Institution of Civil Engineers,* December, 1903. Dr. Stanton used this gage in his experiments on the air resistance of small plates.

The principle of the gage is that of the inclined liquid U-tube, but instead of giving the tube an initial pitch and observing the change of level of the liquid, the Chattock gage is fitted with an elevating screw and micrometer by which the gage is tilted to balance the pressure difference in its two ends. By reading on the micrometer the amount of tilt given, the head in inches of liquid is computed. By this means there is no motion of the liquid in the glass, and errors due to capillarity and viscosity are eliminated. Furthermore, the condition of the surface of the glass has no effect.

The gage (pl. 5 and fig. 2) consists of a glass U-tube mounted on a tilting frame *T*. The pressures to be measured are connected to the bulbs *A* and *C,* which are in communication with each other through a horizontal tube bearing a third bulb *B* at any intermediate point. The bulbs *A, C* and the lower part of *B* are filled with water. The upper part of *B* is filled with castor oil. The water in *B* and *C* is in free communication and hence the oil in *B* is at the pressure of *C*. The water in *A* is led through a thin walled tube through the bottom of *B* extending into the castor oil. An excess of pressure in *A* over the pressure in *C* will cause water to flow from *A* into *B*. A water bubble will then grow at *D* and expand into the oil. The gage can be tilted so that this bubble remains of uniform diameter. The pressures in *A* and *C* are then balanced. To provide this tilting the manometer proper is mounted on a tilting frame *T,* which pivots on the knife edges at *G* and is elevated by the screw *F*. The whole is carried on a bed frame *Z* fitted with three leveling screws *I,* a retaining spring *H,* and a scale *S,* on which may be read the full turns of the screw *F*.

A microscope, *M,* fitted with cross-hairs is mounted on the frame *T* and directed at the bubble *B*. A small mirror on the opposite side illuminates the surface of the bubble.

The screw *F* is fitted with a large drum divided into 100 parts. The screw has 20 threads to the inch. The gage is sensitive to one-half of a division on the drum, and hence to a movement of the screw of 1/4000 inch.



CHATTOCK GAGE AND ALCOHOL GAGE. USED FOR VELOCITY MEASUREMENTS

Before a measurement is taken, the bulbs A and C are opened to the air of the room and the frame tilted by moving the micrometer until the top of the bubble B is brought tangent to the horizontal cross-wire of the microscope. This is the zero reading. The bulbs A and C are then connected to the two parts of a Pitot tube and the frame tilted until the bubble is again on the cross-wire. The amount of tilt is then read on the micrometer.

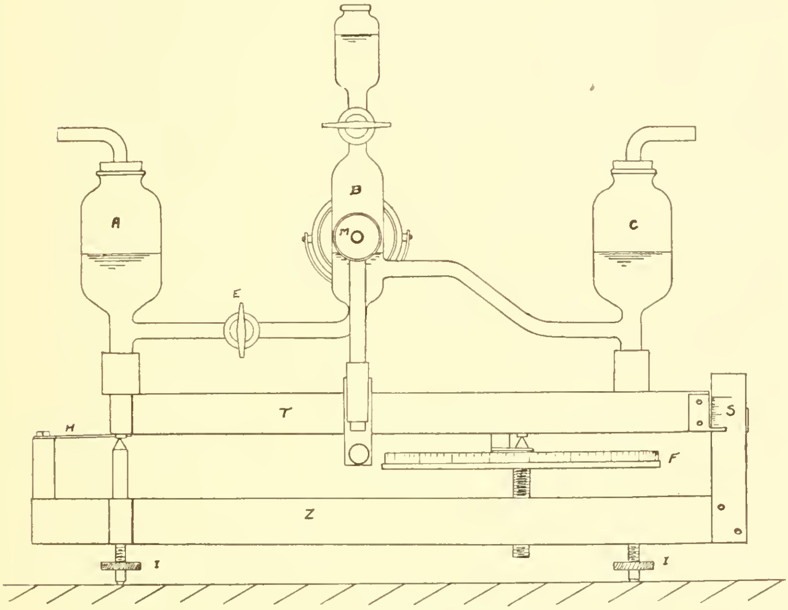


FIG. 2.—Chattock micromanometer.

Naturally too wide an excursion of the bubble will result in its rupture. The loss of a bubble transfers a drop of water from A to B, and hence a new zero reading must be found by balancing up again. To avoid sudden change of pressure and breaking of the bubble, a stop cock at E is fitted. This cock can be closed to make the instrument portable, and in taking a reading an approximate balance is made with E partly opened. The cock is then opened full.

The gage is filled with a solution of salt and water of s. g. 1.06. The addition of a little salt keeps the castor oil from growing cloudy.

Two gages were constructed, one by a skilled glass blower and the other by a student, with a view to determining the effect of workmanship and dimensions. The frame and stand were made identical in the two gages, but the glass work was purposely altered. The tip of the tube at *B* was ground to a knife edge in one gage and in the other ground off square. One tube was .20 inch in diameter and the other .15 inch in diameter.

The two gages were connected to the same static pressure and gave readings identical to 0.25 per cent. It was found that the gage in no way is affected by minor variations in workmanship.

In the gage with the ground knife edge tip it was found that the bubble did not break so readily as in the gage with the square tip. It was suggested by Professor Gill that the tenacity could further be increased by coating the outside of the tube below the bubble with paraffin. This was tried and was found to be of great assistance. A height of bubble from three to four times the diameter of the tube at its base could be allowed without rupture. The reason for this is to be found in the fact that castor oil sticks very tight to glass but will not stick to paraffin. By the use of this wax the bubble could not creep over the edge of the tube and so slide down it causing a break. However, any large excursion of the bubble is to be avoided as tending to cause a slight change in the zero reading. In all tests the zero should be taken at intervals. The effect of the paraffin on the tip could not be detected in the readings of the gage.

The consistency of the gage readings with these various alterations in the base of the bubble as well as in the size of the bulbs and connecting tubes gives great confidence in this type of gage. It was not possible to calibrate this gage experimentally because there was no other gage available to measure it against which was equally sensitive. However, we have Professor Chattock, Dr. Stanton, and the National Physical Laboratory as authority for the calibration of the gage by calculation from the dimensions of its parts, and the density of the liquid. It may be noted that the density of the oil used has no effect on the principle of the gage and is not considered.

For the calculation of tilt, it is then necessary to measure the distance between the centers of the bulbs *A* and *C* and the distance from the knife edge *G* to the screw *F*. An error of 0.1 inch in either of these measurements is an error of 1 per cent in head or 0.5 per cent in velocity. There is no difficulty in getting these distances to the nearest hundredth of an inch. The screw thread was cut so precisely that it was impossible to detect any error in the pitch of the thread. The hole in *Z* was tapped with a standard Brown and Sharp tap. The calculation of the change in level of the surfaces of the liquid in

A and *C* is precise to 0.1 per cent. The density of the solution was taken on a Westfall balance to the same degree of precision.

Since the gage is sensitive to less than 0.1 per cent for heads of more than 0.3 inch, the measurement of velocity depends on the precision of the Pitot tube. The latter is good to probably 0.25 per cent in velocity. However, the air current always has some fluctuation at high speeds so that in the end the velocity measurement is limited in precision by the closeness with which such fluctuations can be averaged. In a very steady current, such as our wind tunnel, it was found that the error in estimating velocity was less than 0.5 per cent. The average of a number of observations is of course better than this.

Change in density of the salt solution is $\frac{1}{3}$ per cent for a change of 60 degrees F. in temperature. A temperature correction is ordinarily unnecessary.

An alcohol gage is a sensitive and consistent instrument, but requires calibration to eliminate errors due to viscosity and capillarity. The question of its suitability for precise work will be discussed later in another paper. It has the great advantage over the Chattock gage in that it requires no delicate manipulation to get a balance, no cross-wire and microscope, and with it it is possible to estimate the mean of fluctuations. The alcohol gage has been successfully used to measure air speeds as low as two miles per hour. It is shown with the Chattock gage in plate 5.

II. NOTES ON THE DIMENSIONAL THEORY OF WIND TUNNEL EXPERIMENTS

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Introduction

The forces which will act between a solid body and a fluid in contact with it in consequence of a relative motion of the two, cannot, except in a few of the simplest cases, be predicted by computation from the size and shape of the body, the relative velocity, and the physical properties of the fluid: the information can be obtained only from experiment. Such experiments may be expensive or impracticable, and it often appears desirable to get the required information, in advance of the final decision on the design of a structure which is to be subject to aerodynamic or hydrodynamic forces, by making preliminary experiments on a small model of the proposed structure.

In order that the results of such observations shall be interpretable as definite statements about the behavior of the full-sized original of which the model is a copy, certain requirements must be satisfied, and when they are satisfied the original and the geometrically similar model are said to be dynamically similar. The conditions for dynamical similarity are bound up with the general question of the possible forms of equations which describe relations subsisting among the physical quantities involved in physical phenomena.

Nature of the Problem to Be Discussed

Let us suppose that a solid body is moving, with the constant velocity S, through a fluid which is itself sensibly at rest at points far distant from the body; and let us consider the forces exerted on the body by the surrounding fluid. Since these forces are evidently due to the relative motion, they would remain unchanged if the body were held at rest and the fluid made to flow past it with the velocity $(-S)$. The boundaries of the fluid are supposed to be so distant from the solid body that no sensible disturbance reaches them, and their nature can then have no influence on the forces with which we are concerned and need not be further referred to. If the fluid is a liquid with a free surface, the foregoing condition requires that the moving body be so deeply immersed as not to cause any surface disturbances.

Let R be any force exerted by the fluid on the body; for example, the component in any specified direction of the force on some particular part of the solid surface; or, to make it more definite, let R be the total head resistance in the direction of motion. Then R will depend on and be completely determined by the relative speed, the size, shape, and attitude of the body, and the mechanical properties of the fluid; and there must be a definite relation connecting these various physical quantities, which can be described by an equation. We wish to consider the nature of this equation in so far as it is fixed by the natures of the separate quantities involved in it.

The Physical Quantities Which Influence Fluid Resistance

Let D be some linear dimension of the body, such as its greatest length. The shape of the body and its attitude, *i. e.*, its orientation with regard to the direction of motion can be specified by stating the ratios of a number of lengths to the particular length D. If these ratios are denoted by $r', r'', r''', \ldots$, etc., the size, shape, and attitude of the body are completely specified by the values of $D, r', r'', \ldots$, etc.

The properties of the fluid which determine its mechanical behavior are its density ρ, its viscosity μ, and its compressibility. Instead of the viscosity, it is generally more convenient to use the kinematic viscosity $\nu = \frac{\mu}{\rho}$ which will do equally well when ρ is given. And similarly, the speed C of sound waves in the fluid is fixed by the density and compressibility so that, conversely, C together with ρ fixes the compressibility. The properties of the fluid which concern us may therefore be specified by stating the values of the density ρ, the kinematic viscosity ν, and the acoustic speed C in the fluid.

We have now enumerated the quantities on which the force R may be supposed to depend, and if nothing has been overlooked there must be a complete relation connecting R with the other quantities. We may state the fact that such a relation subsists by writing the equation

$$f\,(R, S, D, r', r'', \ldots, \rho, \nu, C) = 0, \tag{1}$$

and our first task is to obtain from general principles any information we can about the form of this unknown function f, which will enable us to restrict the amount of experimentation required to finish the work of finding the form of the equation.

Application of the Principle of Dimensional Homogeneity

By the well known "principle of dimensional homogeneity," all the terms of a complete physical equation must have the same dimensions, and this fact enables us to simplify equation (1). Let Π represent a dimensionless product of the form

$$\Pi = R^{\alpha} S^{\beta} D^{\gamma} \rho^{\delta} \nu^{\epsilon} C^{\zeta}, \tag{2}$$

the numerical exponents α, β, γ, etc., being such as to satisfy the dimensional equation

$$[R^{\alpha} S^{\beta} D^{\gamma} \rho^{\delta} \nu^{\epsilon} C^{\zeta}] = [1] \tag{3}$$

when the known dimensions of R, s, D, ρ, ν, and C are inserted. Then it may readily be shown:[1] 1st, that since three fundamental units are needed as the basis of an absolute system for measuring the six kinds of quantity, R, S, D, ρ, ν, and C, the number of possible independent expressions of the form (2) is $6-3$ or 3; and 2d, that if these expressions are denoted by Π_1, Π_2, Π_3, any correct equation involving the quantities which appear in equation (1) and no others, must neces-

[1] Physical Review (2), 4, p. 345, October, 1914.

sarily, in order to have all its terms of the same dimensions, be reducible to the form

$$F(\Pi_1, \Pi_2, \Pi_3, r', r'', \ldots) = 0. \tag{4}$$

In addition to the dimensionless ratios r', r'', etc., there now appear in the equation only three instead of the original six variables, so that the labor of determining by experiment the form of the unknown function is much less than if we had to deal with all six variables.

The More Specific Form of the Equation of Fluid Resistance

The dimensions of the quantities on the familiar mass, length, time, or $[m, l, t]$ system are:

$$[R] = [mlt^{-2}], \qquad [\rho] = [ml^{-3}],$$
$$[S] = [lt^{-1}], \qquad [\nu] = [l^2t^{-1}],$$
$$[D] = [l], \qquad [C] = [lt^{-1}];$$

and we see by inspection that the expressions

$$\Pi_1 = \frac{R}{\rho D^2 S^2}, \quad \Pi_2 = \frac{DS}{\nu}, \quad \Pi_3 = \frac{S}{C}$$

are dimensionless products of the required form (2), and that they are independent. Accordingly, we know that equation (1) must be reducible to the form

$$F\left(\frac{R}{\rho D^2 S^2}, \frac{DS}{\nu}, \frac{S}{C}, r', r'', \ldots\right) = 0. \tag{5}$$

This equation is fundamental to the experimental study of the hydrodynamic or aerodynamic forces acting on totally immersed bodies.

Solving for Π_1 we now have

$$\frac{R}{\rho D^2 S^2} = \phi\left(\frac{DS}{\nu}, \frac{S}{C}, r', r'', \ldots\right), \tag{6}$$

in which the form of the unknown function ϕ remains to be found, if it needs to be found at all, by experiment or by other than dimensional reasoning.

Simplification When Compressibility May be Disregarded

A simplification is possible when the motion is not rapid enough to cause any sensible compression in the fluid. In this event it is immaterial what the compressibility is, so that $\frac{S}{C}$ may be omitted from consideration and equation (6) reduces to

$$\frac{R}{\rho D^2 S^2} = \phi\left(\frac{DS}{\nu}, r', r'', \ldots\right). \tag{7}$$

The approximation attainable when compressibility is thus left out of account depends on the value of $\frac{S}{C}$. If $\frac{S}{C}$ is a small fraction, as it nearly always is with liquids, equation (7) is a satisfactory substitute for (6). The speed of sound in air under ordinary conditions is of the order of 1100 feet per second, or 750 miles per hour. For rifled projectiles $\frac{S}{C}$ may be as high as 2.5 or even 3, so that equation (7) would be entirely misleading if used in studying projectile resistances. But at the speeds which occur in aeronautics, with the exception of propeller tip speeds, the ratio $\frac{S}{C}$ is a sufficiently small fraction that the air acts nearly like an incompressible fluid, *i. e.,* like a liquid of the same density and viscosity; and equation (7) may be used as a sufficiently approximate substitute for the more general equation (6).

Equation (7) supplies the basis for the experimental investigation of the aerodynamic problems which occur in connection with aeronautics and aviation by means of reduced scale methods.

Restriction to Geometrically Similar Bodies

Let us now confine our attention to a series of bodies of various sizes but all of the same shape, and presented to the wind in the same attitude. The bodies are geometrically similar, and any one may be regarded as a reduced or enlarged model of any other. The ratios r', r'',.... are now constants, so that equation (7) assumes the simpler form

$$\frac{R}{\rho D^2 S^2} = \psi\left(\frac{DS}{\nu}\right). \tag{8}$$

in which the form of the unknown function ψ of the single argument $\frac{DS}{\nu}$ remains to be determined by experimenting on bodies of the given series. The nature of this function will depend on the shape and attitude of the bodies but not on their size, if our disregard of compressibility, leading from (6) to (7), was a justifiable approximation.

The obvious procedure, in investigating ψ by analyzing the results of experiments, is to plot observed values of $\frac{R}{\rho D^2 S^2}$ against values of $\frac{DS}{\nu}$ and draw a curve through the points thus obtained. If we are using air of constant density and viscosity the experiments may consist most simply in measuring, by the aerodynamic balance, the force R exerted on a given body at various values of the wind speed S. Variations of $\frac{DS}{\nu}$ may equally well be produced by varying D while S is constant, *i. e.*, by experimenting at a fixed speed but with a series of models of different sizes; or, D, S, ρ, and ν may all be varied simultaneously. But while such experiments furnish a desirable check on the results obtained when S alone is varied, they are not necessary, if compressiblity is negligible; for it is immaterial whether $\frac{DS}{\nu}$ is changed by changing D, S, or ν.

If the plotted points obtained in any of these ways do not all lie on a single curve, within their experimental errors, equation (8) is not accurate enough. And if the models have been exactly geometrically similar, we must conclude that compressibility has played some part in the phenomenon. This means that in the more general equation

$$\frac{R}{\rho D^2 S^2} = \phi\left(\frac{DS}{\nu}, \frac{S}{C}\right) \tag{9}$$

obtained by applying (6) to geometrically similar bodies, the effects of varying $\frac{S}{C}$ are not of entirely negligible importance.

Head Resistance Proportional to S^2; Viscosity Negligible

At ordinary speeds and for bodies that are not too small, experiment shows that in air of standard density, R is very nearly proportional to S^2. It follows that, to the degree of approximation to which equation (8) is valid, $\psi\left(\frac{DS}{\nu}\right)$ is merely a constant and is independent of the values of D, S, and ν. If we write

$$\psi\left(\frac{DS}{\nu}\right) = K,$$

equation (8) reduces to

$$R = K\rho D^2 S^2. \tag{10}$$

As is seen by referring to equation (7), K depends on the values of r', r'', ..., etc.; it is a shape factor for the given series of geometrically similar bodies in the given attitude.

It is to be noted that viscosity does not appear at all in equation (10), so that when the resistance is found to be proportional to the square of the speed, if compressibility is negligible the value of the viscosity is of no importance. This is not equivalent to saying that viscosity plays no part at all in the phenomena; for if viscosity did not exist there would be no eddies of finite size, no dissipation, and at a constant speed no resistance. It means, rather, that the drag on the body by the fluid is due to the continual drain of energy needed to set up anew the turbulent eddying motion about the body; and that when these eddies have once been created it makes no difference how fast they are dissipated by viscosity after the body has left them behind.

The Critical Speed

In the foregoing case of resistance proportional to S^2, the plot of $\frac{R}{\rho D^2 S^2}$ as ordinate against $\frac{DS}{\nu}$ as abscissa gives, of course, a horizontal straight line for bodies of a given series. But if the experiments are carried down to smaller and smaller values of $\frac{DS}{\nu}$, a critical value may be reached where the relation ceases to hold and the character of the fluid motion changes very rapidly, though apparently not discontinuously, so that the function ψ ceases to be a constant for low values of $\frac{DS}{\nu}$. For a given body in a given medium this critical value of $\frac{DS}{\nu}$ corresponds to a critical speed S_C which may be computed *a priori* from the values of D and ν, if the critical value $\left(\frac{DS}{\nu}\right)_C$ has once been determined for bodies of the given shape by varying any one of the variables D, S, and ν, or all together. Eiffel's observations on spheres [1] confirm the foregoing statements when we take into consideration not merely a single speed for each diameter but the whole critical range within which the rapid change in the form of ψ occurs.

Mr. Hunsaker's observations on sharp cornered disks of different diameters, but the same thickness, are very interesting as showing the possible importance of such sharp edges or corners. The disks were not geometrically similar; but the corners at the edges were not only nearly similar but, inch for inch, very nearly identical. Accordingly, that part of the total resistance which may be regarded as due to the sharp corners at the edge reached its critical value always at about the same speed, irrespective of the diameter of the disk which was bounded by the edge. The rapid change in the total resistance near this speed seems to indicate that the "corner resistance" formed a considerable fraction of the whole.

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[1] C. R. 155, p. 1597, December 30, 1912.

The occurrence of a critical speed for a given body in a given attitude is paralleled by the practically much more important phenomenon of the occurrence of a critical attitude at a given speed. Just as the nature of the fluid motion and the law of resistance of a given body change rapidly at a certain critical range of speed, so there are similar rapid changes in the motion and the forces at the critical angle of attack for a given aerofoil at a given speed.

Remarks on the Resistance of Flat Plates Normal to the Wind

From equation (8) it would appear that when R is proportional to S^2 it must also be proportional to D^2, and yet this does not seem to be true for flat plates normal to the wind. On the contrary, while $\frac{R}{\rho D^2 S^2}$ is nearly independent of S, the results of various observers indicate that it increases somewhat as the diameter of the plate increases from a few inches to a few feet. Leaving aside the improbable supposition that this effect is only apparent and due to observational errors, the most obvious explanation is that compressibility may not be entirely negligible. If that is the explanation, equation (9) and not equation (8) is the one to be used, and it is quite conceivable that $\phi\left(\frac{DS}{\nu}, \frac{S}{C}\right)$ might have such a form as to be independent of S without being entirely independent of D. Computations of the amount of compression to be expected at the speeds in question [1] seem to show that the discrepancies are too large to be accounted for in this way. But it may be remarked that in some of the details of the turbulence much higher speeds may occur than the speed of the wind as a whole. Hence compression might occur locally, in some parts of the field about the body, to such an extent as to modify the flow and so affect the resistance, even though computations based on the average speed of the wind might indicate that the effects of compression could not possibly be appreciable under the given experimental conditions.

Mr. Hunsaker's observations on circular disks suggest, however, that there may be another interpretation of the effect in question which does not oblige us to have recourse to the unlikely supposition that compressibility is of importance. If, as appears from these experiments, there is a critical range of speed determined by the form of

[1] See Bairstow and Booth: Rep. British Adv. Committee for Aeronautics, 1910-11, p. 21.

the edge and not dependent on the size of plate, it seems possible that some of the apparent discrepancies between $\frac{R}{S^2}$ = constant and $\frac{R}{D^2}$ = variable may be due to the experimental results of various observers having been influenced by such critical phenomena, which were not, however, sufficiently marked to attract attention.

To decide whether an explanation of this sort is applicable would require an experimental study of the forms of edge which have been used; for until the critical speeds for these edges—if they have any—have been investigated, it is impossible to say whether the speeds at which various experimenters have worked may have overlapped with these critical ranges. Nothing more definite can be said, at present, than that it is well to pay close attention to geometrical similarity; but it seems that a further experimental study of the resistance of flat plates, undertaken with the foregoing possibilities in mind, might lead to interesting results.

Dynamical Similarity

Let us suppose that we are confronted with a problem of design which requires our knowing, in advance, the head resistance, at a prescribed speed, of some body such as an air-ship which is too large for direct experiment. The question is, how to get the desired information from experiments on a small model which can be made at a permissible cost.

Returning to equation (9) or

$$R = \rho D^2 S^2 \phi\left(\frac{DS}{\nu}, \frac{S}{C}\right) \tag{11}$$

we notice that whatever be the form of ϕ, if its two arguments have the same values during two different experiments on geometrically similar bodies, the value of ϕ itself will be the same in both experiments. This observation leads to the notion of corresponding speeds and dynamical similarity.

Let us suppose that we require the resistance R of a body of size D at the speed S in a medium with the properties ρ, ν, C; and that we have a model of the size D_m, which can be run in a medium with the properties ρ_m, ν_m, C_m. Then if we run the model at a speed S_m, such that

$$\frac{D_m S_m}{\nu_m} = \frac{DS}{\nu} \text{ and } \frac{S_m}{C_m} = \frac{S}{C} \tag{12}$$

and observe the resistance R_m, we know by equation (11) that

$$\frac{R}{R_m} = \frac{\rho D^2 S^2}{\rho_m D^2{}_m S^2{}_m}. \tag{13}$$

For when equations (12) are satisfied, $\phi\left(\frac{D_m S_m}{\nu_m}, \frac{S_m}{C_m}\right)=\phi\left(\frac{DS}{\nu}, \frac{S}{C}\right)$, so that ϕ cancels out when we divide equation (11) for the full-sized original by the corresponding equation for the geometrically similar model. Speeds which satisfy equations (12) are "corresponding speeds," and when two geometrically similar bodies are run at corresponding speeds they are "dynamically similar."

