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Many stars are variable, some intrinsically, others by reason of bodies interposing between them and us, as with the eclipsing variables. Our star, the sun, is rarely eclipsed. Astronomers question whether the sun's intrinsic variations are perceptible. I propose to offer observational evidence.

The great obstacle to such an investigation of our star, the sun, is the earth's atmosphere. It offers no such difficulty in measuring variations of the other stars, for the star to be investigated may be compared with many neighboring stars, whose rays suffer nearly the same atmospheric obstruction. Daylight sun drowns out all its neighboring stars.

The planets, which all shine by reflected sunlight, might indeed be compared with neighboring stars. But they are all too imperfect mirrors to yield highly accurate results on the sun's variation. We are therefore forced to make absolute determinations of intensity of the sun's rays, day after day, with proper allowances for the losses they suffer in passing through the atmosphere. Many scientists think it is impossible thus to achieve sufficient accuracy to determine the sun's small percentage variation.

For many years the Smithsonian Astrophysical Observatory has carried on measurements of the intensity of the sun's rays as they would be found at mean solar distance outside our atmosphere. This is called "the solar constant of radiation." The methods and results are described in detail in volumes 2 to 7 of the Annals of the Astrophysical Observatory. We shall not dwell on the methods here. The observations have been made daily, as far as possible, from mountains 6 to 10 thousand feet in elevation, in arid lands, from the year 1918 to the present time. Our stations have been in both hemispheres. Careful comparisons of the measures show no appreciable systematic differences between them, owing to the southern winter occurring in northern summertime. Automatically recorded measurements from balloons at 15 miles elevation show that the solar-constant values are

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reasonable. That is to say, our estimates of atmospheric transmission are sound.

Since the total range of variation found in 10-day mean values of the measures of the solar constant only rarely exceeds 2 percent, and usually is less than 1 percent, scientists, well knowing the seriousness of the obstacle to accuracy presented by variable atmospheric transmission, very reasonably fear that we cannot certainly determine variations of the sun. It is therefore well to present at the outset the evidence on the accuracy of our solar-constant measures.

As stated in volume 6, page 163, of the Annals of the Astrophysical Observatory, a comparison has been made between very large numbers of pairs of daily solar-constant measurements, made thousands of miles apart, in opposite hemispheres, over a period of many years. It yields, from their differences, the result that the probable error of a well-observed solar-constant value, the mean of the results at two stations on a single day, is $\frac{0.164}{\sqrt{2}}$ percent, or $\frac{1}{8}$ percent.

In the work I am about to present on periods in solar variation, 20 out of 31 of the periodic tabulations (excluding 11 tables on a monthly basis, made upon periods of variation exceeding 20 months) comprise tables of 10-day means of the measurements at two or more stations.[1] These tables each contain from 10 to 100 repetitions of such 10-day means. Going back to daily observations, published in volumes 6 and 7 of the Annals, it will be found that most of the 10-day means are means of the average results of two or more stations over at least 4 days, sometimes even 10 days, and of one station during the remainder of the decade. Using the well-known rule that the probable error of the general mean is the probable error of the individual observation divided by the square root of the number of observations, we see that for two-thirds of the tabulations, and excessively so for the periods under 10 months, for which the number of columns of repetition is above 40, the probable error of the individual values in the final column of means is of the order of 1/60 of 1 percent or less. Hence readers should not be skeptical, when smoothly running curves of periodic variation are presented that have a range as small as 0.04 percent. From that minimum, the ranges to be presented in this paper rise to values for some periodic variations of 0.15 to 0.20 percent. In fact, all the periods, some 23 in number, used in my weather papers on precipitation and temperature at various cities, have ranges in solar variation exceeding 0.05 percent. As

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[1] I shall deal with monthly data below.

for weather features, the ranges in precipitation found for these same individual periods run from 5 to 25 percent. Ranges in temperature for individual periods sometimes reach over 2° Fahrenheit.