If the speeds are low enough that compressibility may be disregarded, the value of $\frac{S}{C}$ is unimportant and the condition for corresponding speeds, which ensures dynamical similarity, is merely the first of equations (12). If we use only a single medium so that $\rho_m=\rho$ and $\nu_m=\nu$, the condition for corresponding speeds reduces to

$$\frac{S_m}{S}=\frac{D}{D_m}$$

and geometrically similar bodies will be dynamically similar if their speeds are inversely as their linear dimensions. Any great reduction in scale might therefore involve our running the model so fast as to make the effects of compressibility no longer negligible. But if the original is to be run in air while the model can be run in water, this difficulty may be avoided. For under ordinary conditions the kinematic viscosity of water is from 1/10 to 1/20 that of air, and for a model of given size the speed required for dynamical similarity with the original is reduced in the same ratio as the kinematic viscosity.

In practice the foregoing method of experimentation is usually unnecessary. For under ordinary working conditions the resistances of aeroplanes and their separate structural elements are so nearly proportional to the square of the speed, and the effects of compressibility are so small, that for practical purposes ϕ in equation (11) or in equation (9) may be treated as a constant and equation (10) used for computation, within any ordinary ranges of D and S. *Any* speeds may then be regarded as corresponding speeds, and geometrical similarity suffices by itself for dynamical similarity. If the constant K of equation (10) has been determined by experiments on any body of the given shape at any convenient speed, the same value may be used in equation (10) for computing the value of R for a different speed or a different size or both.

Complete Dynamical Similarity

The experience with flat plates, showing that even though R is proportional to S^2 it may not be to D^2, warns us to be cautious in assuming that equation (10) may be relied on for great accuracy when the size D changes over a very large range; and it seems possible that it may sometimes be desirable to make experiments guided by equation (11) which holds for any series of geometrically similar bodies, whatever the speeds may be.

The conditions for dynamical similarity given by equations (12) can evidently not be satisfied if we work with only a single medium; for if $\nu_m = \nu$ and $C_m = C$, we have $S_m = S$ and $D_m = D$, so that no scale reduction is possible while preserving dynamical similarity. This difficulty may, in principle, be surmounted by running the model in water if the original is to run in air. Suppose, for instance, that the original is an air-ship which is to run 40 miles an hour in air, and let the model be run in water at such a temperature that its kinematic viscosity is 1/15 that of the air. We then have $\nu = 15\nu_m$ and the first of equations (12) gives us

$$15 D_m S_m = DS. \tag{14}$$

The second condition requires that the speed of the model shall be the same fraction of the speed of sound in water as 40 miles per hour is of the speed of sound in air. Since sound travels about four times as fast in water as in air, the model must move at the very high rate of 160 miles per hour, or about 235 feet per second. With this condition that $S_m = 4S$, and the previous condition stated by equation (14), we have

$$D_m = \frac{1}{60} D.$$

A model to 1/60 scale run in water will then be dynamically similar to the original in air, if it is run four times as fast. Having thus satisfied equations (12) we may use equation (13); and if we set $\rho_m = 800\rho$ we have

$$\frac{R}{R_m} = \frac{1}{800} \times 60^2 \times \left(\frac{1}{4}\right)^2 = \frac{9}{32}.$$

The resistance of the original in air will therefore be about one-quarter of the resistance of the dynamically similar 1/60 scale model in water. How soon it will seem worth while to attempt experiments of this sort cannot be predicted, but the notion of dynamical similarity shows how the problem may be attacked.

The Pitot Tube

Hitherto we have let R be the total head resistance of a solid body, but if D is the diameter of the impact opening of a Pitot tube, $\frac{R}{D^2}$ may evidently be regarded as a quantity which is proportional to the impact or velocity pressure p. Hence equation (6), as applied to the Pitot tube at rest in a current of fluid, may be written

$$p=\rho S^2\phi\left(\frac{DS}{\nu}, \frac{S}{C}, r', r'', \ldots\right), \tag{15}$$

and it is interesting to compare this with the known behavior of Pitot tubes and with the Pitot equation as ordinarily given.

In the first place, we know by experience that if the impact opening is the mouth of a long tube pointed up stream, the precise form of the tube and the shape and diameter of its mouth have no appreciable influence on the impact pressure recorded. This means not only that the shape variables r', r'', ..., etc., are of no importance and may be omitted from among the arguments of ϕ, but also that D is likewise of no importance, so that the argument $\frac{DS}{\nu}$ in which it appears may be omitted. Equation (15) thus reduces to the form

$$p=\rho S^2\psi\left(\frac{S}{C}\right). \tag{16}$$

When the fluid is nearly incompressible, like water, the compression caused by the impact pressure p will be so slight that it cannot affect the general behavior of the fluid. Hence compressibility may be left out of account and ψ treated as a constant, so that we have

$$S=\text{const}\times\sqrt{\frac{p}{\rho}}. \tag{17}$$

If p is measured as a head h of the liquid, we have $p=g\rho h$, and equation (17) reduces to

$$S=\text{const}\times\sqrt{gh}.$$

The value of the constant, which cannot be found by dimensional reasoning, is, in practice, $\sqrt{2}$ for a properly constructed tube.

If the fluid is a gas, equation (17) is still applicable when the speed is low. But when the speed is so high that the pressure p causes appreciable compression, $\frac{S}{C}$ cannot be neglected and we must return to equation (16). A form of $\psi\left(\frac{S}{C}\right)$ for high gas speeds may readily be found from thermodynamics, but so many approximations and unproven assumptions have to be made in the course of the argument that the results are not at all convincing.

III. THE PITOT TUBE AND THE INCLINED MANOMETER

By J. C. HUNSAKER

For aeronautical purposes the absolute measurement of velocity is of less importance than the measurement of impactual pressure. For this reason, an anemometer from which the velocity may be deduced from a pressure measurement is preferable to any of the vane anemometers which measure velocity directly, and require a somewhat laborious calculation of the density of the air before the effect of the wind can be evaluated.

The most common as well as the most convenient form of pressure anemometer is the double Pitot tube. Reference may be made to the papers of Taylor[1] and Zahm,[2] in which it is shown that the equation to a stream line in any perfect gas may be simplified in the case of air by considering the air incompressible for velocities below 100 feet per second. The simplified expression connecting pressure and velocity in moving air is then Bernoulli's equation as used in hydraulics:

$$\frac{\rho v_1^2}{2g} + p_1 = \frac{\rho v_2^2}{2g} + p_2,$$

where v_1 and p_1 are velocity and pressure at any point, and v_2 and p_2 are corresponding values for some other point in the same stream line.

Let us choose the point where v_2 is zero, then

$$\frac{\rho v_1^2}{2g} + p_1 = p_2.$$

In air this is the barometric pressure. Let us change the notation so that $\frac{\rho v^2}{2g} + p = p_0 =$ barometric pressure, a constant.

p is now the pressure in the unchecked stream, the "static" pressure, and p_0 is the pressure in the impact tube where the current is brought to rest. This is called "dynamic" pressure.

The Pitot tube is a device for transmitting the pressure difference, $p_0 - p = \frac{\rho v^2}{2g}$, from which the velocity may readily be calculated. The quantity $\frac{\rho v_2^2}{2g}$ is commonly called "velocity" pressure.

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[1] Experiments with Ventilating Fans and Pipes, by D. W. Taylor, Naval Constructor, U. S. Navy, Trans. Soc. Naval Architects and Marine Engineers, 1905.

[2] Measurement of Air Velocity and Pressure, A. F. Zahm, Ph. D., Physical Review, December, 1903.

If one end of an open tube be pointed into a stream of air and the other end be attached to a manometer, the total dynamic pressure will be recorded. On the other hand, if a tube with closed end be pointed into the wind and further fitted with a conical or parabolic tip, the stream line is only slightly deflected and distorted. If then small holes or slots be cut in this tube at a distance well back from the tip, the wind should blow past these openings and the interior of the tube should be subjected to the static pressure of the stream. This pressure can be measured by connecting the tube to a manometer.

It is generally accepted from the results of tests that any open-ended tube of any size, if pointed fairly into the wind, will correctly transmit the dynamic pressure.

It is equally common knowledge that the correct transmission of the static pressure is not so simple. Widely different values are obtained with different forms of tube and static orifice, and many tubes, such as the Dines and the Recknagel, must be calibrated against some standard. It is obvious that the nose of the tube should be of easy form, that the tube should not be large in diameter, and that it should be carefully polished in order that the air stream may pass undisturbed. The best form of entrance will introduce some disturbance, so that such static openings as are used should be placed well back from the nose on the cylindrical portion. The form and size of the openings will be discussed later.

The Pitot tube may consist of two separate tubes or a double tube made up of a pair of concentric tubes, the dynamic tube being enclosed within the static tube. Since the dynamic tube transmits the pressure $p + \frac{\rho v^2}{2g} = p_0$, and the static tube transmits p, it is sufficient to connect the two tubes to the two ends of a U-tube filled with liquid. The reading of the instrument is then proportional to the difference between the pressures transmitted and hence to $\frac{\rho v^2}{2g}$. Knowing the density, the velocity may be computed. The density of air depends on the pressure, temperature, and humidity. Avoidance of the necessity for calculating the density for ordinary aerodynamical tests would be of great assistance.

Elimination of Density of Air

It is generally accepted that the forces produced by a fluid in motion with reference to any solid object depend on the size, shape, and attitude of that object, the velocity, the density of the fluid, and its viscosity, and upon nothing else for ordinary transportation speeds.

The most general expression[1] for this statement which satisfies the theory of dimensions is

$$R = \rho L^2 V^2 f\left(\frac{VL\rho}{\mu}\right).$$

in which

L denotes the length of any linear dimensions of the solid,
V, the relative velocity of solid and fluid,
ρ, the density of the fluid,
μ, the coefficient of viscosity of the fluid,

and f is a function of the single variable $\frac{VL\rho}{\mu}$.

It will be noted that the compressibility of the fluid has been neglected.

The value of $f\left(\frac{VL\rho}{\mu}\right)$ is very nearly constant for bodies of a given shape in a given orientation when the motion of the fluid is sufficiently turbulent. Experimentally it is found that $R \propto V^2$, nearly, and hence not only is $f\left(\frac{VL\rho}{\mu}\right)$ nearly constant, but the influence of viscosity is small. The changes in $f\left(\frac{VL\rho}{\mu}\right)$ with change of scale, density, and viscosity are hence in the nature of a correction.

For objects moving through the air at very low speed, especially objects of easy form, turbulence is not marked and viscosity is of importance. Consequently for such tests the assumption of f constant is not justified.

However, for aeroplane wings, parts, etc., moved through the air at high speeds the resistance to motion is largely due to turbulence, R varies nearly as ρV^2 and $f\left(\frac{VL\rho}{\mu}\right)$ is constant nearly. Therefore, we may assume that for the ordinary work of an experimental wind tunnel, forces to be measured will vary as the density of the air. Likewise, the manometer reading obtained from a Pitot tube will vary as the density of the air.

It has been decided to adopt a standard density for air to be used throughout. Velocity computed from a manometer reading is then referred to this standard air, and forces measured on the balance are

[1] Helmholtz, Wissenschaftliche Abhandlungen, Vol. I, p. 158; O. Reynolds, Phil. Trans. Roy. Soc., 1883, p. 935; Lord Rayleigh, Phil. Mag., 1899, p. 321.

referred to the same standard. Standard air is taken to be dry air under the following conditions:

Barometric pressure, 29.921 inches mercury.
Temperature, 62 degrees Fahrenheit.
Density, 0.07608 pound per cubic foot.

Comparison of Pitot Tubes

Opportunity was taken to compare the National Physical Laboratory standard Pitot tube, calibrated by Bramwell,[1] with several forms of tube in use by engineers in the United States.

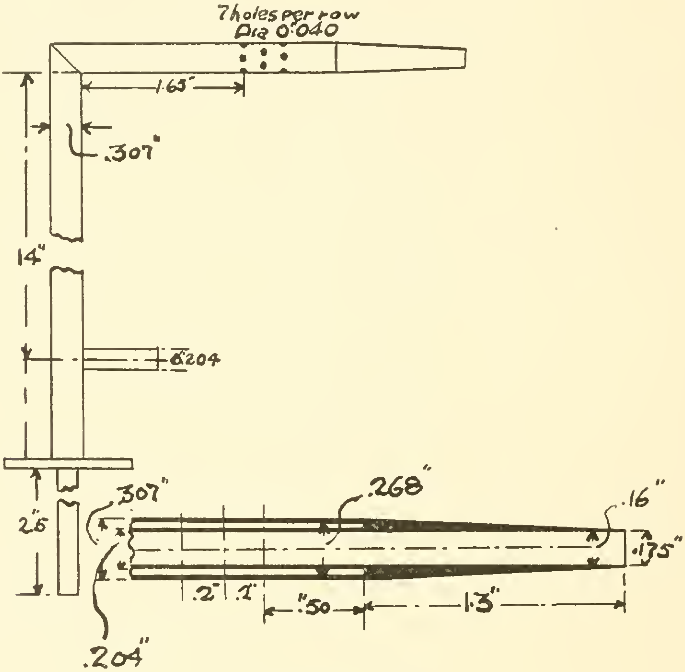

Fig. 3.—N. P. L. tube.

The tube under investigation was mounted in the center of the tunnel and connected by rubber tubing with a Chattock micromanometer. Care was taken to eliminate leaks in the leads, and to point the tube parallel to the axis of the tunnel. A steady wind was then blown through the tunnel and its velocity read from the alcohol gage connected with the side suction plate which had already been calibrated as described in a previous paper. The velocity from the side

[1] Technical Report of the Advisory Committee for Aeronautics, London, 1912-13.

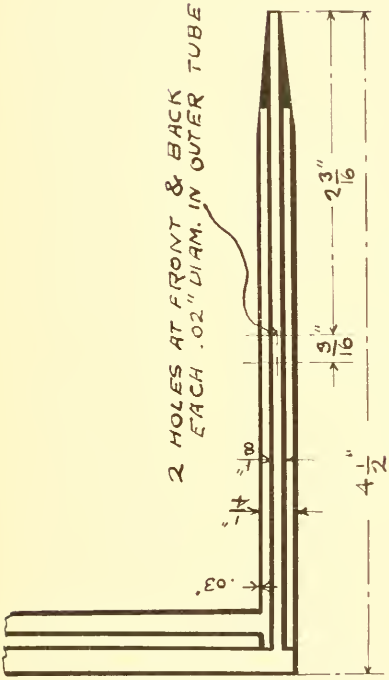

Fig. 4.—A. B. C. tube.

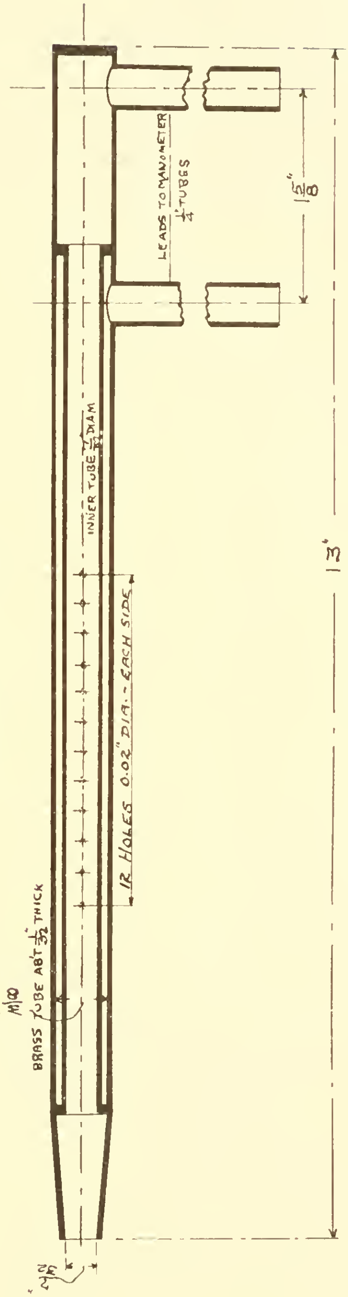

Fig. 5.—Tube A.

plate was then compared with the velocity indicated by the Pitot tube under test. Comparisons were made at a number of speeds for each tube.

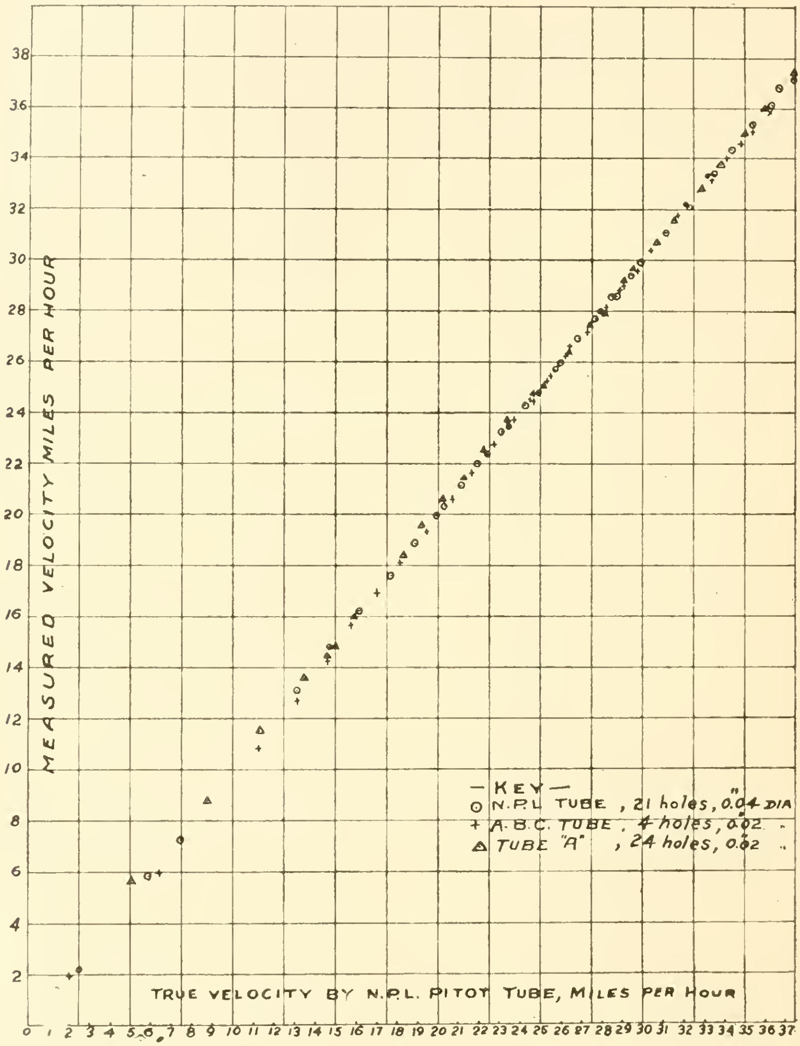

FIG. 6.—Comparison of three forms of Pitot tube. Each point represents one observation, and serves to bring out the sensitivity of the Pitot tube and manometer measurements employed.

It was demonstrated that the velocity is correctly measured by any tube having an easy entrance and a long cylindrical portion parallel to the wind in which a number of small holes are drilled to transmit the static pressure. The arrangement of the holes appeared to have no effect. Long slots in the tube introduced large errors if the tube were not pointed fair into the wind. The size of the tube appeared to be immaterial.

The results of comparison of three tubes with different arrangements of holes from 4 to 24 in number are shown on figure 6. The tubes are shown on figures 3, 4, and 5. The agreement is very close—within $\frac{1}{2}$ per cent at velocities above 10 miles per hour.

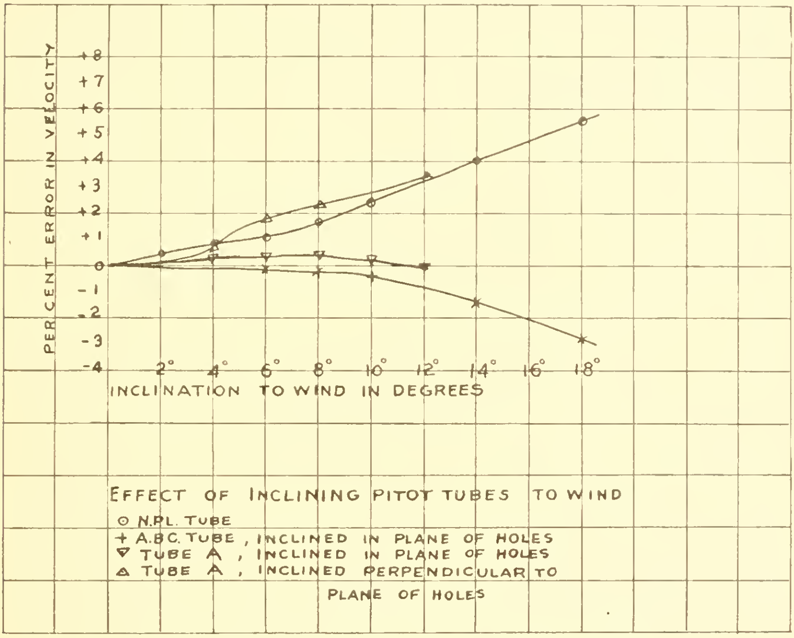

FIG. 7.—Effect of inclining Pitot tubes to wind.

Tests were made to determine the effect of inclining the tubes to the wind. The tubes with holes show an error of but 1 per cent for an inclination of 4 degrees. The results are shown on figure 7.

The following conclusions may be drawn from these tests:

(1) Static openings should be small holes about 0.03 inch in diameter to minimize effects of bad alignment or turbulence.

(2) An error of 2 degrees in aligning the tube causes no important change in velocity reading.

(3) If a tube is correct at two speeds it remains correct at all others within the range of our experiments.

(4) Holes should be symmetrically distributed on the cylindrical portion of the tube.

(5) Any new type of tube should be calibrated against a standard.

The Inclined Manometer

Granted that a Pitot tube is at hand which will correctly transmit the static pressure, measurements of velocity are no better than the manometer used. The ordinary U-tube filled with water, gasoline, alcohol, or other light liquid shows a head of less than 1 inch for ordinary velocities. To obtain the velocity head within 1 per cent, it would be necessary to read the displacement of the meniscus to within 0.01 inch. This is hardly practicable, and various devices are used to magnify the reading. It is at once apparent that if the U-tube be canted at an angle of 1 in 20, a 1-inch head of liquid corresponds to 20 inches on the scale. With an inclined tube, the diameter of the tube must be kept small in order to obtain a good meniscus for reading. On the other hand, in any gage in which the imperfections in the glass produce changes in capillarity, the liquid sticks at some places. A large tube tends to reduce this source of error. The American Blower Company recommend an inclined U-tube filled with gasoline for use with the Pitot tube. This type involves the simultaneous reading of the meniscus level in each leg of the tube, a somewhat difficult feat in an unsteady current.

The German "Krell" manometer is filled with alcohol colored to give a visible meniscus. One leg of the U-tube is an inclined glass tube, and the other is a reservoir bottle whose section is some 400 times the section of the tube. Hence, as liquid rises in the glass tube the depression in the reservoir is unimportant. Only one meniscus level then need be read.

An inclined tube manometer on the Krell principle was constructed, and an investigation made of its errors by comparing it with a Chattock manometer known to be nearly correct. This alcohol manometer is shown in figure 8. It is seen to include a reservoir R mounted on a hinged plate with leveling screw. By means of the latter, the liquid in the tube is brought to the zero of the scale at the beginning of a test, thus making a zero correction unnecessary. The glass tube T is likewise mounted on a brass plate pivoted at the knife edge K, and adjusted in pitch by the screw S. To the brass plate are attached permanently two small machinists' spirit levels L_1 and L_2, set at 3 degrees and 6 degrees to the axis of the tube. The corresponding pitch is roughly 1 in 10 and 1 in 20. For a low velocity measurement (below 30 miles per hour) the screw S is turned until the level L_1 shows horizontal. The tube is then inclined 3 degrees. The instrument is thus quite independent of the leveling of the table or bench on which it may be used. Connection between the reservoir and glass tube is made by a short piece of rubber tube. Displacement of the liquid in T is read on a scale of 600 half millimeters attached to the frame. This manometer was made by a skilful instrument maker, and great care was taken to set the spirit levels at the correct angles. The best grade of German glass tubing was used, and each tube was carefully cleaned with strong sulphuric acid and potassium bichromate.

If there are no appreciable errors in the leveling, the correct head of liquid (alcohol, 95 per cent, stained red with fuchsine dye) is

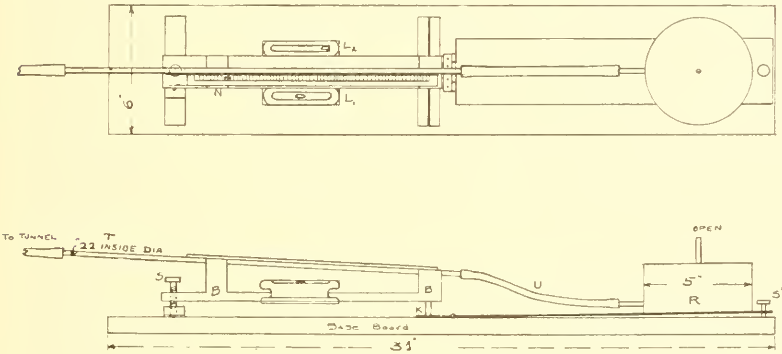

FIG. 8.—Alcohol manometer.

given from the geometrical construction. Thus: Head of liquid = displacement in $T \times$ sine of inclination. A small correction can be made for the depression of the liquid in R as the level in T rises. This also can be computed from the dimensions.

The density of the alcohol was taken on a chemist's "Westfall Balance" to a precision of 0.1 per cent. The effect of surface tension is to cause the level in T to be slightly higher than the level in R when the two ends of the manometer are under the same pressure. This is not an error in the instrument, since the zero setting takes account of it.

Tests were made by connecting both the reservoir end of the alcohol gage and one leg of the Chattock gage to the same static pressure made by a water column. In this way errors due to fluctuations of pressure were eliminated. The Chattock gage readings were taken as a standard for reference. The same Chattock gage was used in all tests. The alcohol gage was fitted with a straight glass tube 0.15 inch in diameter. The tube was clean and dry. The velocity calculated from the alcohol gage was found to be 12 per cent low. The tube was then wet by blowing the liquid to the top of the scale and then allowing it to settle back to zero. Readings taken subsequently were only 4 per cent low. The experiment was repeated using a glass tube 0.17 inch in diameter, clean and wet. The velocity recorded was 4 per cent low. A different tube, but of same diameter, was then put in the alcohol gage. Its average readings were found to be 10 per cent low. Examination of the glass tube showed two minute cracks in the glass hardly to be seen with the naked eye. A glass tube 0.2 inch in diameter was then tested and read 2 per cent low. A tube 0.22 inch in diameter read 1.5 per cent low. A tube 0.25 inch in diameter could not be used on the 3-degree pitch, as the alcohol would not form a meniscus.

In all, some 1,000 check observations were made, and the following conclusions drawn:

(1) The inclined type of liquid gage as commonly employed in ventilation work is not an instrument of precision.

(2) For consistent results, the glass tubing used must be free from all slight flaws on the inner surface which might cause changes in capillarity throughout the bore.

(3) The tube must be uniform in diameter.

(4) The tube must be as large as it is possible to use and still get a good meniscus.

(5) For alcohol at 3 degrees inclination an internal diameter of 0.22 inch is suitable.

(6) The maximum precision with such a gage used to measure air speeds from 4 to 40 miles per hour is about 1.5 per cent on velocity.

(7) The alcohol gage properly constructed is consistent and very sensitive.

(8) The alcohol gage may be used as an instrument of precision when calibrated against a standard.

In its final form with 0.22-inch tube, this alcohol gage was found to measure speeds within 1.5 per cent. Such precision is ample for engineering work, and this type of gage is recommended for a cheap portable instrument. For a laboratory standard, however, an error of 1.5 per cent cannot be accepted. Since the gage responded to changes of velocity of less than ¼ per cent, its sensitivity is such that it may be calibrated against a better manometer, and when calibrated, may be as precise as the standard.

IV. ADJUSTMENT OF VELOCITY GRADIENT ACROSS A SECTION OF THE WIND TUNNEL

By H. E. ROSSELL, Asst. Naval Constructor, U. S. Navy, and
D. W. DOUGLAS, S. B.

In any wind tunnel experiments, it is important that the velocity of the air striking different parts of the model shall be the same. Consequently, after developing precise methods for measuring velocity, the cross-section of the tunnel was explored to detect variations in velocity from point to point.

The procedure was as follows: The side plate described above (Report I, page 10) was connected to the Chattock gage. One observer by regulating the motor field rheostat kept the velocity as nearly constant as possible, indicated by keeping his gage reading constant. A speed of about 28 miles per hour was selected as standard. The Pitot tube (National Physical Laboratory tube) was mounted on a standard and moved parallel to itself along vertical lines 6 inches apart. Great care was taken to point the tube in the axis of the wind. The Pitot tube was connected to the alcohol gage. A velocity reading was taken every 6 inches by a second observer. The same two observers made the entire test.

The first preliminary tests showed the velocity over the section for a constant static pressure, as shown on the Chattock gage to be far from uniform. The velocity near the sides was higher than near the center. Such a result could be caused by the honeycomb at the suction mouth of the tunnel. The air entered the mouth in converging lines of flow as was shown by the direction taken by fine silk threads. The honeycomb was at the very end of the tunnel, and probably straightened out this flow too soon to allow a sufficient volume of air to reach the center.

To assist the air to flow more to the center, the honeycomb was shoved 1 foot into the tunnel. The effect was satisfactory, but not enough.

The honeycomb was then shoved 1 foot farther into the tunnel. The exploration of velocity over the section showed that a fair result had been obtained, and it was concluded that no advantage would be gained from further movement of the honeycomb.

Since models not greater than 18 inches in span are to be used, the useful part of the tunnel is included within a square 2 feet on a side. The parts of curves, drawn through the experimental points, which passed through the 2-foot square, show an average velocity of 27.8

miles per hour. The per cent difference between the velocity at each point and the average velocity of 27.8 miles per hour was then calculated. The results are shown marked on the section of the tunnel in figure 9. It is seen that the variation in velocity over the useful part of the tunnel is from + 1.1 to − 1.2 per cent. It is believed that this

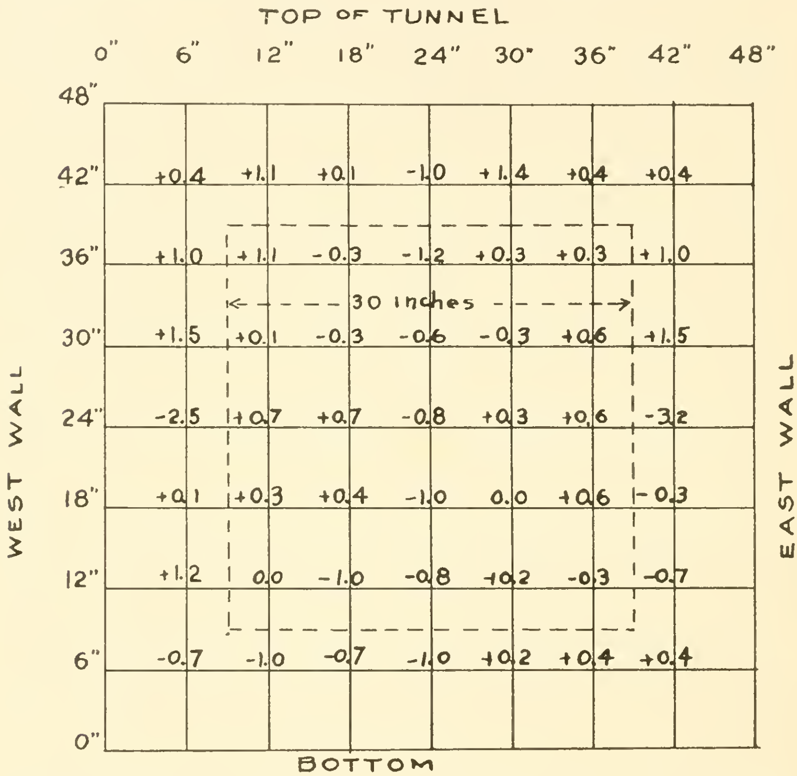

Fig. 9.—Variation of velocity across section of wind tunnel. Points represent per cent above or below mean velocity in dotted square which is 27.8 miles per hour.

variation is not too great for our purposes, and that the uniformity of flow compares well with that to be found in other wind tunnels.

With regard to the effect of the variation in velocity across the section, it may be stated that such a small variation in velocity is only of importance in tests on the moment tending to turn a model aeroplane away from or into the wind. An excess of velocity on one wing would give a tendency of the model to show a "lee helm." However, if the model be reversed and the experiment repeated, the average of the measurements taken in both tests will have eliminated any error due to lack of symmetry in the flow.

Rotation or twist of the air in the tunnel has not been detected, and if present must be small. The honeycomb at entrance and baffles at exit are designed to prevent the air spinning with the propeller.

V. CHARACTERISTIC CURVES FOR WING SECTION, R. A. F. 6

By H. E. ROSSELL, Asst. Naval Constructor, U. S. Navy, C. L. BRAND, Asst. Naval Constructor, U. S. Navy, and D. W. DOUGLAS, S. B.

In order to furnish a final check upon the calibration of instruments, the alignment of tunnel and balance and general methods of testing, it was considered desirable to repeat the determination of the aerodynamical constants published by the British Advisory Committee for Aeronautics, Report 1912-13, for the wing profile, designated as R. A. F. 6.

Two models 18 inches span by 3 inches chord were cut in brass, and carefully filed and scraped to form. The surface was highly polished to remove tool marks.

Each model was mounted vertically in the wind tunnel and its "lift" and "drift" forces measured on the balance for angles of the chord to the wind from -4 degrees to $+18$ degrees.

The moment of the resultant force about the vertical axis of the balance was measured on the torsion wire. It was then possible to determine the direction of the resultant from the ratio $\frac{\text{lift}}{\text{drift}}$, the magnitude by $\sqrt{\text{Lift}^2+\text{Drift}^2}$, and its line of action from: $\frac{\text{observed moment}}{\text{magnitude}}=$ perpendicular distance from axis.

The center of pressure is usually defined arbitrarily as the intersection of the resultant force with the plane of the chord. This point was calculated for each incidence.

On figure 10 are plotted the values of lift and drift coefficients, defined by:

$$K_y=\frac{\text{Lift}}{AV^2},\quad K_x=\frac{\text{Drift}}{AV^2},$$

where A is the wing area in square feet, and V the velocity of the wind in miles per hour. Lift and drift are in pounds force, hence K_y

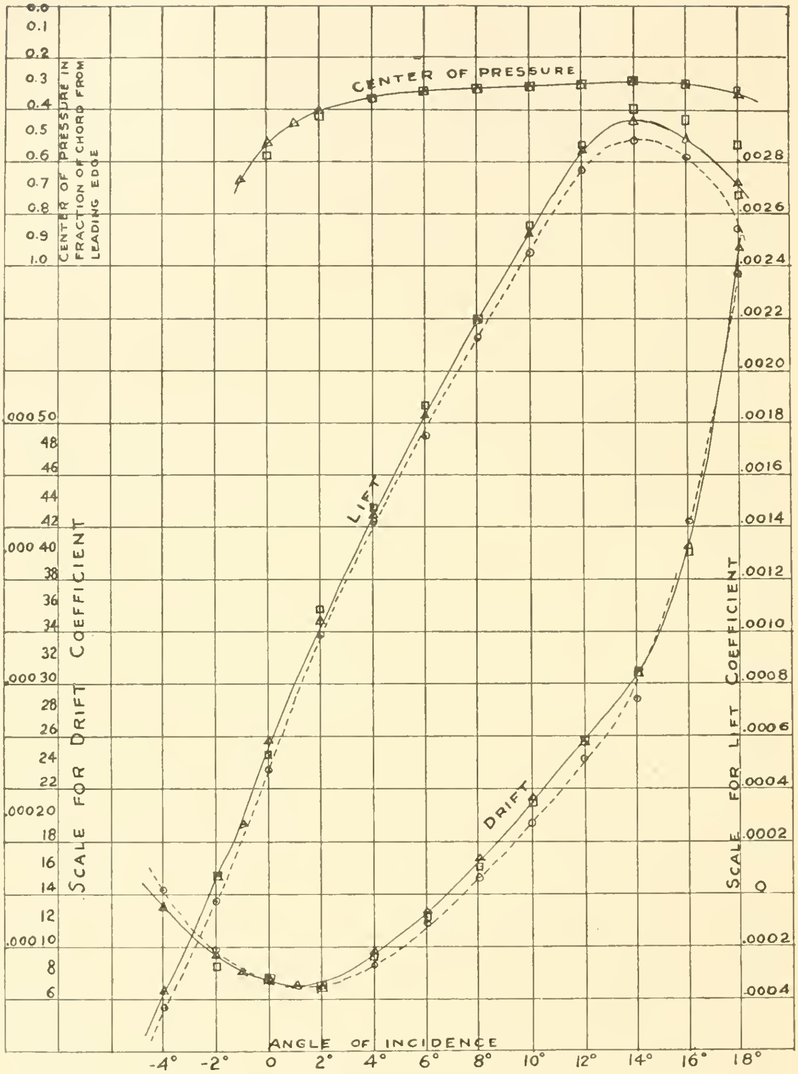

FIG. 10.—Wind characteristics.

Aerofoil R. A. F. 6.
Velocity 29.85 miles per hour.
Density of air 0.07608 pound per cubic foot.
⊡ Nat. Phys. Lab. observations.
—— Δ Mass. Inst. Tech. observations, model A.
- - - ⊙ Mass. Inst. Tech. observations, model B.

is the force in pounds on 1 square foot due to a wind of 1 mile per hour, of air of standard density (*i. e.*, .07608 pound per cubic foot).

The "center of pressure" is also shown on figure 10 in terms of distance from leading edge in fraction of chord.

On the same sheet are shown the experimental points published by the National Physical Laboratory for this wing, using the same size model and the same speed, *i. e.*, 29.85 miles per hour.

It is seen that there is only slight discrepancy between the lift observations up to an incidence of 14 degrees, the useful range in aviation. The English points lie from 1 to 3 per cent higher than the corresponding points for model B, and coincide with those of model A.

Similarly for the drift observations there is very good concordance up to 14 degrees.

The curve of center of pressure coefficient is in practical coincidence with the English observations.

It appears that undetected differences in workmanship and finish between two models may cause a change in coefficients of not more than 3 per cent. Actual observations are precise within one-half of 1 per cent. Consequently, our results may be considered sufficiently precise for purposes of aeroplane design.

VI. STABILITY OF STEERING OF A DIRIGIBLE

By J. C. HUNSAKER

When floating in the air with no way on, a dirigible takes an attitude such that the center of gravity lies on the vertical passing through the center of buoyancy of the envelope or gas bag. The ship is then in stable equilibrium. When under way, the thrust of the propellers is balanced by the resistance of the air. In order that the attitude shall not be changed, the moment of the propeller thrust and the moment of the air resistance taken about any point must balance one another. It is convenient to take axes of coordinates, vertical, transverse, and horizontal, located at the center of buoyancy. If it be assumed that the ship is in equilibrium on her course, then the component forces along and moments about the three axes through the center of buoyancy are each zero.

This is equivalent to the statement that the motion of the ship is one of pure translation in the direction of the fore and aft axis of the envelope, which axis is horizontal.

Any angular deviation of this axis will call into play moments about the axes chosen above as well as forces along those axes. The forces will cause the trajectory of the center of buoyancy to be deflected, and the moments will swing the ship's axis. The motion is stable if the forces and moments tend to restore the original attitude and state of motion.

If we consider only the moments produced by angular deviation of the ship's axis, we may say that the ship is stable if the moments tend to restore the original attitude, and unstable if they tend to magnify any initial deviation.

With a model held fixed in a current of air by a spindle passing through the center of buoyancy, the stability of any attitude is measured by the moments about the spindle.

An elongated ellipsoid, as can be shown by hydrodynamic theory,[1] has three positions of equilibrium in a wind, corresponding to the directions of the three axes. Only one position, however, is stable, and the body tends to place itself broadside to the wind. For torpedo-shaped bodies, the stable position is intermediate between the broadside-on position and the desired bows-on position, and for any such body there is a stable "drift-angle" at which the body tends to hold its major axis to the wind. This angle may be between 50 and 90 degrees to the wind.

A feathered arrow with weighted head is stable for a translation along the direction of its shaft. In general, it is not practicable to fit sufficient fin surface at the stern of a dirigible envelope to give it such weather-cock stability, but it is possible to reduce the "drift-angle" to 20 degrees.

A dirigible must be steered both in a vertical and in a horizontal plane and its stability of route requires both horizontal and vertical fins and rudders.

A wooden model of a dirigible hull was fitted with rudders and fins in accordance with usual practice and tested in the wind tunnel at 35 miles per hour. The fin and rudder area was then adjusted until a satisfactory combination was obtained.

The principal interest in the research lies in the fact that it is generally possible to base the designed fin and rudder area upon such experimental wind tunnel tests instead of rule of thumb.

The model was mounted in the wind on a vertical spindle passing through the center of buoyancy as calculated from the plans. The

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[1] Lamb, Hydrodynamics, p. 121.

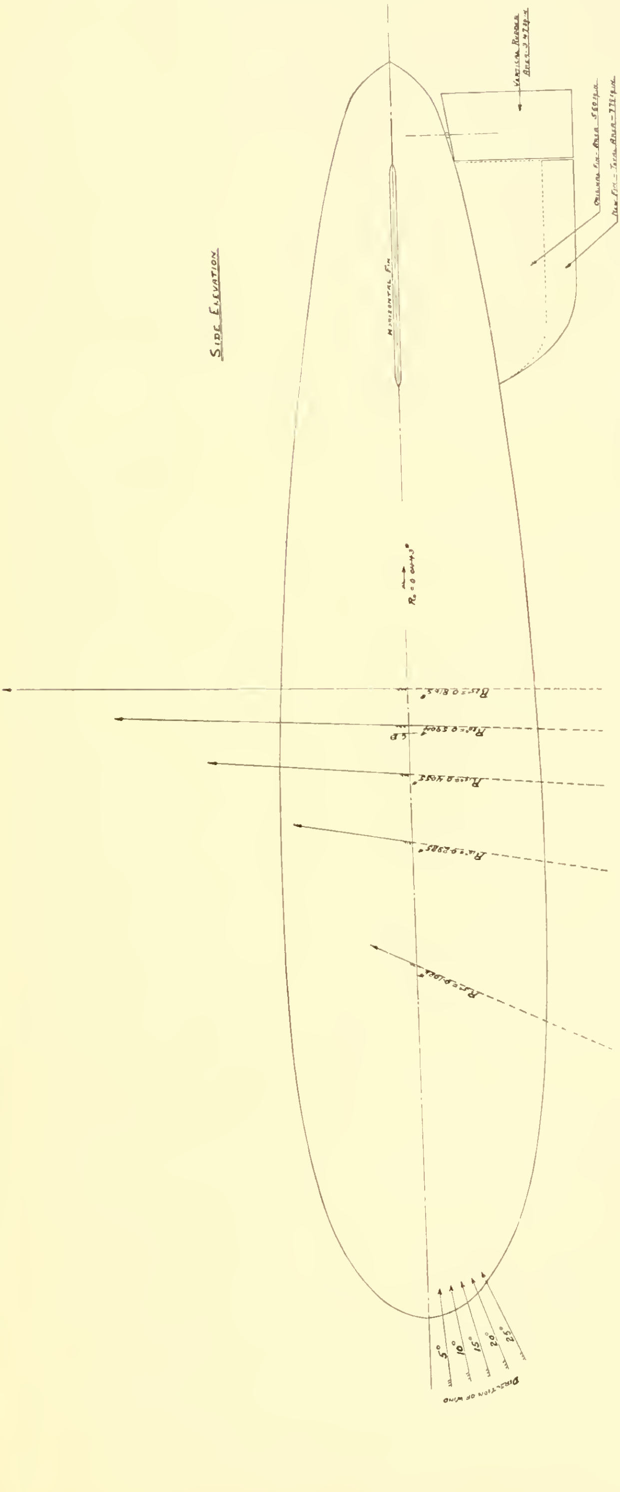


Fig. 11.—Resultant forces on model dirigible envelope for angles of pitch. Forces in pounds for 35 M. P. H., standard air (dry air), 62° F., 29.921" barometer.

moment about the vertical axis through the spindle, M, was measured on a torsion wire with the model inclined 0, 5, 10, 15, and 25 degrees to right and left of the tunnel axis. Similarly the longitudinal and lateral components of resultant wind force in the horizontal plane were observed, *i. e.*, R_x and R_y.

The total resultant force is then

$$R = R_x^2 + R_y^2.$$

The direction of this force is at the angle

$$\theta = \tan^{-1} \frac{R_y}{R_x}$$

measured from the axis of the tunnel. The resultant force R has an arm A, the perpendicular distance from the center of buoyancy to the line of action of R. Thus

$$A = \frac{M}{R}.$$

The force R is then determined in magnitude, direction, and line of application, and is laid out graphically on the model drawing for each angle of inclination.

Experiments were made with the model swung on its vertical axis through the center of buoyancy to obtain the yawing moments, and with the model on its transverse axis to obtain the pitching moments.

The resultant forces for angles of pitch are shown graphically on the side elevation of the model, figure 11. It appears that with the horizontal fins fitted, the resultant forces pass forward of the center of buoyancy. The model is, therefore, unstable when moving in the direction of its longitudinal axis, and if left to itself will take a "drift angle" of about 20 degrees up or down. In practice this pitching moment is counterbalanced by the powerful righting couple due to the weight of the car suspended beneath the envelope.

For example: If the center of gravity of the whole ship be a distance d below the center of buoyancy, the righting couple for a pitch of θ degrees is

$$M_s = Wa \sin \theta$$

where W is the total weight.

On the other hand, the upsetting moment on the envelope due to wind forces is a function of the inclination, as shown, and the velocity squared, or

$$M_e = KV^2 f(\theta)$$

where K is a constant.

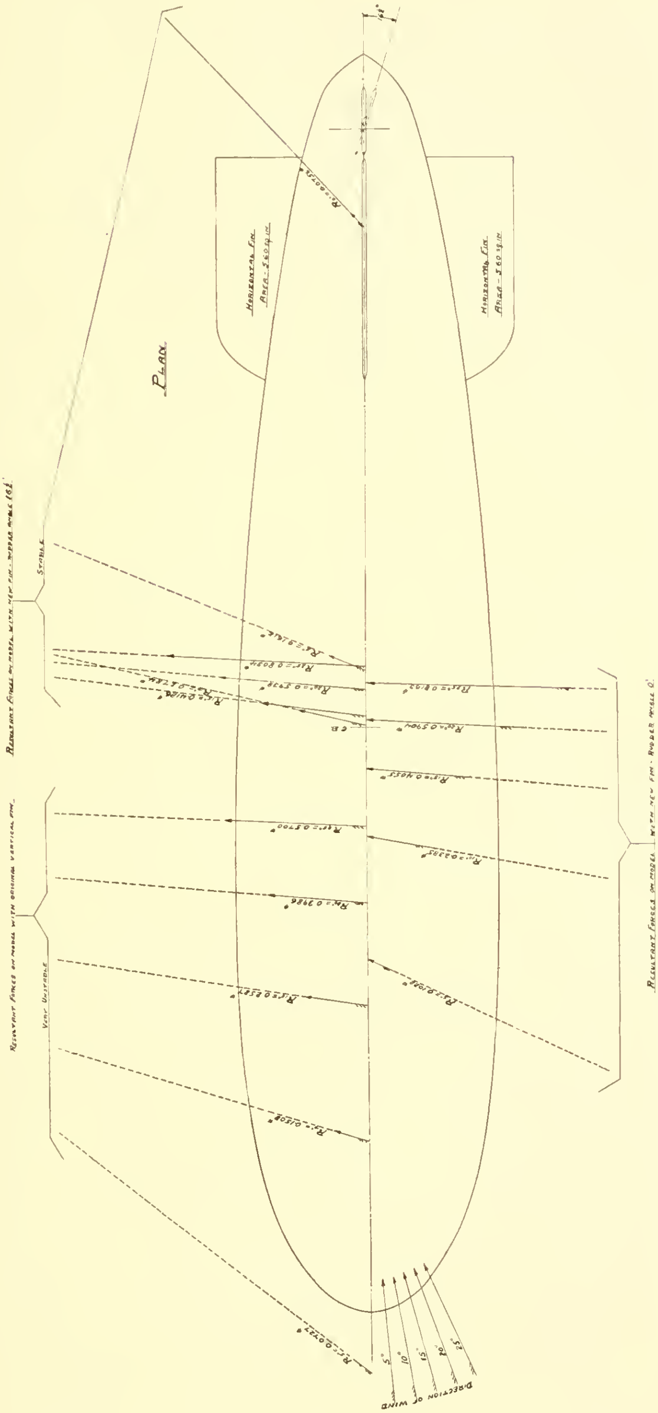


FIG. 12.—Resultant forces on model dirigible envelope for angles of yaw. Forces in pounds for 35 M. P. H., standard air (dry air), 62° F., 29.921" barometer.

For longitudinal stability, the following inequality must be satisfied:

$$M_s > M_e$$
$$Wa \sin\theta > KV^2 f(\theta).$$

For given incidence, $f(\theta)$ and $\sin\theta$ are constant, and hence the righting moment due to weights is constant. The upsetting moment of the wind increases as the square of the velocity. At some critical velocity the upsetting moment may preponderate and the ship become unmanageable.