In an earlier paper, "Periodicities in the Solar-Constant Measures" (Smithsonian Misc. Coll., vol. 117, No. 10, 1952), I used nearly all of the observed solar-constant values, from August 1920 to December 1950. This interval included the unique depression of the solar constant of about 5 percent, which was found in 1922 and 1923. It also included the widely varying values found from 1920 to 1922. In this present research I desired to obtain the highest attainable accuracy. Both in length and range I aimed to determine the best average values of periodic variations in solar radiation. I therefore purposely omitted those exceptional years, and began with September 1923, ending with December 1952.

There is a weightier reason than this improved selection of data for revising my former study of solar radiation. I have become more and more convinced, by my researches on precipitation and temperature, that a simple tabulation of a suspected period, in a long series of monthly or 10-day values, is unconvincing as to the reality of periodicity. For there is a large family of periods that have been found in various phenomena, all members of which are aliquot parts of $22\frac{3}{4}$ years. Not only does a long succession of monthly or 10-day means contain numerous members of this family, but the individual members, if segregated merely by simple tabulations, each contains, in its mean expressions, several shorter periods which are aliquot parts of the period being investigated.

Unless these shorter periods are removed, the mean expression of the desired period is so marred in form as to be unconvincing. No elimination of these overriding shorter periods, aliquot parts of the periods determined, was done in the publication just cited.

In the present publication I shall disclose a considerable number of new members of the family, found overriding known periods, and all integrally related to $22\frac{3}{4}$ years. Many of these new periods are so short as to be better expressed in days than in months. They include some of the periods discovered in basal pulse rates by my friend Dr. F. P. Marshall.

After a short statement regarding the solar-constant data, specimens of the treatment of it will follow, and after that the conclusions reached will be given in tables and graphs.

In my former publication (cited above), I gave, in table 4 in the appendix, a list of solar-constant values of the years 1920-1950. For convenience of tabulation these values were given in the form of departures from 1.90 calories per square centimeter per minute. They were given, not in calories, but in fractions of our mean values of the solar constant, which is 1.946 calories per square centimeter per minute. Thus an observed value, for instance 1.950 calories, would be computed for tabulation as a departure, $\frac{1.950-1.900}{1.946}$, or 2.58 percent of the mean solar constant. To save printing, the decimal point is omitted in the table. When the range of a period in solar variation is found (if, for example, it is 7 on this scale) we know at once that its range is 0.07 percent of the mean solar constant. In table 1 of the present paper I add to the former table mentioned above the departures in solar-constant values from volume 7 of the Annals, which represent the years 1951 and 1952.

As I said above, comparisons of daily solar-constant measures, made in different hemispheres over many years, show that they go along together without systematic departures. Their accidental departures indicate a daily probable error of only $\frac{1}{8}$ percent when two stations cooperate. It is not material to our investigation of *solar variation* that our results should be expressed on precisely the true scale of heat units. Hence, though interesting, it is not disturbing that our accepted mean solar-constant value differs from the new determination of the solar constant by F. S. Johnson.[2] He used, in part, recent rocket observations, and his result is 2.00 ± 0.04 calories. It differs by 2.7 percent from ours, which is 1.946 calories. But our work on *solar variation* is not thereby prejudiced.

I now proceed to illustrate, by examples of a short and a long period in solar variation, the work presented. For the short period I take 7 months, which is $1/39 \times 273$ months. By tabulations and graphs, this 7-month period and its superriders will be given in considerable detail. For the long period I take $68\frac{1}{4}$ months. To save printing, the $68\frac{1}{4}$-month period will be given mainly by graphs. The method for it will have been apparent from the treatment of the 7-month period.

Table 2 gives the means of three groups of tabulations with 21 10-day mean values in each group, making an average 7-month period. They therefore summarize 51 repetitions. They include, respectively, 17 repetitions covering the interval September 1923 to December 1932; 17 repetitions January 1933 to November 1942; and 17 repetitions December 1942 to October 1952. The next step with each group is to compute the average march of thirds of the mean summaries. The justification for this will be apparent from the triple

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[2] Journ. Meteorol., vol. 11, No. 6, December 1954.