This is the critical velocity first pointed out by Colonel Renard, which led to serious difficulties in the early dirigibles.

For a given design, it is possible from wind tunnel tests to determine this critical velocity, and by suitable additions to the horizontal fin area or lowering of the car to insure that in operation the critical velocity can never be reached.

The tests as described above were repeated for angles of yaw. In the first series, a single vertical fin of 5.6 square inches area was fitted as shown in figure 11. The resultant forces are drawn on figure 12. The model is very unstable, and tends to swing to the right or left of its course until the axis makes an angle of about 40 degrees to the wind. The fin area is obviously insufficient. It will be noticed that the resultant force for 5 degrees yaw passes outside the model. This is no doubt due to the arbitrary mechanical definition of resultant force. The resultant force as drawn merely represents that force which, acting along the line shown, will have the same moment about the center of buoyancy as the complicated distribution of pressure about the model. This experimentally observed moment might as well be represented by a couple.[1]

To improve the stability of steering, a larger vertical fin and a vertical rudder were next fitted and the test repeated. The rudder was fixed in the plane of the fin. The resultant forces are shown on the lower part of figure 12. It appears that the drift angle has been reduced from 40 to 20 degrees, but that the ship is still unstable. It is not practicable to fit more fin surface, and the remaining instability must be met with the rudder.

The rudder was then set at $16\frac{1}{2}$ degrees to the keel line and the test repeated. The resultant forces shown on figure 12 are seen to lie

[1] This position of the resultant force at small angles was predicted by Sir George Greenhill, and later verified experimentally by L. Bairstow, Tech. Report of the Advisory Committee for Aeronautics, p. 35, London, 1910-11.

abaft the center of buoyancy. The ship is therefore stable. At 10 degrees angle of yaw, the resultant force passes nearly through the center of buoyancy, and if the ship should get in this position the pilot would have to use slightly more than $16\frac{1}{2}$ degrees rudder angle to bring her back.

The conclusion from these tests is that with the size rudder and fin fitted, 7.79 and 3.47 square inches, the ship can be held on her course by the use of not more than $16\frac{1}{2}$ degrees of rudder.

The importance of an adequate vertical rudder is apparent, since the weights give no restoring moment for yawing as they do for pitching.

It appears impossible in practice to give sufficient vertical fin area to hold the ship on her course without use of the helm.

VII. PITCHING AND YAWING MOMENTS ON MODEL OF CURTISS AEROPLANE CHASSIS AND FUSELAGE, COMPLETE WITH TAIL AND RUDDER, BUT WITHOUT WINGS, STRUTS, OR PROPELLER

By J. C. HUNSAKER and D. W. DOUGLAS

A wood model of the Curtiss tractor body, complete with all appendages except wings, was constructed to a scale 1 inch to the foot and 24.5 inches actual length. The model was held in its ordinary position in the wind tunnel by a vertical spindle attached to the balance. The angle of yaw was varied, and observations were made to determine the components of force directed down stream and across the stream, as well as the twisting moment about a vertical axis passing through the supporting spindle. The axis of the model was kept in a horizontal plane during this test.

The model was then removed from the spindle shown in figure 13, side elevation. A new spindle was set into the side of the model as shown in figure 13, plan. The model was again placed in the wind tunnel but now lay on its side. Rotation about the new spindle axis made it possible to set the model at various angles of pitch, the angle of yaw being held constant. Measurements were made as above of down stream and cross stream components of force, and moment about the axis of the spindle.

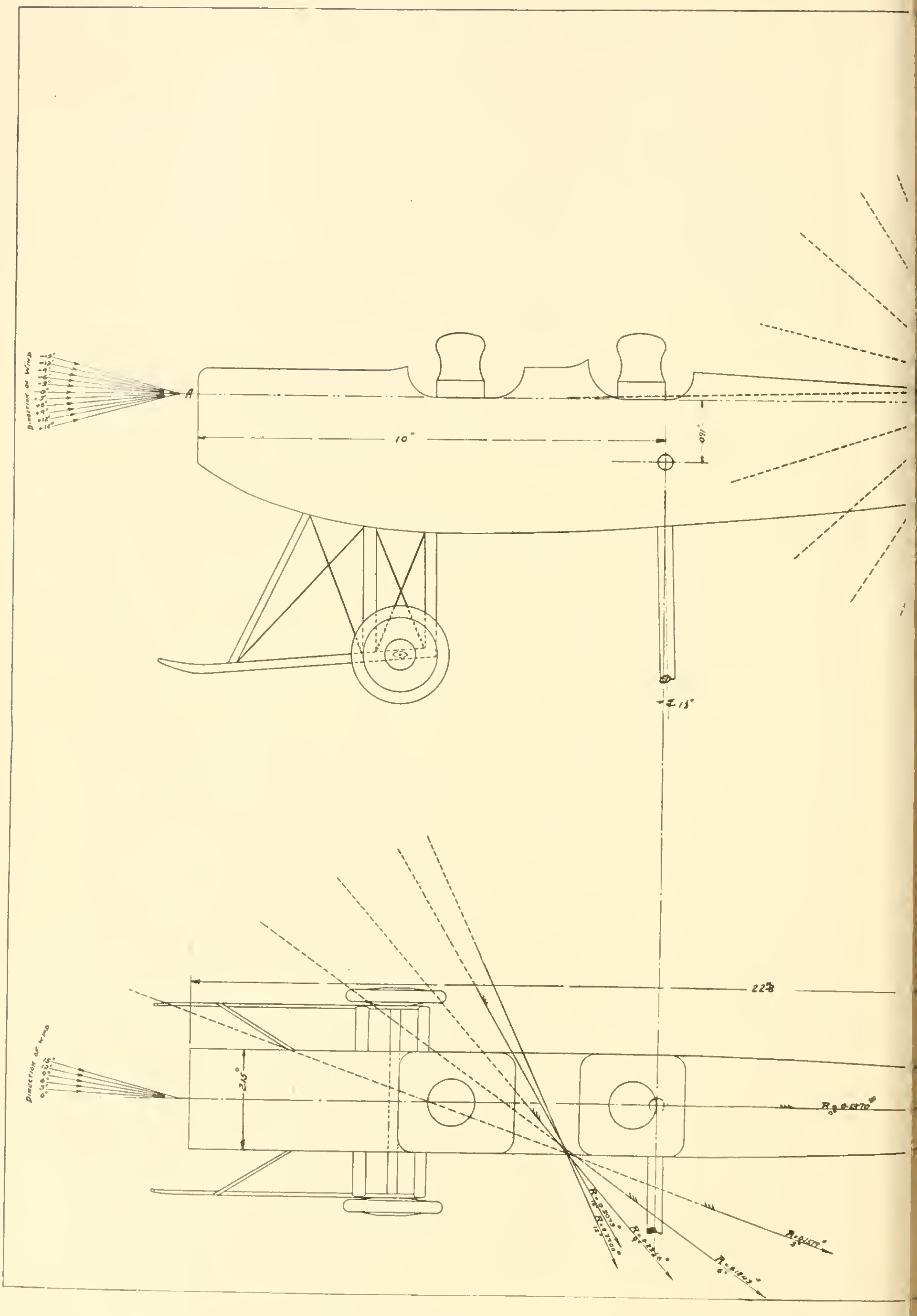
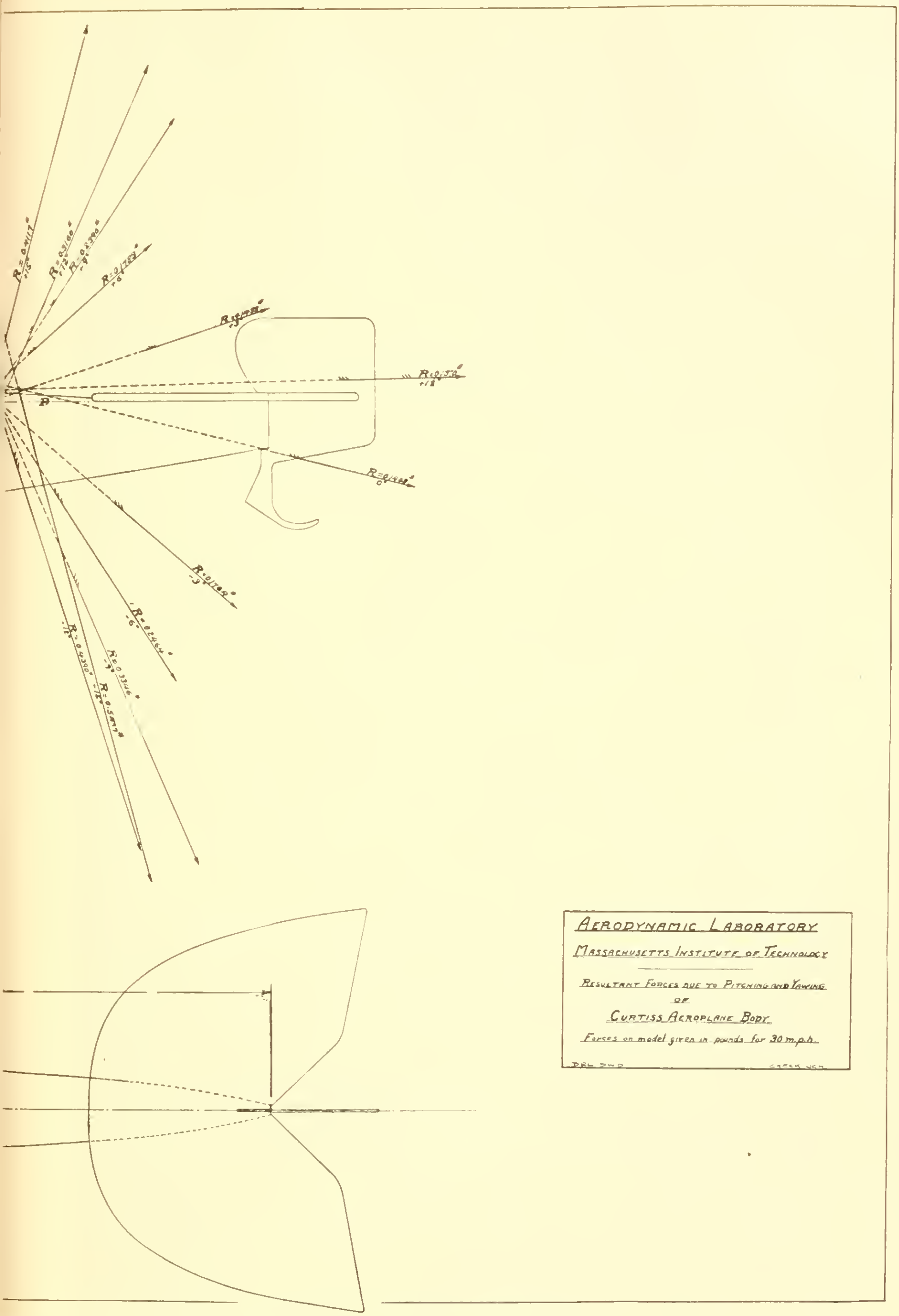


FIG. 13.—Resultant forces due to pitching and yawing of Curt



plane body. Forces on model given in pounds for 30 M. P. H.

The forces and moment measured in the first test are called

R_x, drift component,
R_y, cross wind component,
M_z, yawing moment;

and in the second case,

R_x, drift component,
R_z, lift component,
M_y, pitching moment.

Forces are measured directly in pounds and moments in pounds-inches on the model for a wind velocity of 30 miles per hour. Density of air is 0.07608 pound per cubic foot.

For any angle of pitch we may substitute for the lift, drift, and pitching moment a resultant force vector defined as that force which is the mechanical equivalent of these. It is to be noted that we here deal only with forces in the plane of symmetry of the aeroplane.

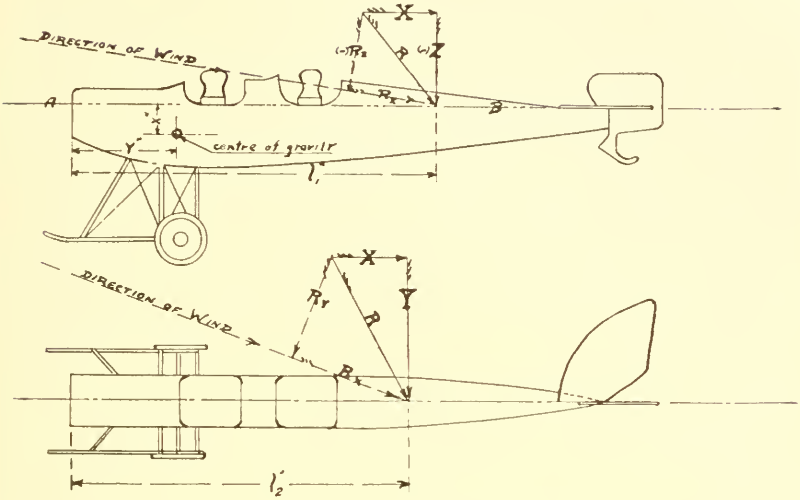
Similarly in the second test, where the angle of pitch was kept constant at 1.5 degrees, the drift, cross wind force, and yawing moment are represented by a vector in the horizontal plane passing through the intersection of the body axis and vertical spindle.

These two resultant force vectors may be called for convenience pitching resultant and yawing resultant. They are shown in position, magnitude, and direction on the two views of figure 13.

The artifice of representing a force and a moment by a vector makes it immaterial where the axes of support of the model were originally taken. In aeroplane design, the axis of the propeller usually is made to pass near the center of gravity, and it is hence necessary to locate the center of gravity at such a point that angular deviations from normal flight attitude will produce moments about the center of gravity tending to restore the original attitude.

The usual location of the center of gravity well forward in the body will insure that the pitching resultants pass to the rear of the center of gravity and that the pitching is stable. See figure 13, side elevation, where the pitching resultants are shown graphically.

For convenience, the resultant force on the model is given on figure 14 in terms of lift and drift components marked R_z and R_x. The resolution of the forces is shown on the upper figure. It appears from figure 15 that the drift R_x is practically constant from $+2$ degrees to -2 degrees pitch, but the lift R_z is zero only for $1\frac{1}{2}$ degrees pitch. It would be of some advantage to fly the machine at full speed with the body "tail heavy" $1\frac{1}{2}$ degrees.



ANGLE OF PITCH	R_X	R_Z	X	Z	l_1''
0°	0.1365	-.0340	.1365	-.0340	18.56
+3°	.1383	+.0370	.1362	+.0442	16.80"
+6°	.1453	+.1050	.1335	+.1196	16.54"
+9°	.1615	+.1760	.1321	+.1991	16.82"
+12°	.1854	+.2570	.1279	+.2898	17.05"
+15°	.2227	+.3460	.1256	+.3918	16.80"
-3°	.1395	-.1080	.1336	-.1151	17.00"
-6°	.1543	-.1920	.1334	-.2071	17.00"
-9°	.1784	-.2830	.1319	-.3084	17.00"
-12°	.2180	-.3810	.1340	-.4182	16.95"
-15°	.2727	-.4750	.1405	-.5293	17.51"

PITCHING MOMENT "# (about C.G.) = $X \times x'' - Z \times (l_1'' - Y'')$

ANGLE OF YAW	R_X	R_Y	X	Y	l_2''
0°	0.1365	0	0.1365	0	—
3°	.1454	0.0440	.1429	0.0515	5.12"
6°	.1599	.0915	.1495	.1077	6.72"
9°	.1796	.1525	.1535	.1788	7.18"
12°	.2034	.2255	.1521	.2628	7.51"
15°	.2358	.2850	.1541	.3364	7.61"

YAWING MOMENT "# (about C.G.) = $Y \times (l_2'' - Y'')$

NOTE :- ALL FORCES ON MODEL ARE IN POUNDS, AT 30 M.P.H.

FIG. 14.

It is also possible to resolve the force R into components X along the axis of the machine and Z perpendicular thereto. Since a force may be resolved at any point in its line, we choose the intersection of the line of action with AB. This point is a center of pressure and is a distance "l_1" from the nose of the machine. See upper figure of figure 14. A force is completely defined by X, Z, and l_1, and these quantities are given in the upper table. The component X has no moment about the center of gravity if the center of gravity be on line AB, and if the center of gravity be a distance y from the nose, the pitching moment is $(l_1-y)Z$. Upward forces are plus. In general, for a center of gravity with coordinates x, y as shown on figure 14, the pitching moment is:

$$X_x-Z(l_1-y).$$

By use of figure 14 and this formula, a curve of pitching moments can be readily obtained for any assumed position of the center of gravity.

In a similar manner the model was held at a pitch angle of $1\frac{1}{2}$ degrees to the horizontal wind, and placed at angles of yaw from 15 degrees right to 15 degrees left. Observations right and left have been averaged. The resultant forces are shown in direction, magnitude, and application in the lower figure of figure 13. The cross wind and drift components R_y and R_x are tabulated on figure 14.

As above, the components along the axis of the body X, and at right angles Y, are also tabulated. The intersection of the line of the resultant force with the vertical plane of symmetry is taken as a center of pressure and is a distance l_2 from the nose of the body. The yawing moment about the center of gravity placed a distance y from the nose is hence

$$Y(l_2-y).$$

It is apparent from the last test that the aeroplane is directionally unstable, unless the center of gravity be well forward of the forward passenger's seat. The addition of a propeller and the fin effect of biplane struts will augment this tendency to instability of steering, and require a still farther forward position of the center of gravity.

Forces are given in pounds on the model for a wind speed of 30 miles. If the model is to a scale of 1 inch to the foot, the forces on the full-size body will be 144 times as great and at 60 miles four times greater. Thus the resistance of this body at 60 miles would be about

$$0.137\times 144\times 4=79 \text{ pounds},$$

on the assumption that the resistance varies as the square of the speed. The true resistance would be less than this on account of the exponent of V being somewhat less than two. On the other hand, in

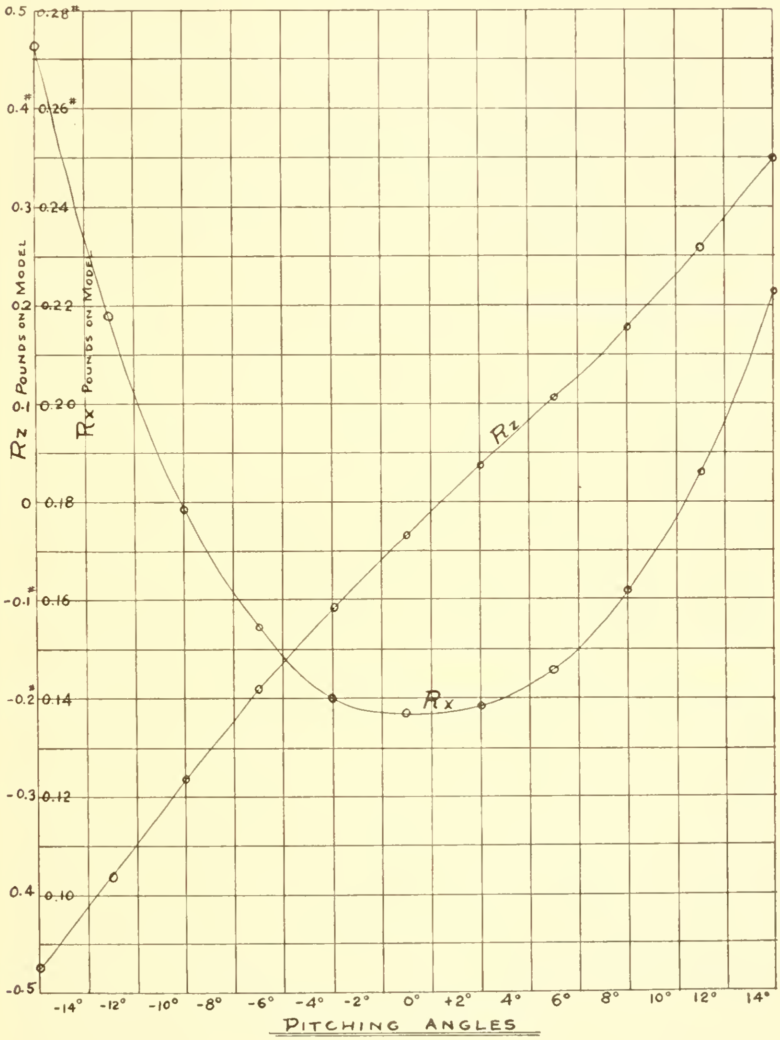


Fig. 15.

a tractor type aeroplane, the body resistance may be considerably augmented by the slip stream from the propeller. The assumption of the " square law " is usual in design.

To show the precision of the experimental work, curves of R_x, R_y, and R_z, plotted on pitch and yaw are given on figures 15 and 16. The

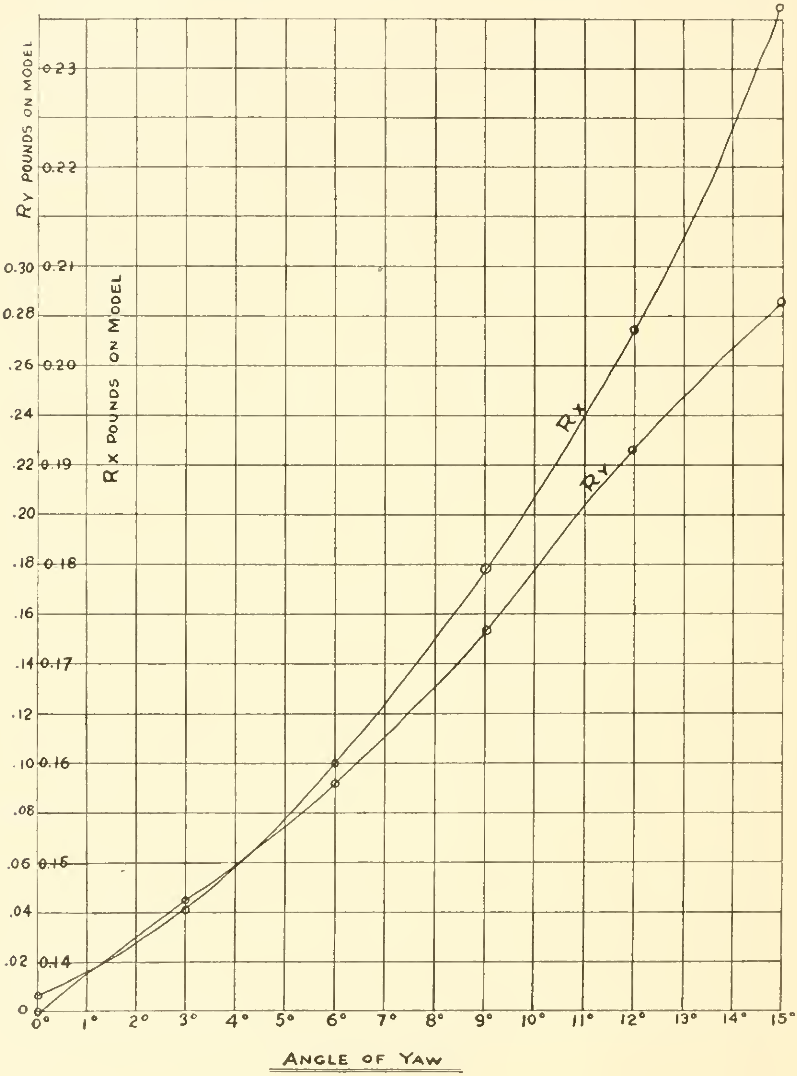


Fig. 16.

individual observations are there shown by the small circles which appear to permit a fair curve to be passed through them.

VIII. SWEPT BACK WINGS[1]

By H. E. ROSSELL and C. L. BRAND, Asst. Naval Constructors, U. S. Navy

Part I.—Lift, Drift, and Center of Pressure

To determine the effect on the aerodynamical properties of a wing of sweeping back the wing tips, as in the so-called German "Pfeil" aeroplanes, a series of tests was made on the model described as R. A. F. 6. The right and left halves of the wing were swept back 10, 20, and 30 degrees to the normal position as shown in figure 17.

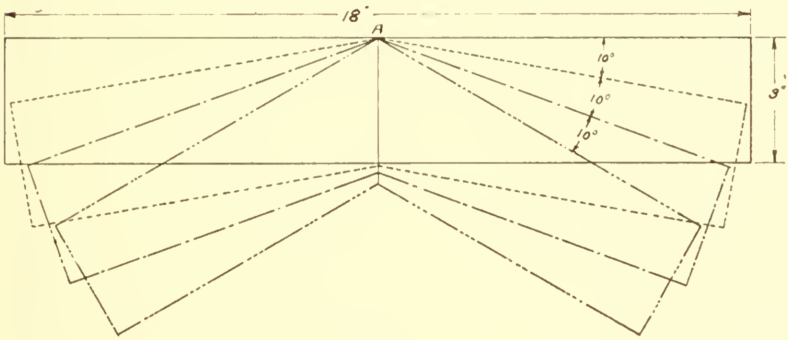


Fig. 17.

I.
II.
III.
IV.

The first part of the investigation deals with the variation in lift and resistance coefficients, and the movement of the center of pressure for the different models as the angle of incidence changes. The wind velocity of 29.85 miles per hour of standard air[2] remains in the plane of symmetry of the wing.

The curves of the above characteristics for the four wings are given in figures 18, 19, 20.

The ratios of lift to resistance coefficients are given in figure 21.

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[1] Abstract from a research submitted for the degree of Master of Science in the Department of Naval Architecture, at the Massachusetts Institute of Technology.

[2] Standard air is of density 0.07608 pound per cubic foot.

The following key will serve to identify the wings:

Wing	Swept Back
I	0 degrees
II	10 degrees
III	20 degrees
IV	30 degrees

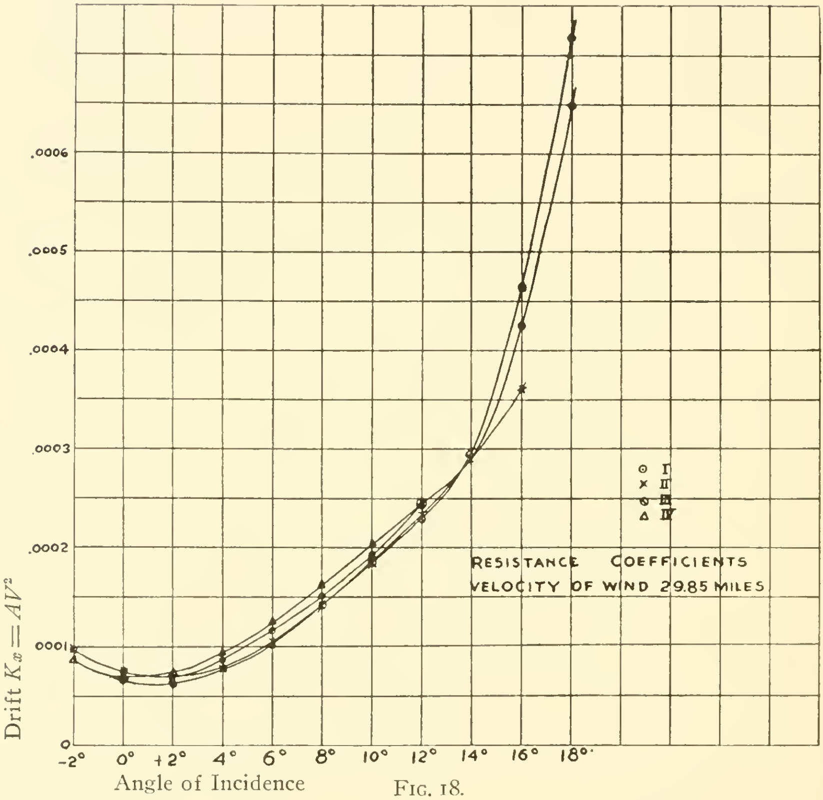
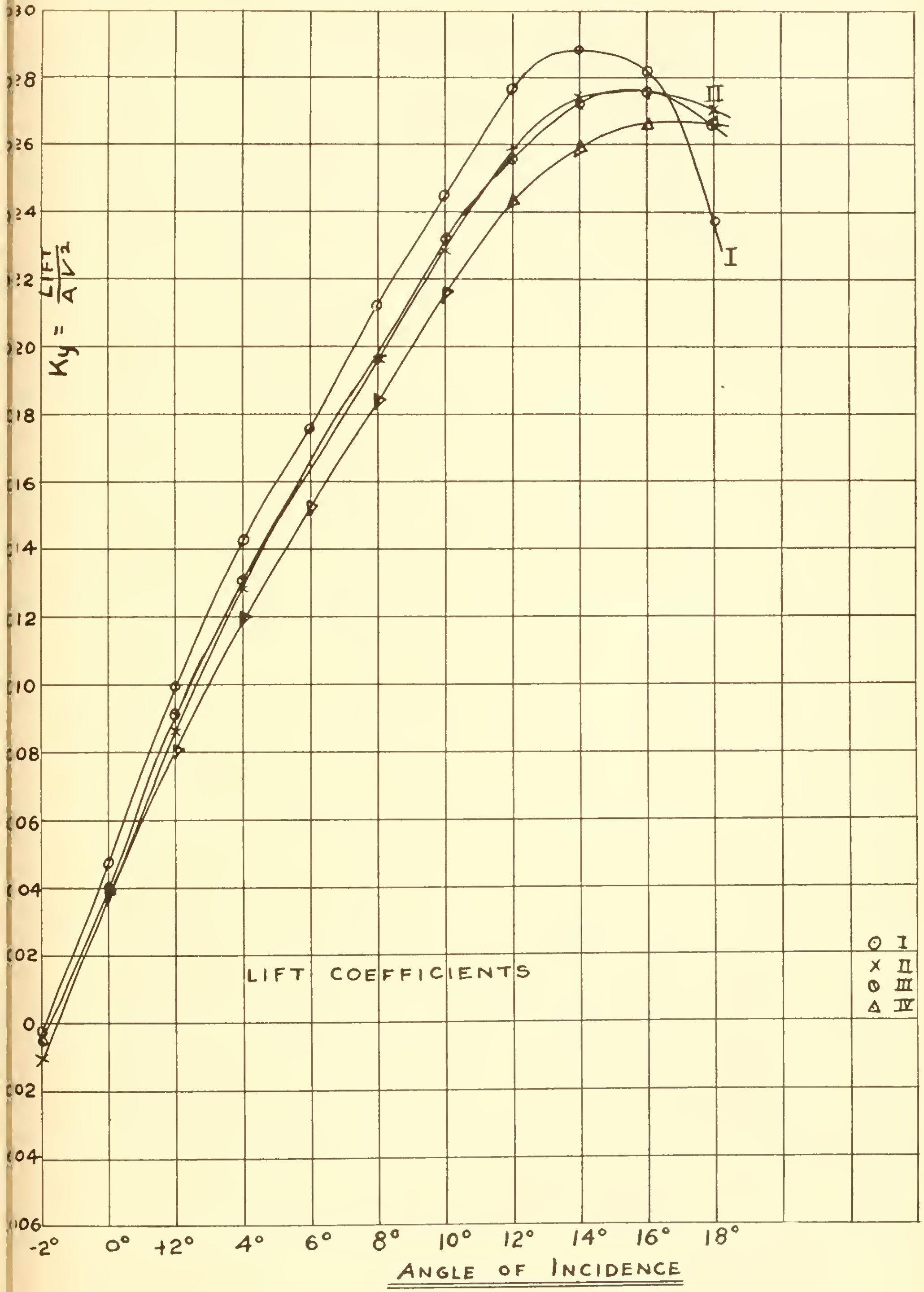


FIG. 18.

Coefficients are given as pounds force per square foot per mile per hour velocity.

It appears that a sweep back of 10 degrees and 20 degrees is of no disadvantage from considerations of lift and resistance. There is, however, no gain. For a sweep back of 30 degrees, the lift coefficient is diminished and the ratio of lift to resistance is seriously reduced. What advantages there may be as regards lateral sta-



·30
·28
·26
·24
·22
·20
·18
·16
·14
·12
·10
·08
·06
·04
·02
0
·02
·04
·06
$Ky = \frac{LIFT}{AV^2}$
LIFT COEFFICIENTS
II
I
⊙ I
× II
⊗ III
△ IV
-2°
0°
+2°
4°
6°
8°
10°
12°
14°
16°
18°
ANGLE OF INCIDENCE

FIG. 19.

bility for wings swept back 30 degrees are therefore paid for by a considerable loss in effectiveness. The ratio $\frac{K_y}{K_x}$ of 16.5 for the normal wing and for No. II is reduced to 12.9 for No. IV.

The center of pressure motion is shown by the curves of figure 20, in which it appears that the motion is similar for all the wings tested.

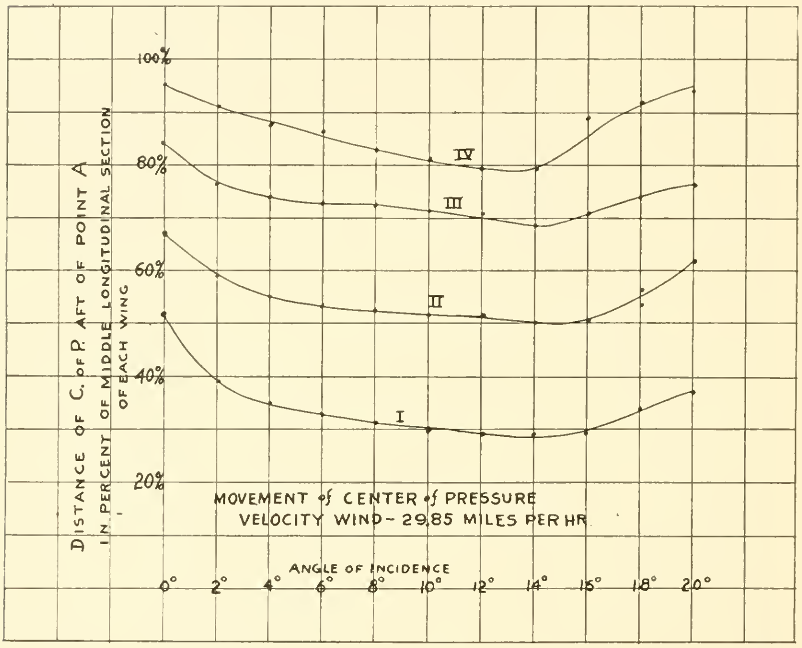
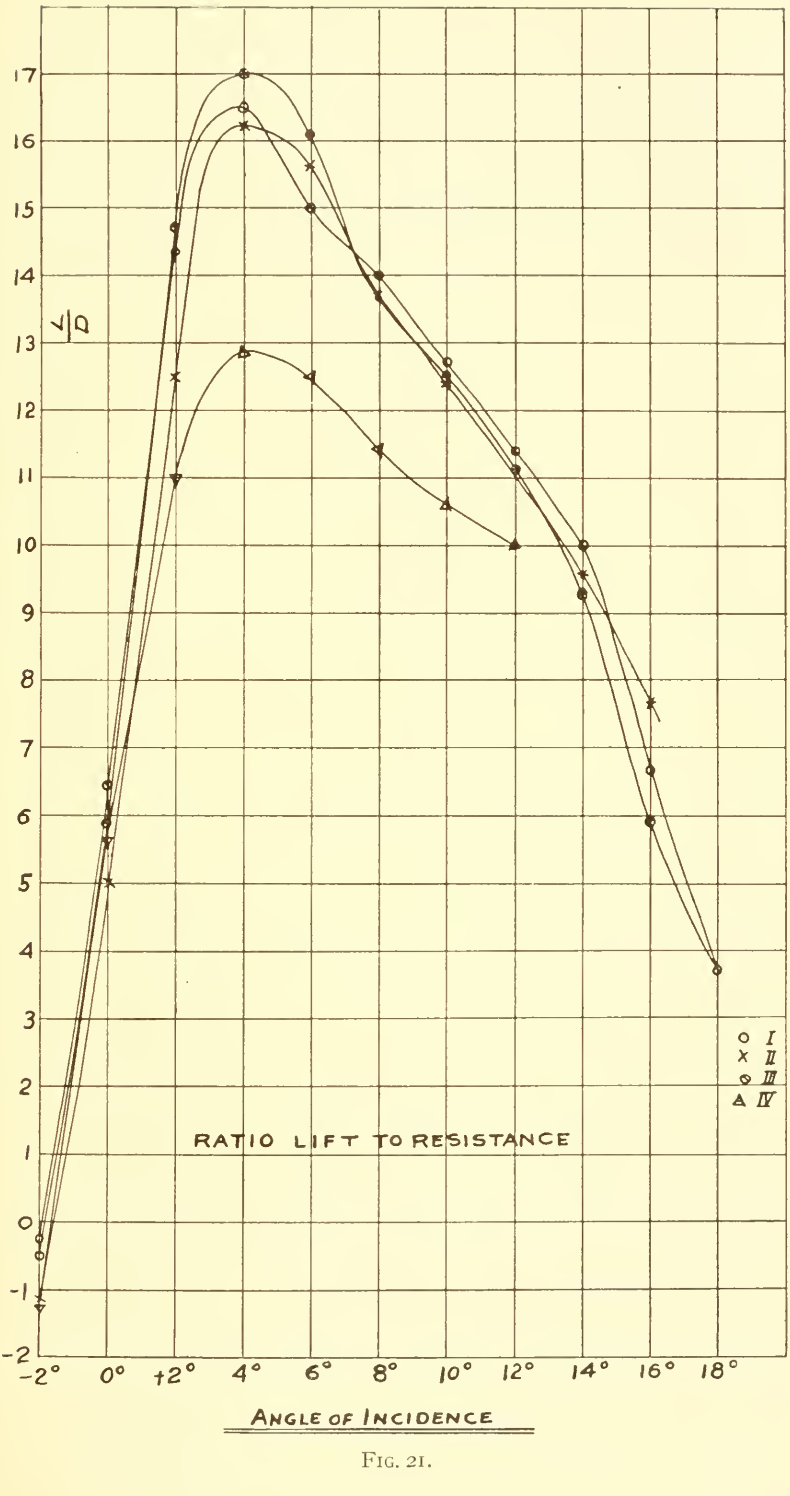


FIG. 20.

The center of pressure motion gives rise to the same degree of longitudinal instability. The center of pressure is referred to the forward point of the middle longitudinal section of each wing, and it appears that for a given angle of incidence the center of pressure is thrown to the rear by about two-tenths of the chord for a sweep back of 10 degrees, and four-tenths of the chord for a sweep back of 20 degrees. The center of gravity of an aeroplane will, therefore, have to be placed farther to the rear if swept back wings are used.



17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-1
-2
L/D
RATIO LIFT TO RESISTANCE
o I
x II
⊘ III
△ IV
-2°
0°
+2°
4°
6°
8°
10°
12°
14°
16°
18°
ANGLE OF INCIDENCE

FIG. 21.

Part II.—Rolling, Pitching, and Yawing Moments

The object of this test is to determine the effect on stability of sweeping the wing back to various angles, as described in Part I of this report.

The wing uncut was first mounted horizontally on the balance, and forces and moments as enumerated below were measured for angles

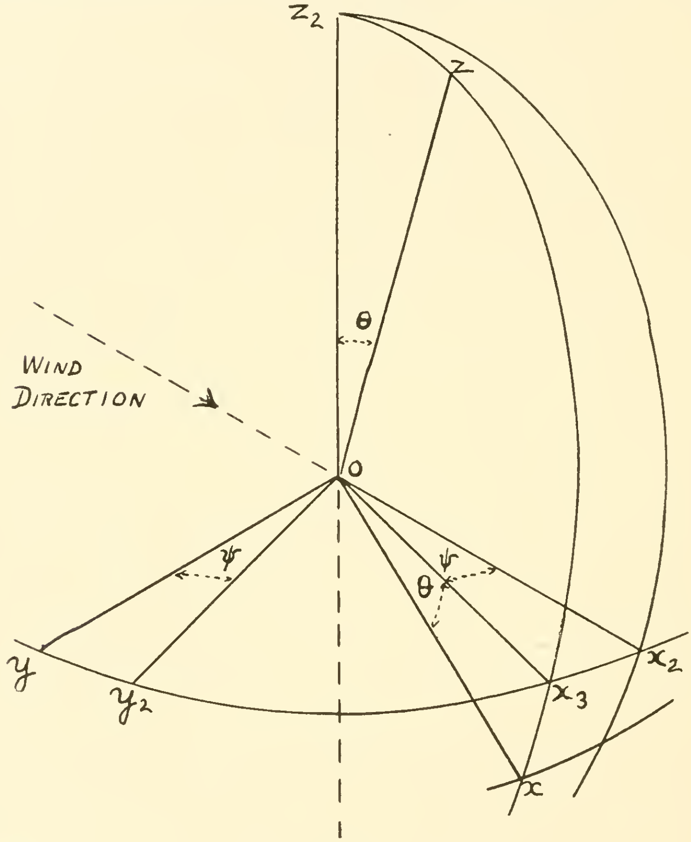


Fig. 22.

of yaw of 0, 5, 10, 15, 20, 25, and 30 degrees in both directions for each of the following angles of incidence: 0, 3, 6, and 9 degrees. The velocity of wind was kept constant at 29.85 miles per hour. The same measurements were made with wing models swept back to 10, 20, and 30 degrees. The method of manipulating the balance to make these measurements is given in detail in Report 68, Technical Report of the Advisory Committee on Aeronautics, 1912-13 (London).

It is convenient to calculate the forces and moments about moving axes fixed relative to the wing. The axes are lettered OX, OY, and OZ. In figures 22 and 23, point O we have located at the middle of the leading edge of each wing. OX is the fore and aft, OY the transverse, and OZ the normal axis of the wing. When the wing has zero incidence and yaw, the position of the axes is shown in figure 22 by

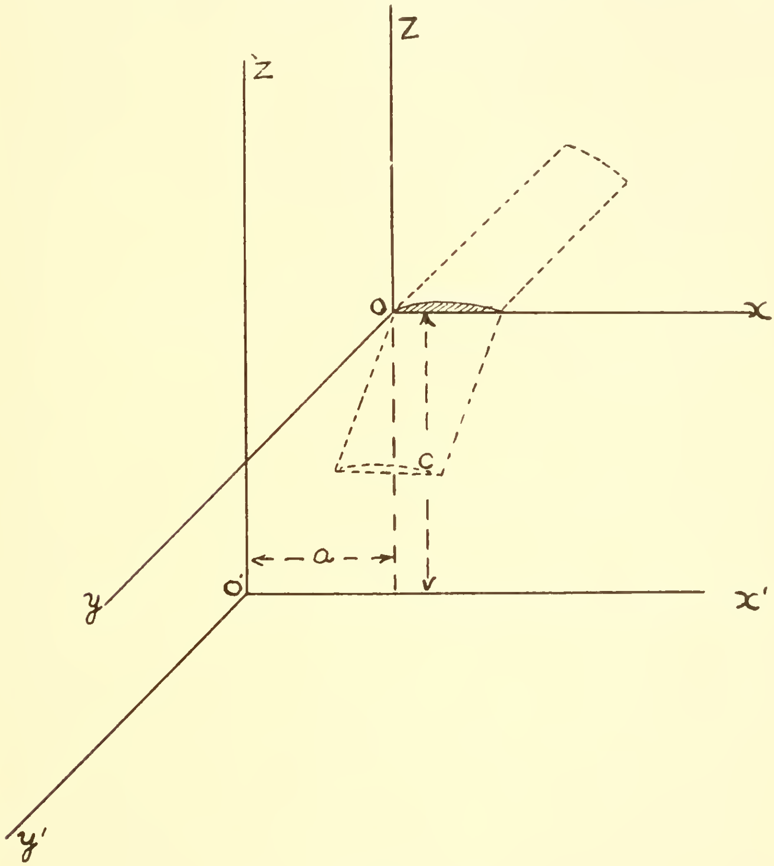


FIG. 23.

OX_2, OY_2, and OZ_2. Now suppose the wing to be turned about OZ_2 through a positive angle of yaw ψ. Then the axes will be in the position, OX_3, OY, and OZ_2. Now if the wing is given an angle of incidence of θ about OY, the axes will swing into the positions OX, OY, OZ.

In making the calculations it is easier first to refer the forces and moments to a set of axes parallel to OX, OY, and OZ, which we have designated as $O'X'$, $O'Y'$, and $O'Z'$. The origin of these axes is coincident with the origin of axes about which rolling and pitching moments were measured.

Forces along axes OX, OY, OZ are called AX, AY, AZ, A being the area of model in square feet; and moments about these axes are called L, M, and N. Forces along and moments about axes $O'X'$, $O'Y'$, and $O'Z'$ are represented in the same way, *i. e.*, AX', AY', etc.

The notation used for the measured forces and moments is as follows:

(1) Force along $O'Z_2' = V_F$. Measured on vertical force lever.
(2) Moment about $O'Z_2' = M_Z$. Measured on torsion wire.
(3) Moment about $O'Y' = V_P$. Measured on vertical force lever.
(4) Moment about $O'X_3' = V_R$. Measured on vertical force lever.
(5) Moment about an axis parallel to $O'Y_2'$ and distant l below it $= M_D$. Measured on drift beam.
(6) Moment about an axis parallel to $O'X_2'$ and distant l below it $= M_C$. Measured on cross wind beam.

The forces and moments desired can then be calculated by the following equations:[1]

$$L' = V_R \cos\theta - M_2 \sin\theta.$$
$$M' = V_P.$$
$$N' = V_R \sin\theta + M_Z \cos\theta.$$
$$AX = -V_F \sin\theta + (M_D \cos\phi - M_C \sin\phi - V_P)\frac{\cos\theta}{l}.$$
$$AY = \frac{V_R - M_C \cos\phi - M_D \sin\phi}{l}.$$
$$AZ = V_F \cos\theta + (M_D \cos\phi - M_C \sin\phi - V_P)\frac{\sin\theta}{l}.$$
$$L = L' + cAY.$$
$$M = M' - cX + aAZ.$$
$$N = N' - aAY.$$

Where a and c are the X and Z coordinates of the origin O taken from axes $O'X'$, $O'Y'$, and $O'Z'$.

Figures 24 to 31 show the forces X, Y, Z, and moments L, M, N plotted on angles of yaw as abscissæ for constant angles of incidence of 0, 3, 6, and 9 degrees. At any incidence the chord of the wing at its

[1] Technical Report of the Advisory Committee for Aeronautics, London. 1912-13. Report No. 75.

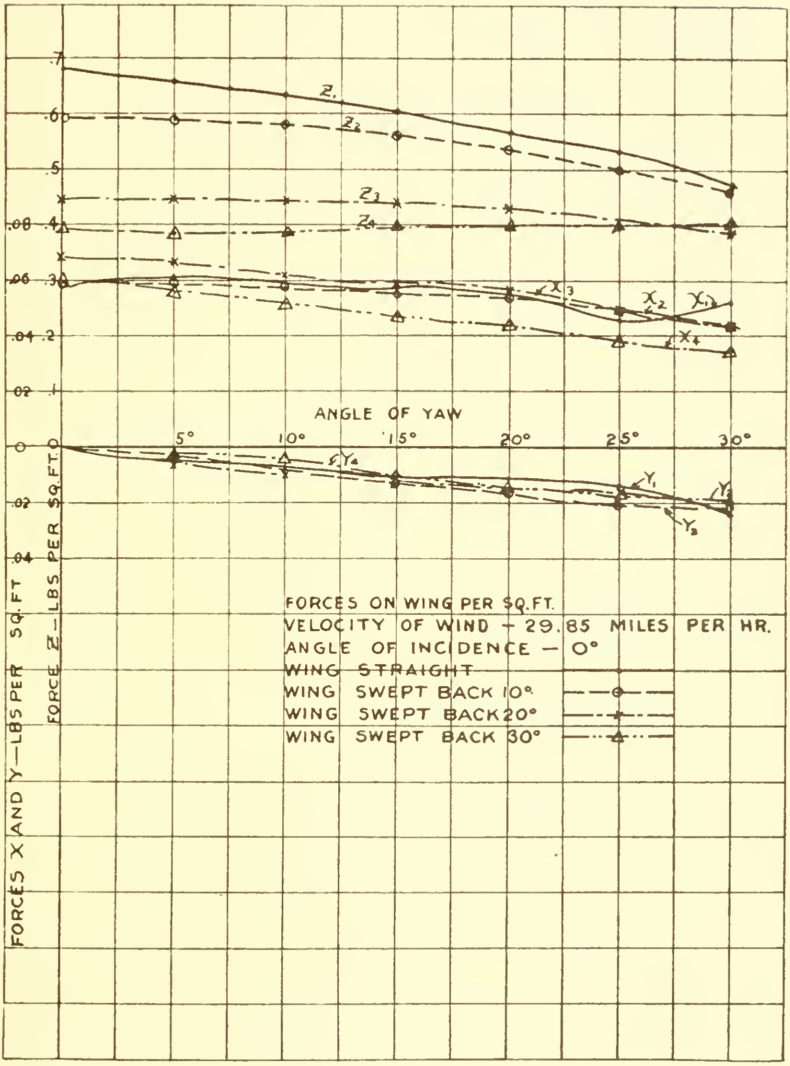

ANGLE OF YAW
FORCES X AND Y—LBS PER SQ. FT
FORCE Z—LBS PER SQ. FT.
FORCES ON WING PER SQ. FT.
VELOCITY OF WIND — 29.85 MILES PER HR.
ANGLE OF INCIDENCE — 0°
WING STRAIGHT
WING SWEPT BACK 10°
WING SWEPT BACK 20°
WING SWEPT BACK 30°

Fig. 24.

center coincides with the *OX* axis. In figure 32, *X, Z,* and *M* are plotted with angles of incidence as abscissæ and with 0 degrees yaw.

The notation used is as follows:

Subscript 1—wing straight.
Subscript 2—wing swept back 10 degrees.
Subscript 3—wing swept back 20 degrees.
Subscript 4—wing swept back 30 degrees.

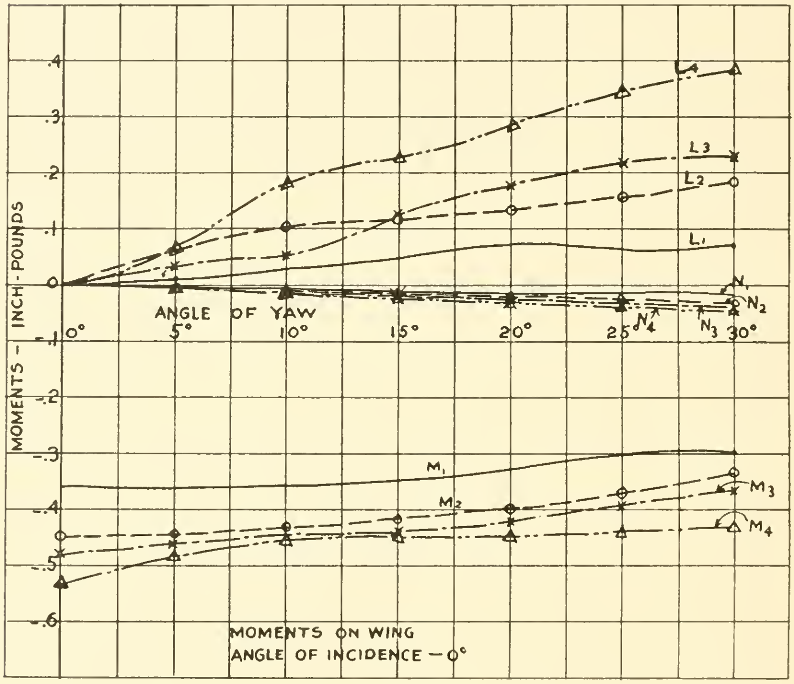


FIG. 25.

For the application of the theory of the dynamical stability of aeroplanes, reference should be made to the excellent papers by Mr. L. Bairstow.[1]

It is shown that for longitudinal stability the rate of change of the quantities *X, Z,* and *M* with angle of incidence determines the so-called resistance derivatives. It appears from the observations made on swept back wings that there is no appreciable change in the slope of curves of *X, Z,* and *M* plotted on angle of incidence for the

[1] Technical Report of the Advisory Committee for Aeronautics, London, 1912-13. Report No. 79.

different wings. Figure 32 shows some difference in form for curves M plotted on angle of incidence. It must be borne in mind, however, that the origin of these curves is at a greater distance from the center of pressure the farther the wing is swept back. A better idea of the variation in pitching moment with change in incidence can be obtained by plotting the curves with the estimated center of gravity

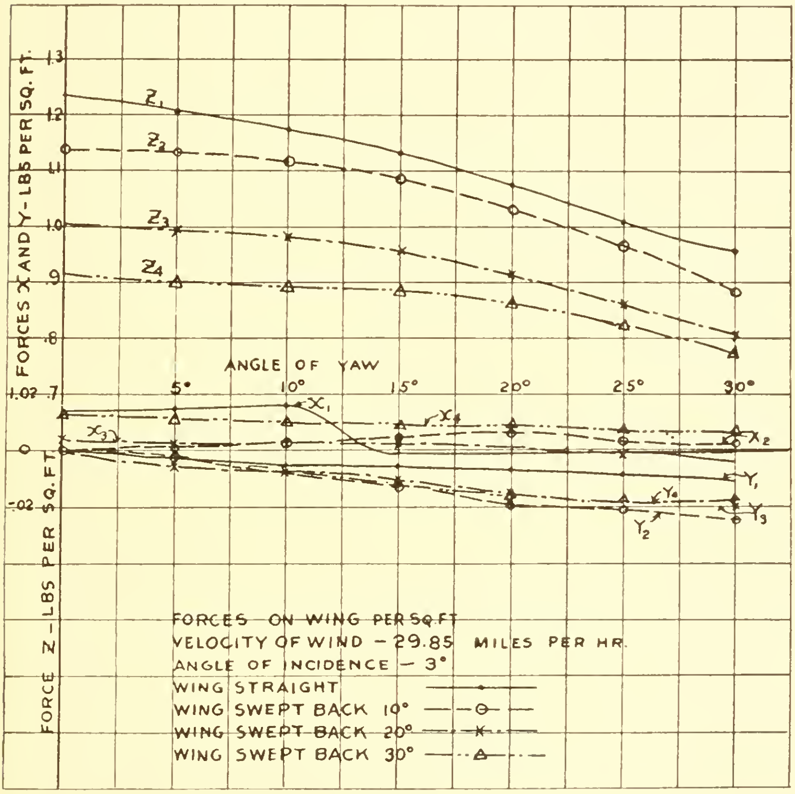


Fig. 26.

of the machine as the origin. Figure 20, Part I of this report, shows the movements of the centers of pressure to be similar for all wings. The conclusion is that sweeping back wings has no effect on the corresponding derivatives of X, Z, and M, and consequently no effect on longitudinal stability. It is of course to be expected that swept back wings may slightly increase the radius of gyration for pitching and so affect the motion indirectly. Aerodynamically, however, no advantage is to be expected.

For lateral stability the derivatives affected by the wings are the rate of change of Y, L, and N with angle of yaw. From considerations of symmetry, Y, L, and N are not affected by angles of roll.

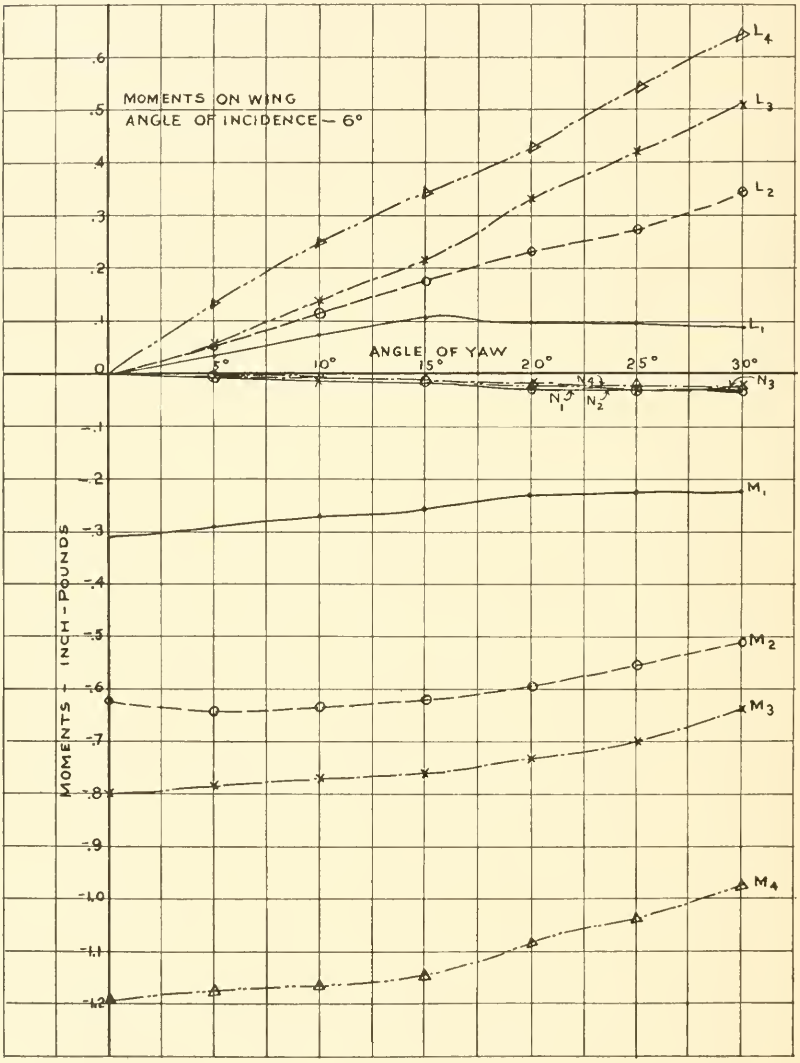


FIG. 27.

The curves of lateral force Y give evidence of difficulties in experimentation. The force Y on the model was in the neighborhood of 0.004 pound. It does not appear that there is any consistent change in Y due to sweep back which can be brought out. In any case the force Y is so small that small changes would be of no interest.

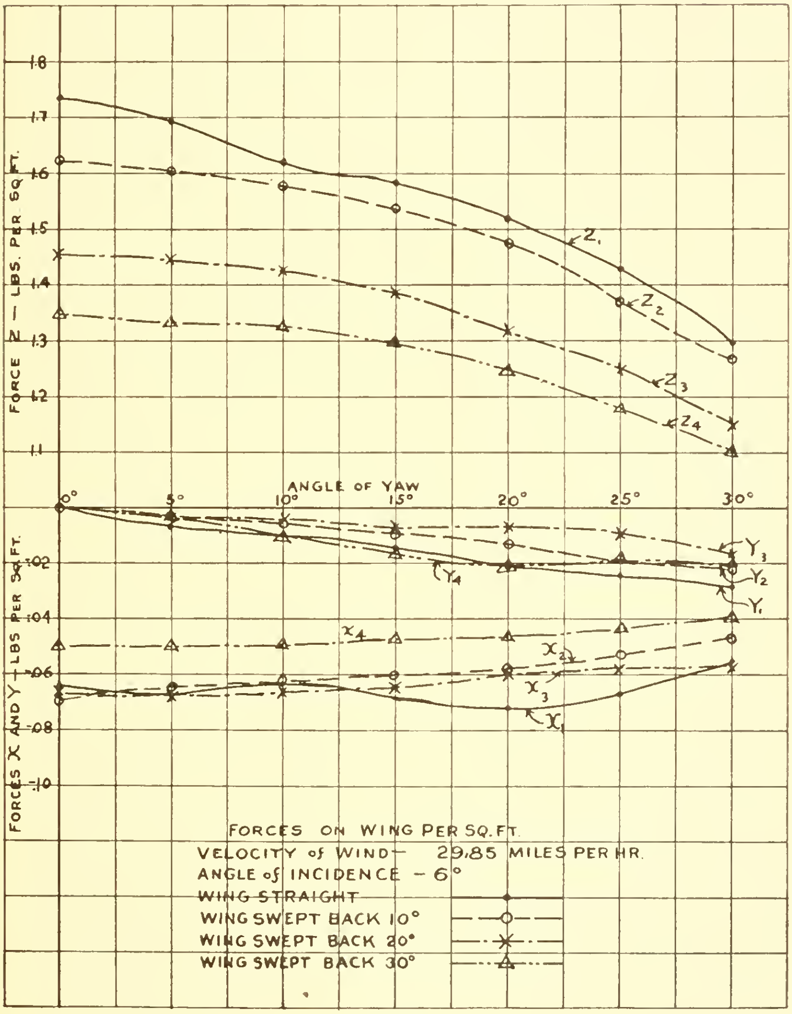


FIG. 28.

The curves of yawing moment N with angle of yaw show an extremely small moment, which was obviously impossible to measure with sufficient precision to detect changes between the various wings.

Indeed, as in the case of lateral force Y, the effect of changes in N produced by the wing is of no account.

On the other hand, L, the rolling moment produced as the wing is yawed out of its course, is large and of the greatest importance, as it

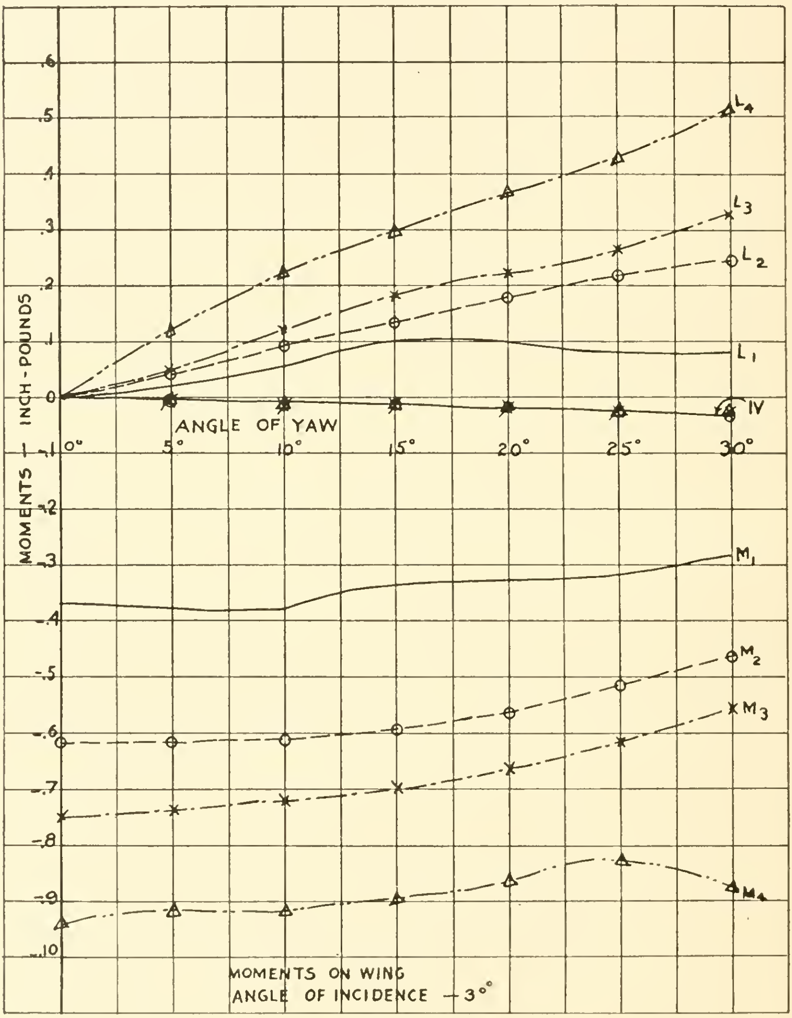


FIG. 29.

interconnects directional and lateral stability. For a normal wing flying at any reasonable incidence a deviation from the course causes a small rolling moment tending to bank the machine in a manner appropriate to making the turn. The couple may be called a " natural banking" moment. This "natural banking" moment is increased nearly 100 per cent if the wing has a sweep back of but 10 degrees. The moment for a sweep back of 20 degrees is still greater.

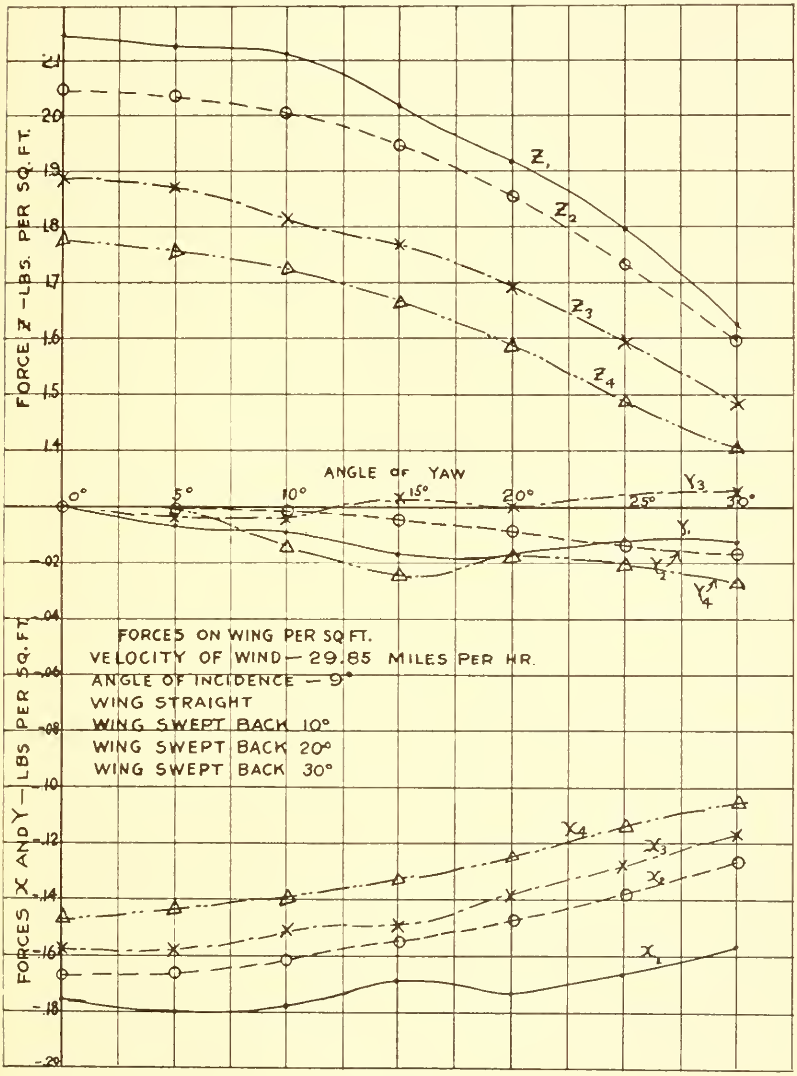


Fig. 30.

In view of the structural difficulties in making strongly swept back wings, and the loss of effectiveness as carrying surface for a sweep of more than 20 degrees, one would conclude that if any sweep back be used, about 10 or 15 degrees is sufficient.

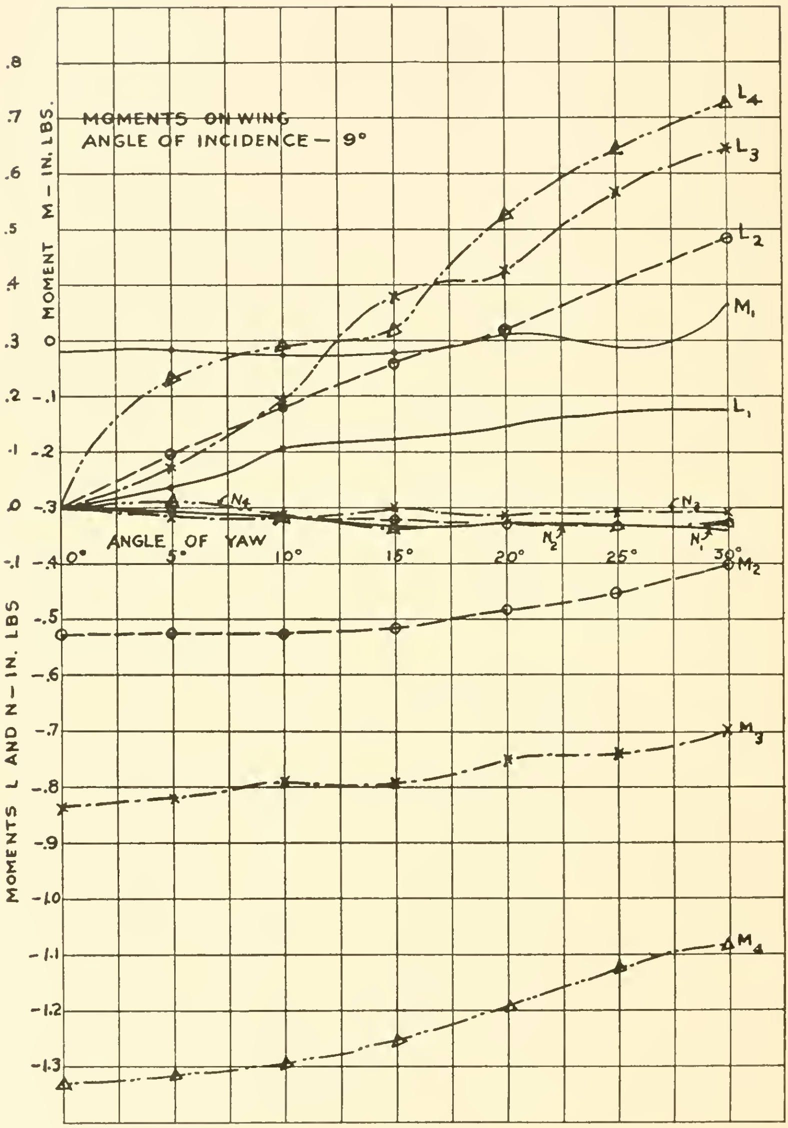

MOMENTS ON WING
ANGLE OF INCIDENCE — 9°
MOMENT M — IN. LBS.
MOMENTS L AND N — IN. LBS
ANGLE OF YAW
L4
L3
L2
M1
L1
N4
N3
N2
N1
M2
M3
M4
.8
.7
.6
.5
.4
.3
.2
.1
.0
-.1
0
-.1
-.2
-.3
-.4
-.5
-.6
-.7
-.8
-.9
-1.0
-1.1
-1.2
-1.3
0°
5°
10°
15°
20°
25°
30°

FIG. 31.

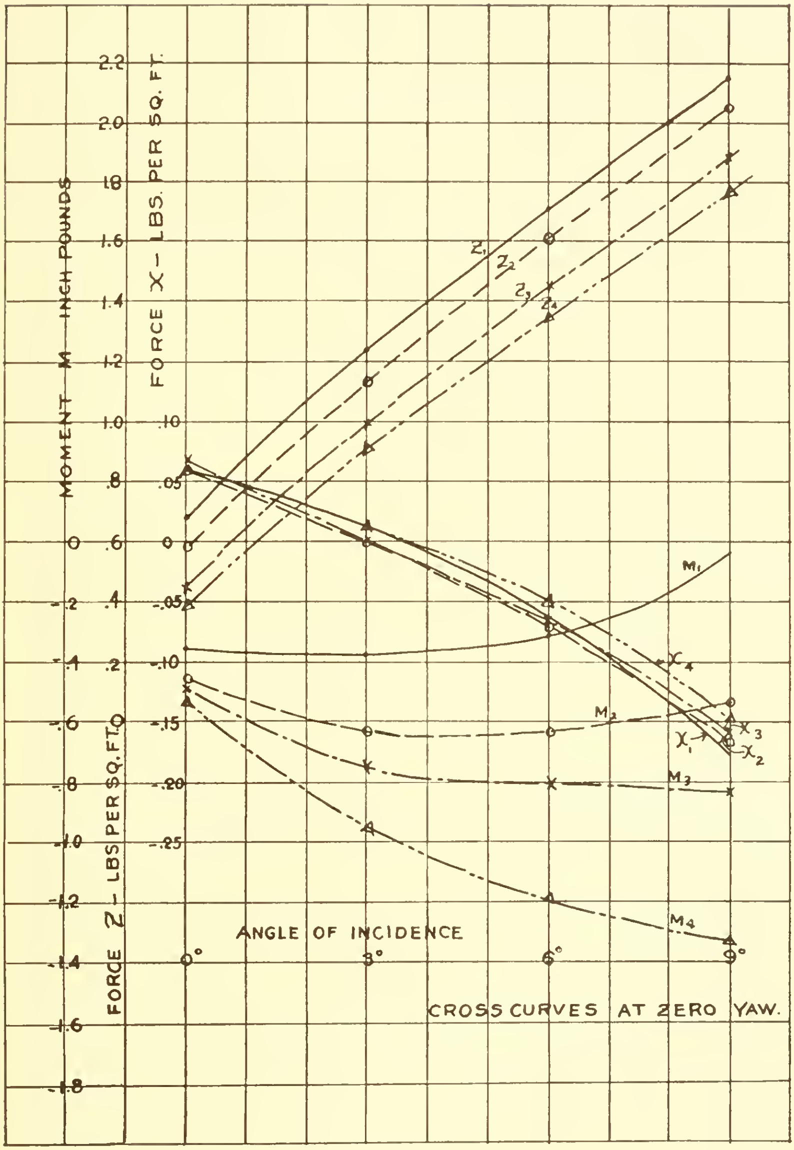

MOMENT M - INCH POUNDS
FORCE X - LBS. PER SQ. FT.
FORCE Z - LBS PER SQ. FT.
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
.10
.05
0
-.05
-.10
-.15
-.20
-.25
-.2
-.4
-.6
-.8
-1.0
-1.2
-1.4
-1.6
-1.8
Z1
Z2
Z3
Z4
M1
M2
M3
M4
X1
X2
X3
X4
ANGLE OF INCIDENCE
0°
3°
6°
9°
CROSS CURVES AT ZERO YAW.

Fig. 32.

The banking moment is appreciable on the aeroplane, as may be shown by the following calculation:

Incidence 6 degrees, yaw 15 degrees, sweep 10 degrees, moment L_1 on model .18 pound-inch at 29.85 M. P. H., moment on biplane of 400 square foot area L_2 at 60 M. P. H.

$$\frac{0.18}{12} \times 2 \left(\frac{200}{.364}\right)^{1.5} \times \left(\frac{60}{29.85}\right)^2 = 1{,}560 \text{ pound-feet.}$$

The greatest value of swept back wings is to be found on a side slip. If by any accidental cause an aeroplane with swept back wings is heeled over, it begins to side slip toward the low side. The apparent wind is no longer from dead ahead, but is a little to one side as compounded from the velocity of side slip and velocity of advance. The effect is the same as if the machine had yawed from its course, and with swept back wings the " natural banking " moment becomes a much greater righting moment than with straight wings. The machine then has a degree of lateral stability.

For lateral stability, swept back wings can be made to give a righting couple without introducing a lateral force or yawing couple. The absence of a yawing couple and lateral force is of advantage if the machine is to be kept on its course as it rolls.

Naturally, it is purely a matter of judgment whether too large a righting couple is disadvantageous. In a gusty wind, a machine with swept back wings will tend to roll as side gusts strike it. This may be uncomfortable for the pilot, but if his aileron control be powerful, he can always overcome the rolling moment due to the wings. Approaching a landing, this is especially necessary. In the air, it would be both unnecessary and fatiguing for him to fight the natural rolling of his machine.

A " natural banking " moment can be obtained by the use of a vertical fin above the center of gravity, or by giving the wings an upward dihedral angle. The equivalence of these methods has not yet been determined.

These tests bring out simply the fact that with a sweep back of 10 degrees an appreciable righting moment may be expected without change in any of the other aerodynamical properties of the straight wing.

No reference is made here to the " rotary derivatives " or changes in forces and moments produced by angular velocity. The damping effect of wings on such oscillations can hardly be appreciably affected by sweep back, and accordingly this question has not been investigated.

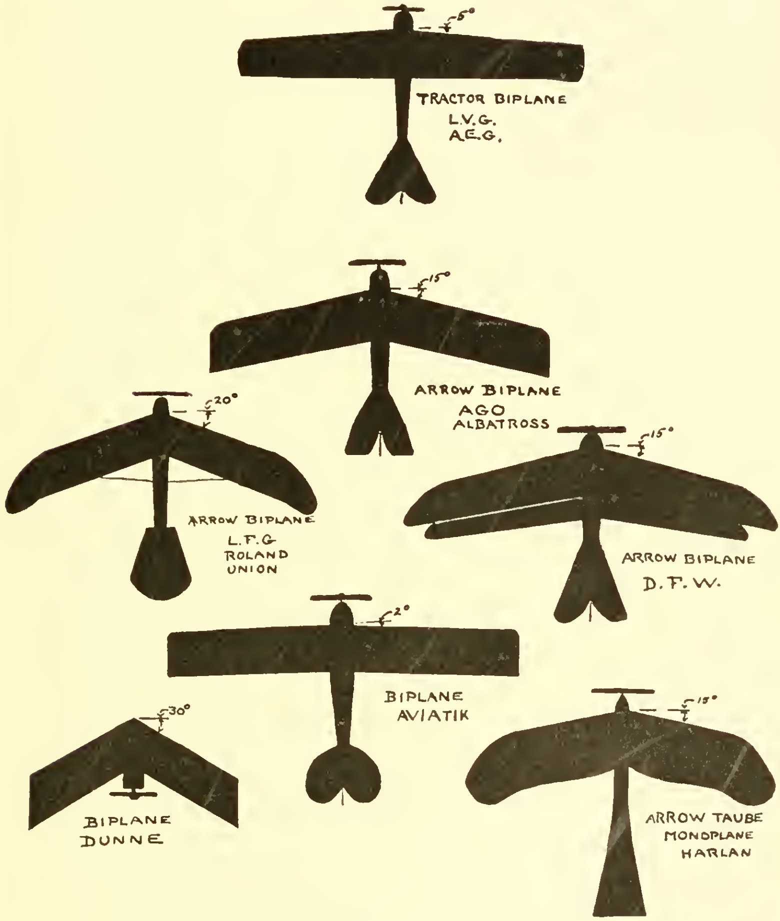


Fig. 33.—Recent aeroplanes.

Figure 33 shows some types of aeroplanes used in Germany at the present time, and also the English Dunne machine. Both the German machines and the Dunne are reported to be laterally stable in normal flight.

IX. EXPERIMENTS ON A DIHEDRAL ANGLE WING

By J. C. HUNSAKER and D. W. DOUGLAS

Following up experiments by Rossell and Brand showing the effect on the lateral stability of sweeping back the wings of an aeroplane, additional tests have now been made to determine whether the righting moment given by the above procedure cannot be better obtained by another method. To this end, a brass model 18 inches by 3 inches having curvature, known as R. A. F. 6,[1] was made identical in every way with the straight wing tested by Rossell and Brand, except that each half of the wing was inclined upwards and outwards $2\frac{1}{2}$ degrees from the horizontal. This gave a dihedral angle upward of 175 degrees. The use of this amount of angle has come into general practice in aeroplane design.

The wing was mounted horizontally on the balance and forces and moments were measured for angles of yaw of 0, 5, 10, 15, 20, 25, and 30 degrees to right and left and at angles of incidence of 2, 4, 6, and 9 degrees. The velocity of the wind was kept constant at 30 M. P. H. standard air. The method of manipulating the balance to make the necessary measurements is described fully in Report 68, Technical Report of the Advisory Committee for Aeronautics, 1912-13 (London). The calculations, from the measured forces and moments, to obtain the final results, were made as is outlined in the preceding report on Swept Back Wings, by Rossell and Brand (page 61).

The axes along and about which the calculated forces and moments act are as follows:

OX—longitudinal axis coincident with chord at middle section.

OY—transverse axis of wing, perpendicular to OX.

OZ—normal axis of wing, perpendicular to OX and OY.

The origin O is the intersection of the axis OX with the leading edge of the wing.

The forces are:

X—acting along OX, positive when in direction of wind.

Y—acting along OY, positive when tending to increase side slipping to left.

Z—acting along OZ, positive when acting upwards.

The moments are:

L—acting about OX, rolling moment, positive when wing tends to take proper bank for turn to right.

[1] Technical Report of the Advisory Committee for Aeronautics, London, 1912-13.

M—acting about *OY*, pitching moment, positive when wing tends to stall.

N—acting about *OZ*, yawing moment, positive when wing tends to turn to right away from wind.

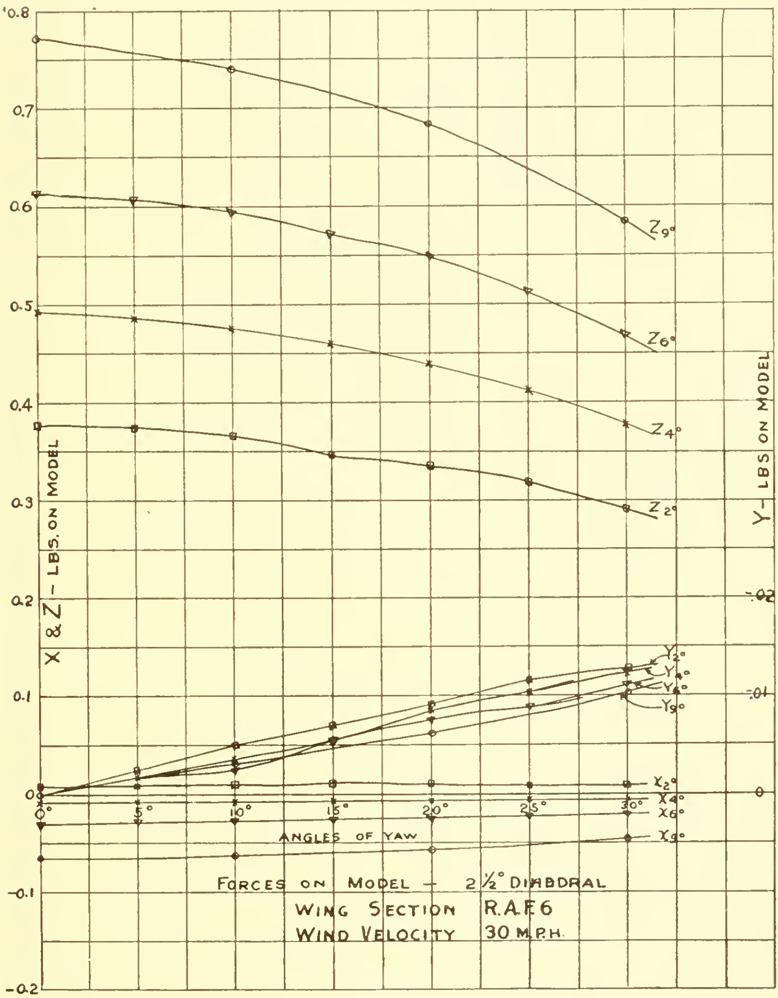


Fig. 34.
(Subscripts designate angle of incidence.)

In figure 34 we can see the changes of the forces *X*, *Y*, and *Z* with yaw and incidence. It will be noted that *Y* is plotted to a scale of ordinates ten times greater than *X* and *Z*. This force, although negative and hence acting to resist a side slip, is so small in magnitude that it is negligible. *Z* falls in magnitude with increase of yaw angle as is to be expected, and as was the case with the swept back wings.

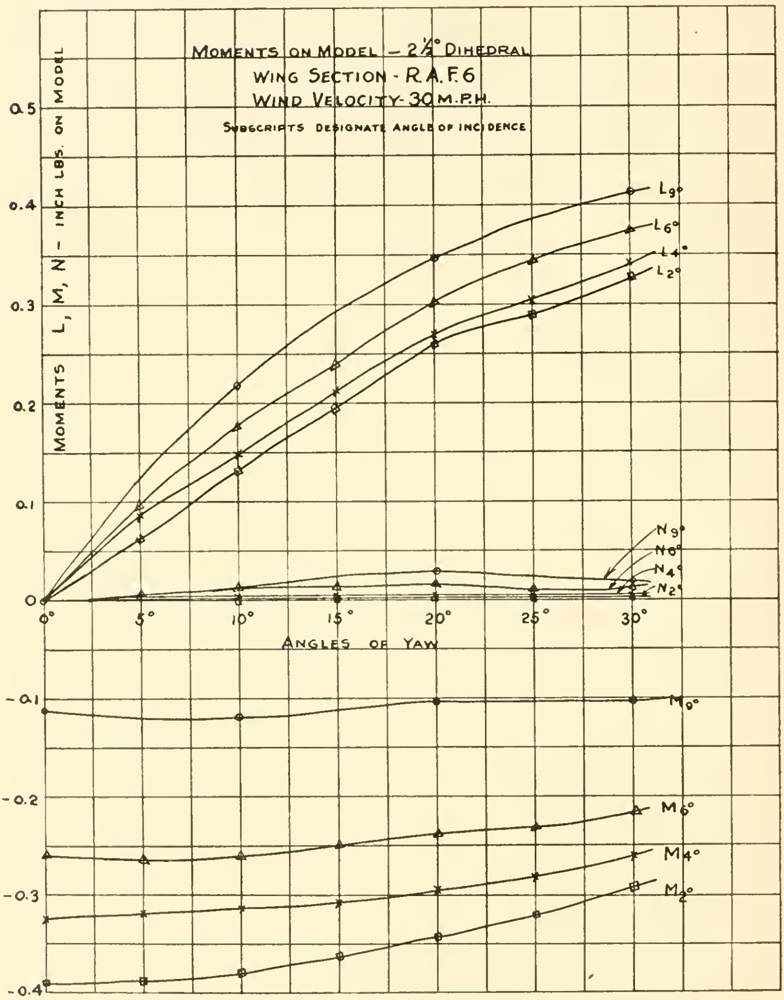


FIG. 35.

Figure 35 shows the moments on the wing. *N,* the yawing moment, is very small and does not change appreciably with the incidence of the wing.

M, the pitching moment, shows at the smaller angles of incidence a tendency for the machine to stall in side slipping. At the larger angle of incidence of 9 degrees, however, this stalling tendency disappears. This is a good feature, as it shows that the wing should not get into a bad stalling position when side slipping. The dihedral angle seems in this respect superior to the swept back wing, which has a stalling tendency at all incidences.

The curves for rolling moments, L, show for all incidences a rolling moment which increases rapidly with angle of yaw and with the incidence. This moment is a natural banking moment, and hence is one which is favorable for lateral stability. Comparing the rolling moments curve at 6 degrees incidence, for a normal wing, with the dihedral curve, it is seen that the magnitude is from two to three times greater in the case of the dihedral. Comparing this same dihedral rolling moment curve with those for the swept back wings (fig. 29), it is seen that the dihedral gives a rolling moment of magnitude about equal to that obtained with 20 degrees swept back wings, except at a large angle of yaw.

As it is much more difficult structurally to build a 20-degree swept back wing than a dihedral, and as the latter is as effective, it seems that the dihedral is of more value for purposes of lateral stability.

It is of interest to note in this connection that Professor Langley's "aerodrome" of 1902, as well as his previous power-driven models, were given dihedral angle wings inclined upwards by about 6 degrees.

X. CRITICAL SPEEDS FOR FLAT DISKS IN A NORMAL WIND

By J. C. HUNSAKER

1. Prefatory Note on Normal Flow Past a Circular Disk

By E. B. WILSON, Professor of Mathematics, Massachusetts Institute of Technology

Theoretical hydrodynamics cannot yet give a very satisfactory quantitative account of Mr. Hunsaker's experiments to which this note is attached. The classical theory treats two cases which are qualitatively somewhat like nature: First, symmetric continuous flow; second, asymmetric discontinuous flow. In both cases the flow is irrotational and the fluid incompressible. In the first case, owing to the symmetry of the lines of force, there is theoretically no resultant pressure to urge the disk down stream. Such a condition may possibly approximate to nature in cases of slow flow under high pressures. In the second case, as the method of solution depends on the theory of a complex variable, the problems treated are those of two dimensional flows, and here the whole fluid is divided by a surface of discontinuity (for velocity) into a moving and a stationary portion with the stream lines in the moving portion diverging instead of closing in behind the obstructing object. This state of affairs may be found in nature to a certain approximation in the case of jets. It is clear that the theory of the continuous solution is entirely inapplicable in the discussion of the pressure exerted on the disk by the fluid, and that for that problem the discontinuous solution would be necessary. And it is equally clear that for a discussion of the possibility of a critical velocity the discontinuous solution is disadvantageous, and the continuous solution must in the present state of our knowledge be used for what information it may afford.

We shall henceforth assume that the fluid is incompressible, that the motion is irrotational, symmetric fore and aft of the disk, and identical in all planes through the axis of the disk; that the eddies which are known to form are to be disregarded; and that the viscosity may be neglected. We know that the ordinary continuous symmetric solution must break down at least as soon as cavitation appears, and we may with reasonable safety assume that the velocity $u=U$ of the stream sufficient to induce incipient cavitation at the edges of the disk will be an upper limit for the critical velocity found by Mr. Hunsaker. (There is unfortunately no assurance that the upper limit may not seriously exceed that critical velocity.) Now if the disk has a perfectly sharp edge, cavitation will take place for any velocity of the general stream, no matter how small that velocity may be. To get any result of value we must therefore replace the disk by a body of rounded contour, such as an ellipsoid of revolution. The theoretical problem which we shall solve is therefore this: To find the velocity U at which cavitation begins in the case of an extremely flat ellipsoid of revolution whose axis coincides with the general course of the stream; and for this we shall determine a very simple approximate expression in terms of the axes of the ellipsoid. For an ellipsoid 6 inches by 1/16 of an inch we shall find $U=22$ foot-seconds, or thereabouts, and this result will be discussed in the light of the experiments.

As the motion is irrotational, by assumption, there is a velocity potential ϕ, and as the density is assumed constant, the velocity potential satisfies Laplace's equation,

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0.$$

We have to find a solution of this equation which is the same at all points of circles about the axis of rotation (owing to the assumed rotational symmetry), which reduces to ux at all points at great distances from the ellipsoid (the axis of x being taken along the axis of revolution and u being the general velocity of the stream), and which satisfies the condition that the normal derivative $\frac{d\phi}{dn}$ vanishes at each point of the ellipsoid (as the flow must be tangential to the ellipsoid).

We first introduce cylindrical coordinates with the axis of revolution as axis. Then

$$y=r\cos\theta, \quad z=r\sin\theta,$$

and

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}=0.$$

As we are interested only in solutions which do not depend on θ, Laplace's equation reduces to

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}=0. \tag{1}$$

We next replace x and r, the rectangular coordinates in a plane through the axis, by a system of coordinates (ζ, μ) derived from the system of ellipses and hyperbolas in the xr-plane confocal with the ellipse in which that plane cuts the ellipsoid.[1] If c be the distance from focus to center, we write

$$x=c\mu\zeta, \quad r=c\sqrt{1-\mu^2}\sqrt{\zeta^2+1}. \tag{2}$$

The elimination of μ and ζ respectively gives

$$\frac{x^2}{c^2\zeta^2}+\frac{r^2}{c^2(\zeta^2+1)}=1 \text{ and } \frac{r^2}{c^2(1-\mu^2)}-\frac{x^2}{c^2\mu^2}=1. \tag{3}$$

A small value of ζ gives a narrow ellipse; a large value, a large ellipse. The set of ellipses may therefore be represented by values of ζ from 0 to ∞. A small value of μ gives a sharp hyperbola nearly coincident with $x=0$, a value of μ equal to 1 gives the line $r=0$, the axis of revolution. By assigning different signs to μ and to one of the radicals in (2) we may represent all points (x, r) of the plane; but the symmetry of the figure is such that we may work only in the

[1] This system of coordinates is that used by Lamb, Hydrodynamics, 2d ed. (1895), p. 150, for treating the motion of an ellipsoid of revolution in a fluid at rest at infinity. We could use Lamb's analysis and make a correction to bring the ellipsoid to rest in a moving fluid. It seems as easy to solve our problem independently with all the simplifications it admits.

quadrant in which x and r are positive, and hence deal only with positive values of μ and the radicals.

In terms of the coordinates (ζ, μ) equation (1) becomes

$$\frac{\partial}{\partial\zeta}\left[(\zeta^2+1)\frac{\partial\phi}{\partial\zeta}\right]+\frac{\partial}{\partial\mu}\left[(1-\mu^2)\frac{\partial\phi}{\partial\mu}\right]=0, \tag{4}$$

as a straightforward change of variable will show.[1] We follow the usual method of integration and try for particular solutions of the form[2]

$$\phi=Z(\zeta)M(\mu),$$

a product of a function of ζ by a function of μ. Then (4) becomes

$$\frac{1}{Z}\frac{d}{d\zeta}\left[(\zeta^2+1)\frac{dZ}{d\zeta}\right]+\frac{1}{M}\frac{d}{d\mu}\left[(1-\mu^2)\frac{dM}{d\mu}\right]=0.$$

Here the variables are separated and the equation could not hold identically unless the two parts were equal and opposite constants. If we set

$$\frac{d}{d\mu}\left[(1-\mu^2)\frac{dM}{d\mu}\right]+n(n+1)M=0, \tag{5}$$

$$\frac{d}{d\zeta}\left[(\zeta^2+1)\frac{dZ}{d\zeta}\right]-n(n+1)Z=0, \tag{6}$$

we see that the first is Legendre's equation and the second a slight modification of it. For $n=0, 1, 2, \ldots$, the polynomial solutions of (5) are respectively constant multiples of $1, \mu, 3\mu^2-1, \ldots$; and of (6), $1, \zeta, 3\zeta^2+1, \ldots$.

A consideration of (2), or of the figure made up of the confocal ellipses and hyperbolas, shows that for large values of ζ, *i. e.*, in the distant portions of the plane, $c\zeta=\rho$ and $\theta=\cos^{-1}\mu$ are approximately polar coordinates with the x-axis as polar axis. Then in these regions we have approximately

$$\frac{\partial\phi}{\rho\partial\theta}=-\sin\theta\frac{\partial\phi}{\rho\partial\mu}=-\text{(tangential velocity along } \rho=\text{const.)}.$$

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[1] It is easier to transform (4) into (1), and still easier to express directly in terms of ζ and μ the condition of continuity; for if ds_e and ds_h are elements of arc along the ellipses and hyperbolas, the velocities are $-\frac{\partial\phi}{ds_e}$ and $-\frac{\partial\phi}{ds_h}$, and the flux is

$$\frac{\partial}{\partial s_h}\left(r\frac{\partial\phi}{\partial s_e}\right)+\frac{\partial}{\partial s_e}\left(r\frac{\partial\phi}{\partial s_h}\right)=0,$$

which is readily expressed in terms of ζ, μ.

[2] See, for example, Wilson's Advanced Calculus, Chapter XX.

By the hypothesis this velocity is $-u \sin \theta$, where u is the velocity of the stream. Hence, when ζ is large, we must have for all values of μ approximately

$$\frac{\partial \phi}{c\zeta \partial \mu} = \frac{Z}{c\zeta} \frac{dM}{d\mu} = -u, \text{ independent of } \mu. \tag{7}$$

It follows that M cannot be of higher order than 1, and the only possibilities for n are 0 and 1. The solution for ϕ must therefore be of the form

$$\phi = Z_0 + \mu Z_1, \tag{8}$$

where Z_0 and Z_1 are solutions of (6) for $n=0$ and 1 respectively. Moreover, for the radial velocity in distant regions we have

$$-\frac{\partial \phi}{c \partial \zeta} = -\frac{dZ_0}{cd\zeta} - \mu \frac{dZ_1}{cd\zeta} = u \cos \theta = u\mu. \tag{9}$$

We have next to express the condition that flow along the ellipsoid shall be tangential, *i. e.*, that the normal derivative $\frac{d\phi}{dn}$ shall vanish. The normals to the elliptical section of the ellipsoid are the hyperbolas along which μ is constant. The normal dn is

$$dn = \sqrt{dx^2 + dr^2} = c\left(\mu^2 + (1-\mu^2)\frac{\zeta^2}{\zeta^2+1}\right)^{\frac{1}{2}} d\zeta = c\sqrt{\frac{\mu^2+\zeta^2}{\zeta^2+1}}\, d\zeta.$$

The particular ellipse which is the profile of the disk is determined by some value ζ_0 of ζ. Then

$$\frac{d\phi}{dn} = c\sqrt{\frac{\zeta_0^2+1}{\mu^2+\zeta_0^2}}\left(\frac{dZ_0}{d\zeta} + \mu \frac{dZ_1}{d\zeta}\right)_{\zeta=\zeta_0} = 0.$$

As this equation holds for every μ, we have

$$\left(\frac{dZ_0}{d\zeta}\right)_{\zeta=\zeta_0} = 0, \left(\frac{dZ_1}{d\zeta}\right)_{\zeta=\zeta_0} = 0. \tag{10}$$

The equations (7), (9), (10) should suffice to determine what values of Z_0 and Z_1 are needed in (8) to represent the flow.

The general solution of (6) for $n=0$ may be obtained at once by integration,

$$Z_0 = C_1 + C_2 \tan^{-1} \zeta. \tag{11}$$

The general solution for $n=1$ may be obtained from the abovementioned particular solution ζ by the substitution $Z = Z'\zeta$, which gives

$$Z_1 = K_1 (\zeta \tan^{-1} \zeta + 1) + K_2 \zeta. \tag{12}$$

From (10), (11), (12) we find:

$$\frac{C_2}{\zeta_0^2+1} = 0, \quad K_1\left(\tan^{-1}\zeta_0 + \frac{\zeta_0}{\zeta_0^2+1}\right) + K_2 = 0.$$

Hence $C_2=0$, and the ratio $K_1:K_2$ is determined. From (7) and (9) we have the approximate equations

$$\frac{K_1(\zeta\tan^{-1}\zeta+1)+K_2\zeta}{c\zeta}=-u,\quad K_1\left(\tan^{-1}\zeta+\frac{\zeta}{\zeta^2+1}\right)+K_2=-cu.$$

Hence $K_1\frac{\pi}{2}+K_2=-cu$ and

$$K_1=\frac{-cu}{\cot^{-1}\zeta_0-\frac{\zeta_0}{\zeta_0^2+1}},\quad K_2=\frac{cu}{\cot^{-1}\zeta_0-\frac{\zeta_0}{\zeta_0^2+1}}\left(\tan^{-1}\zeta_0+\frac{\zeta_0}{\zeta_0^2+1}\right).$$

As the solution for Z_0 reduces to a constant by $C_2=0$, the value of ϕ may be taken as merely μZ_1 in (8) ; or

$$\phi=\mu\left[\frac{cu}{\cot^{-1}\zeta_0-\frac{\zeta_0}{\zeta_0^2+1}}\right]\left[-\zeta\tan^{-1}\zeta-1+\zeta\left(\tan^{-1}\zeta_0+\frac{\zeta_0}{\zeta_0^2+1}\right)\right].$$

We are dealing only with very small ellipsoids and hence ζ_0 is small. Hence ϕ reduces approximately to

$$\phi=\frac{c\mu u}{\frac{\pi}{2}}\left[-\zeta\tan^{-1}\zeta-1+2\zeta_0\zeta\right].$$

The velocity along the ellipse $\zeta=\zeta_0$ may be obtained like the normal velocity. The element of arc ds is

$$ds=\sqrt{dx^2+dr^2}=c\left[\zeta_0^2+\frac{\mu^2(\zeta_0^2+1)}{1-\mu^2}\right]^{\frac{1}{2}}d\mu=c\sqrt{\frac{\zeta_0^2+\mu^2}{1-\mu^2}}\,d\mu,$$

$$-\frac{d\phi}{ds}=\frac{2u}{\pi}\sqrt{\frac{1-\mu^2}{\zeta_0^2+\mu^2}}[1].$$

The value of this velocity is greatest when $\mu=0$, *i. e.*, at the ends of the ellipse, as was to be expected; and its value is then $\frac{2u}{\pi\zeta_0}$. The value of ζ_0 may be expressed in terms of the axes of the ellipse. From (3),

$$a=c\sqrt{\zeta_0^2+1},\quad b=c\zeta_0.$$

Indeed the value is $\zeta_0=\frac{b}{c}=\frac{b}{a}$ approximately. Hence the velocity is approximately

$$\text{Maximum velocity}=\frac{2u}{\pi}\frac{a}{b}.$$

By Bernoulli's principle for a stream line or Kelvin's theorem on irrotational motion we have

$$\frac{p}{\rho}+\frac{1}{2}v^2=\text{const.}$$

When cavitation begins, $p=0$, $v=2a\frac{U}{\pi b}$. The value of $\frac{p}{\rho}+\frac{1}{2}v^2$ at large distances from the disk may be taken as $\frac{p_0}{\rho}$, where p_0 is the atmospheric pressure in pounds per square foot. Hence we find

$$\frac{p_0}{\rho}=\frac{1}{2}\frac{4a^2U^2}{\pi^2b^2}, \quad U=\frac{\pi b}{a}\sqrt{\frac{p_0}{2\rho}} \tag{13}$$

As numerical data we may take $p_0=(22\times 12)^2$, $\rho=.08$. Then

$$U=2100\frac{b}{a} \text{ foot-seconds.} \tag{14}$$

In the case of an ellipsoid 6 inches by 1/16 inch, the velocity U is about 22 foot-seconds. For smaller disks of the same minor axis the velocity U should be larger in inverse ratio to the size of the major axis.

The upper limit of 22 foot-seconds here found for the critical velocity is not surprisingly above the actual range of 10 to 20 foot-seconds found by Mr. Hunsaker. But a noteworthy result shown on his diagrams is that the critical velocity does not vary proportionately to the reciprocal of the diameter of the disk—it is practically constant—and had we taken to test the theory his smallest disk, we should have had an upper limit decidedly above the experimental value. This lack of accord between his experiments and the present theory can hardly be regarded as surprising. His disks were all equally sharp upon the edge, and all considerably sharper than an ellipsoid of the same length and breadth, particularly in the smaller disks. Once the edge is sharp enough to start cavitation at a given velocity of the general stream, the motion becomes such that we can no longer expect our theory to hold, and it is entirely possible that the thickness of the disk is unimportant above a certain value. The effect of a 6-inch disk 1/16 of an inch thick may not differ appreciably from that of one 1/32 of an inch thick. The effects of the changing density of the air and of viscosity might also, and probably would, be of some importance. On the whole we may consider the correlation of experiment and theory as fairly satisfactory; at least it is good enough to indicate strongly the existence of a critical velocity for these disks at reasonably small velocities of the stream.

2. Critical Speeds for Flat Disks in a Normal Wind

The value of tests on the resistance of small models in a wind tunnel depends upon the precision with which such results may be applied to larger models at different speeds. The resistance of thin disks normal to the wind is not of great practical interest in aeronautics, but certain theoretical conclusions of general importance may be drawn from experiments with disks.

The resistance of the air depends upon the density, viscosity, and compressibility of the fluid. Except in gunnery, the velocities in common use do not approach the velocity of sound in air, and hence compressibility is neglected.

Two well-defined types of flow are recognized. The first is the so-called stream line flow in which the fluid flows around the body in steady lines, closing in behind it so that no turbulent wake is formed. Such flow may be expected about a fish-shaped body at low speeds. The second form is called the discontinuous or turbulent flow, in which eddies are formed on the body and are carried down stream as a turbulent wake. Such flow may be expected about bodies of abrupt form moving at high speeds.

If no eddies are formed, the resistance to motion is a viscous drag depending on the viscosity of the fluid, a linear dimension, and the velocity.

With the formation of eddies, additional resistance is caused by the energy lost in imparting kinetic energy to the fluid in the eddies. It is observed that, in general, eddies are left behind and so represent a loss of energy. The eddy making resistance should depend on the kinetic energy imparted to the fluid and hence should vary as the density, the square of the velocity, and the extent of the disturbance. The latter should be proportional to the square of a linear dimension of the body, or the cross-section of the wake.

In any real case both viscous and eddy resistance are present. For bodies of easy shape, the turbulence is not great and we should expect viscosity to play an important part, and the resistance to vary less rapidly than V^2.

However, for a thin disk with its face normal to the wind we should expect eddy making to be violent and the resistance to vary with the density of the air, area of face of disk, and square of velocity. Under such conditions the viscous drag might be negligible.

At very low speeds, if it be possible for the fluid to turn the corner and close in behind the disk, eddy making may be so much reduced that the viscous drag is important. An entirely different form of flow may then exist, and the resistance should no longer vary as the square of the velocity. The change from turbulence to steady flow should be marked by a critical velocity. The resistance of disks, aero-

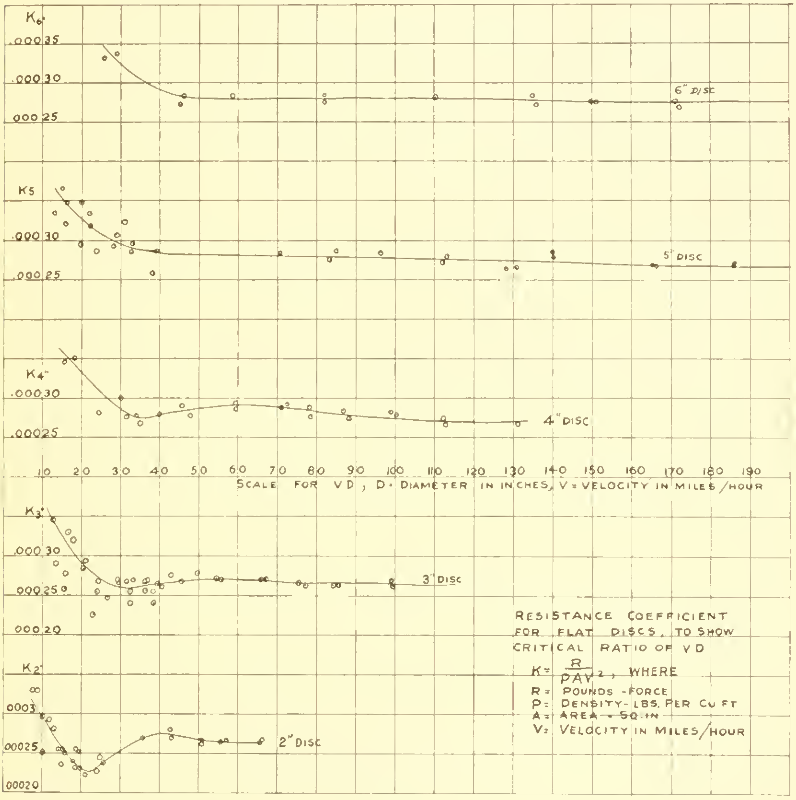


FIG. 36.

plane wings, and all sharp-edged objects is usually represented by a formula as follows:

$$R = K\rho AV^2,$$

where

- R is resistance in pounds,
- ρ is density in pounds per cubic foot,
- A is area in square inches,
- V is velocity in miles per hour,
- K a coefficient assumed constant.

To verify this law of resistance, it was decided to test in the wind tunnel a series of thin disks placed normal to the wind. Disks of sheet brass 1/16 inch thick with square edges and diameter 2, 3, 4, 5, and 6 inches were used. The resistance of each disk was measured for a series of speeds. The measurements were repeated with the face of each disk reversed and the results averaged.

If resistance be correctly represented by the formula above, the constant K computed from the observed resistances should remain constant. In figure 36 the values of K computed from each observation are plotted on the product VD as abscissæ, when D is the diameter in inches. It appears that K does not remain constant even for a given disk. The points represent actual observations, and it is seen

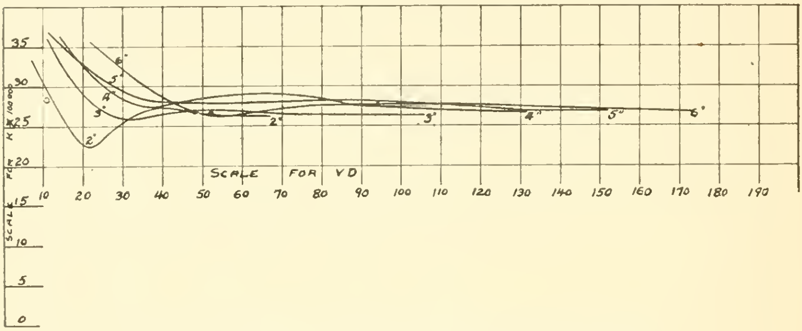


FIG. 37.—Resistance coefficient as a function of VD.

$$K = \frac{R}{\rho A V^2}$$

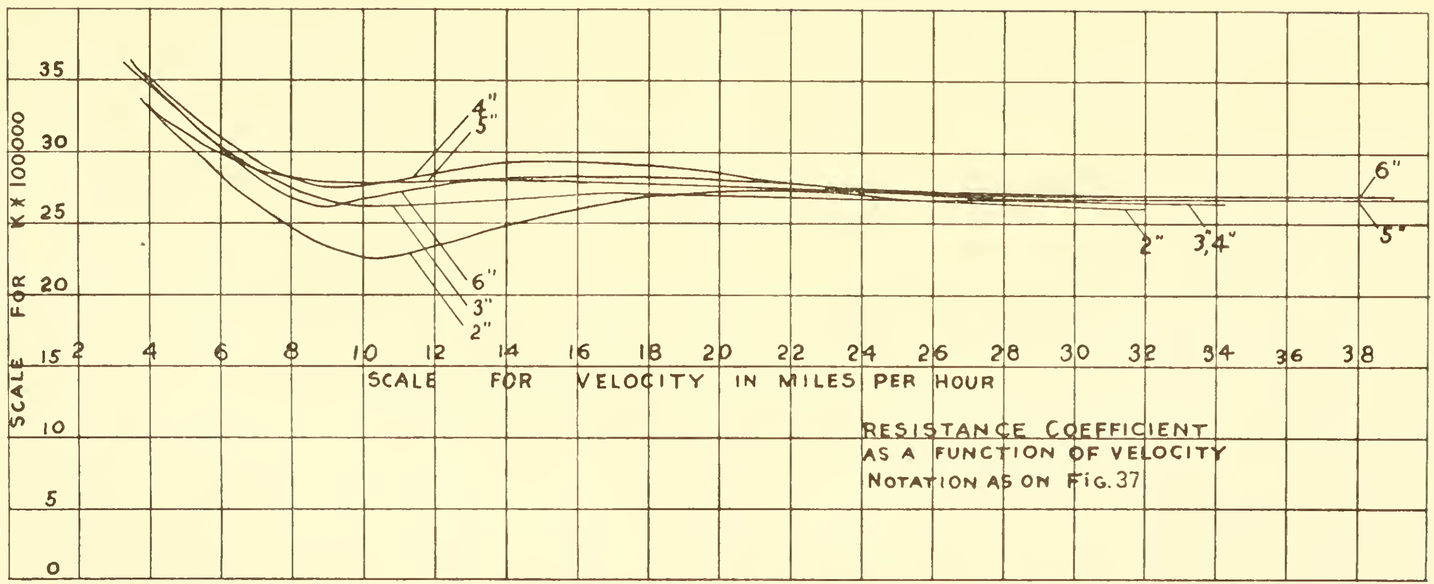
where

$R =$ force in pounds.
$P =$ density in pounds per cubic foot.
$A =$ area in square inches.
$V =$ velocity in miles per hour.
$D =$ diameter in inches.

that below a value of VD of 40, K becomes very erratic. A great many check observations were taken in this region, but the flow seems to be unstable and K cannot be determined with precision. The mean curves are replotted on figure 37 on VD as abscissæ, and again on figure 38 on V as abscissæ. The critical velocity apparently discovered seems to be about 9 miles per hour for all the plates.

Theoretically, if the resistance due to viscosity be important the coefficient K should be a function of the product VD. Thus, Lord Rayleigh has suggested[1] that where both density and viscosity are

[1] Phil. Mag., p. 66, July, 1904.



SCALE FOR K × 100000
35
30
25
20
15
10
5
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
SCALE FOR VELOCITY IN MILES PER HOUR
4"
5"
6"
3"
2"
2"
3",4"
6"
5"
RESISTANCE COEFFICIENT
AS A FUNCTION OF VELOCITY
NOTATION AS ON FIG. 37

FIG. 38.

important, the most general form of the resistance equation which satisfies dimensional requirements is the following:

$$R=\rho AV^2 f\left(\frac{VL}{\nu}\right),$$

when L is a linear dimension, such as diameter of disk; ν the kinematic viscosity of the air, taken constant; and f an unknown function of the single variable $\left(\frac{VL}{\nu}\right)$.

This expression may be written

$$R=\rho AV^2\phi(VD), \text{ when } \nu \text{ is sensibly constant.}$$

The coefficient K should then be a function of VD and the curves of figure 37 should coincide. This is not the case, and it may be concluded that the effect of viscous drag is not important, or that the disks are not geometrically similar since the thickness remains constant. The critical velocity appears by figure 38 to be about the same for each disk.

For velocities above 27 miles per hour, the value of K remains practically constant for all speeds and all disks. The usual formula can, therefore, be applied if model tests are run at speeds greater than 27 miles per hour. Our wind tunnel testing will in future be conducted at speeds between 27 and 40 miles per hour.

The values of K for larger disks appear to be larger than those for the smaller disks. This discrepancy may be eliminated when it is considered that the 6-inch disk obstructs 1.25 per cent of the tunnel and should cause us to underestimate the mean velocity past the disk by about 1.25 per cent, and consequently K as computed will be 2.5 per cent too large. The actual discrepancy between values of K for the 2-inch and 6-inch disks is about 3.5 per cent, leaving 1 per cent to be laid to experimental errors.

It seems safe to conclude that for speeds above 27 miles the coefficient K remains constant and the same for all disks.

The same conclusion was reached by Stanton[1] and by Riabouchinski,[2] but Eiffel[3] states that K is greater for larger surfaces. His tests in the wind tunnel certainly show an increase of K with area, but

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[1] T. E. Stanton, Proceedings of the Institution of Civil Engineers, Vol. 156, Part II, London, 1904.

[2] Bulletin de l'Institut de Koutchino, Moscow, 1912.

[3] La Resistance de l'Air et l'Aviation, Paris, 1912.

this may largely be accounted for by the effect of the walls.[1] M. Eiffel's tests in the open air with large plates dropped from the Eiffel Tower indicate still larger values of K, but here velocities were as great as 90 miles per hour. In addition to the extreme difficulty of obtaining precise measurements of resistance and speed with this method, a further complication is injected into the problem from the fact that the velocity of fall was accelerating.

It seems obvious that the resistance to accelerated motion should be greater than that due to uniform motion, since the energy system accompanying the disk is being built up.

In conclusion, then, within the limits of these tests, the resistance of a flat disk normal to the wind for speeds above a certain minimum may be correctly represented by

$$R = K\rho AV^2.$$

The existence of a critical velocity for disks has not been detected in previous experiments. M. Eiffel's tests were not run at sufficiently low velocity, Riabouchinski's tests were not very precise; but the experimental work of Stanton or Föppl might have been expected to bring out a critical velocity.

One explanation may lie in the fact that the critical velocity at which eddy making begins to become a stable phenomenon depends largely on the quality of the wind in the tunnel. It is reasonable to suppose that a turbulent wind would cause the critical velocity to be reached sooner than it would be with a more steady wind.

Experiments at Göttingen on spheres showed a marked critical velocity at a certain point. When the wind was made more turbulent by a fish net up stream, the critical velocity came at about two-thirds of the former value.[2]

The Göttingen tunnel in which Föppl's tests were made is a closed circuit in which the wind is guided around the corners by vanes, strained through diaphragms and wire mesh and forced to flow in horizontal lines where the model is placed. The result of such forcing the air to take unnatural lines of flow must introduce a degree of

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[1] Lord Rayleigh, "On the Resistance Experienced by Small Plates Exposed to a Stream of Fluid," Phil. Mag., July, 1915. A simple yet delicate experiment showed that the resistance of two small plates was equal to that of a single large plate of the same area for the same speed. This is in agreement with the results presented above for disks, and indicates that if there be a critical velocity, this velocity is the same for both large and small areas. Lord Rayleigh used disks cut from cardboard, which had presumably a square edge resembling that of the brass disks.

[2] Zeitschrift für Flugtechnik und Motorluftschiffahrt, May 16, 1914.

turbulence into the translational motion that is finally realized. The turbulence of the wind may make the flow about the model turbulent at all speeds used. Hence no critical velocity would be found.

Stanton's tests were made with a Pitot tube placed 1 foot up stream from the model. The tube was made up of two ⅛-inch tubes. The disks were then placed in the wake of these tubes and we might expect the critical velocity to be affected. That the effect of such an arrangement is appreciable was determined from a simple test. In the course of our test on the 5-inch disk above, a glass tube 3/16 inch in diameter was run across the tunnel 1 foot up stream from the disk. The resistance of the disk was found to be reduced by about 6 per cent. Increasing Stanton's value of *K* by 6 per cent brings it into agreement with our mean value of *K*. The following table summarizes the values of *K* for flat plates normal to the wind given by the several experimenters. The notation and units used are as as above.

Authority	Diameter Plate Inches	Velocity Miles	*K*	Remarks
Eiffel	4	25	.000240	4.9 ft. wind tunnel.
"	6	25	.000243	4.9 ft. " "
"	10	25	.000247	4.9 ft. " "
"	15	40–90	.000263	Open air.
"	20	40–90	.000275	"
"	28	40–90	.000284	"
"	39	40–90	.000290	"
Riabouchinski..	0.5	3–15	.000255	3.9 ft. wind tunnel.
" ..	1	3–15	.000252	3.9 ft. " "
" ..	2	3–15	.000252	3.9 ft. " "
Föppl.........	7.9	15–30	.000255	6.7 ft. " "
Author........	2	35	.000265	4.0 ft. " "
"	3	35	.000265	4.0 ft. " "
"	4	35	.000265	4.0 ft. " "
"	5	35	.000265	4.0 ft. " "
"	6	35	.000268	4.0 ft. " " corrected for influence of walls.
Stanton........	5	13	.000243	2.0 ft. wind tunnel.
"	1	13	.000250	2.0 ft. " "
"	1.5	13	.000248	2.0 ft. " "
"	2	13	.000246	2.0 ft " "
"	3	13	.000268	2.0 ft. " "

It has been suggested that the critical velocity found by us is not that of the disks but a critical velocity for the tunnel. It is well known that there is a marked critical velocity for the flow of air or water in pipes where the fluid becomes suddenly turbulent.

From the experiments of Stanton and Pannel[1] on the flow of air, oil, and water through pipes, it was concluded that turbulence began

[1] Phil. Trans. Roy. Soc., 1914, p. 200.

for a value of the "Reynolds Number" $\frac{VD}{\nu}=2500$. Here D is diameter of pipe, V is velocity, and ν the coefficient of kinematic viscosity. A very rough calculation for the square 4-foot wind tunnel, using the above figure, indicates a critical velocity of 1/16 mile per hour. For the 3-inch pipes of the honeycomb, the critical velocity is about 1 mile per hour. Our experiments were conducted well above these speeds.

Similarly there is a limiting velocity for the flow of air at atmospheric pressure to be deduced from St. Venant's equation for motion of a compressible fluid.

For air at ordinary temperatures this limiting velocity is about 770 miles per hour.

A more practical condition which may occur at ordinary speeds may account for the existence of an apparent critical velocity at which the fluid refuses to turn a corner due to its inertia. Thus, for true stream line motion about a disk, the air is required to turn sharply over the edge and close in on the back of the disk. The stream line has a finite radius of curvature and a finite velocity at any point, and there is consequently a centrifugal force on each particle of the fluid. Unless the pressure gradient is sufficient to balance this centrifugal force, the curvature of the stream line cannot be maintained. A dead water then forms at the rear of the disk which is dragged away by the viscosity of the moving air in contact with it, thus setting up a turbulent wake. A considerable increase in resistance might be expected to take place when turbulence is set up.

Our experiments show an abrupt change near 13 feet per second for thin disks, with unstable flow for velocities between 10 and 20 feet per second. This range is of the same order of magnitude as the critical velocity for a flat ellipsoid deduced by Mr. E. B. Wilson in the prefatory note above.

NOTE ON AN EMPIRICAL EQUATION TO EXPRESS THE EXPERIMENTAL RESULTS

Within the limits of these experiments the normal resistance of a thin disk may be represented by the expression

$$R=.0018\,D^2V+.00103\,D^2V^2,$$

in which

R is total force on disk in pounds,
D, diameter in feet, and
V, wind velocity in feet per second.

This leads to the equation to a straight line drawn on figure 39,

$$\frac{R}{D^2V} = .0018 + .00103V.$$

It appears that for velocities above 20 feet per second the values of

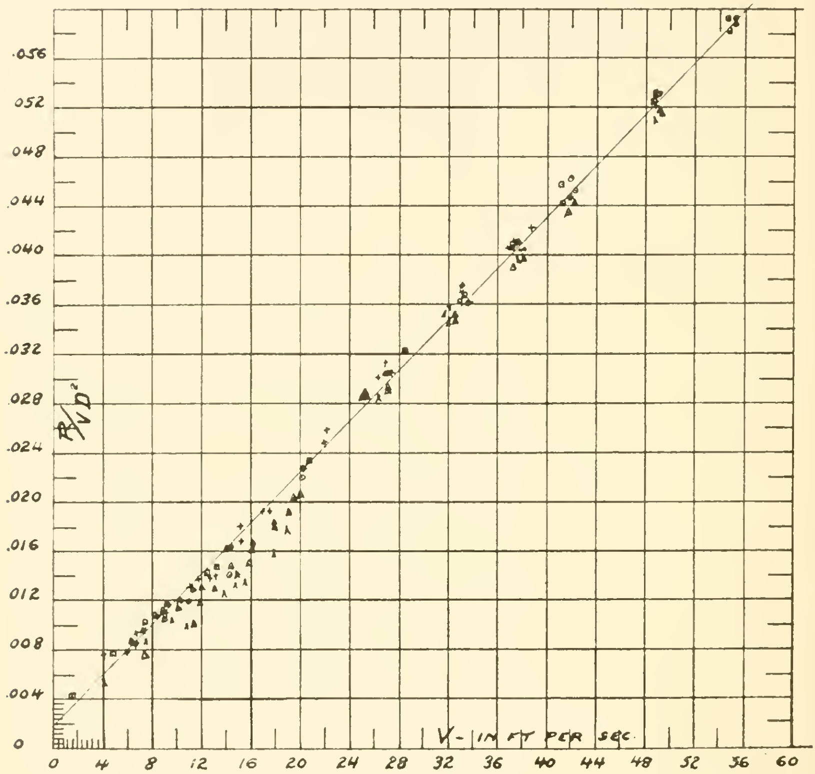

FIG. 39.—Units.

R = Pounds, force.
D = Feet, diameter.
V = Feet per second, velocity.

⊙ 6 ft. disk.
⊡ 5 " "
× 4 " "
Δ 3 " "
Λ 2 " "

$\frac{R}{D^2V}$ as observed for all disks lie near the line. It should be noted that the above expression is not dimensionally homogeneous, and it is therefore not safe for extrapolation. However, it was pointed out that the disks used were of uniform thickness and hence not geometrically similar.