TABLE 1.—*Solar-radiation changes, 1951-1952*
(A continuation of table 4 of Smithsonian Misc. Coll., vol. 117, No. 10, Publ. 4088)

Year	Month	Decade	Year	No.	Value	Mean No.	Year	Month	Decade	Year	No.	Value	Mean No.
5	1	I	1	1096	224		5	1	I	2	1132	211	
		II		97	315	260			II		33	268	247
	—	III		98	240			—	III		34	263	
5	2	I	1	99	257		5	2	I	2	35	195	
		II		1100	232	272			II		36	206	210
	—	III		01	327			—	III		37	228	
5	3	I	1	02	292		5	3	I	2	38	173	
		II		03	280	283			II		39	206	190
	—	III		04	276			—	III		40	189	
5	4	I	1	05	213		5	4	I	2	41	236	
		II		06	223	224			II		42	227	230
	—	III		07	235			—	III		43	222	
5	5	I	1	08	236		5	5	I	2	44	221	
		II		09	182	212			II		45	225	243
	—	III		10	219			—	III		46	282	
5	6	I	1	11	212		5	6	I	2	47	230	
		II		12	202	221			II		48	251	243
	—	III		13	248			—	III		49	244	
5	7	I	1	14	244		5	7	I	2	50	200	
		II		15	248	243			II		51	202	209
	—	III		16	237			—	III		52	226	
5	8	I	1	17	228		5	8	I	2	53	237	
		II		18	256	253			II		54	217	225
	—	III		19	276			—	III		55	220	
5	9	I	1	20	271		5	9	I	2	56	191	
		II		21	238	240			II		57	239	214
	—	III		22	210			—	III		58	212	
5	10	I	1	23	229		5	10	I	2	59	222	
		II		24	220	227			II		60	231	223
	—	III		25	232			—	III		61	209	
5	11	I	1	26	245		5	11	I	2	62	209	
		II		27	199	219			II		63	216	209
	—	III		28	212			—	III		64	202	
5	12	I	1	29	244		5	12	I	2	65	253	
		II		30	225	235			II		66	229	244
5	—	III	1	1131	236		5	—	III	2	1167	251	

maximum forms of curves in figure 1. The curves A_1, A_2, A_3 are the summaries, and the curves B_1, B_2, B_3 the mean forms of the periods of one-third of 7 months. Subtracting the latter from the former leaves C_1, C_2, C_3, which are the 7-month summaries deprived of the periodicities $1/117 \times 22\frac{3}{4}$ years. The results are tabulated in column 3 of table 2. We now perceive in curves C_1, C_2, C_3 a period of one-fourth their length. Removing it, as before, we obtain curves D_1, D_2, D_3, and columns 4 and 5, table 2. There is now seen, indistinctly it is true, except in D_1, a period of one-seventh of 7 months. Removing it as before, the curves E_1, E_2, E_3 result, and also columns 6 and 7 of table 2. No other superriders being seen in curves E_1, E_2, E_3, and all three of these curves being in the same phase and of similar forms, we conclude that there is no further correction required to the period 7.0 months. The three curves E_1, E_2, E_3, are therefore combined into the general mean curve F and column 8 of table 2. So there is clearly shown a period of exactly 7 months ($1/39 \times 273$ months) in solar variation. The 7-month period has a range of 0.08 percent of the solar constant. In association with it there are discovered three periods, 1/117, 1/156, and 1/273 of $22\frac{3}{4}$ years, having lengths of $2\frac{1}{3}$, $1\frac{3}{4}$, and 1 month, and ranges of about 0.06, 0.04, and 0.02 percent of the solar constant. The last of these three periods is perhaps doubtful, both on account of its small range, and of the divergence of phases in the three determinations of it. The other two superriders seem well determined, having similar forms and nearly similar phases and ranges in the three determinations. These superriders are, to be sure, less strongly developed in range in the middle curves than in the first and third sets, but the phases are nearly alike in all three.

Seven months in days is $7/12 \times 365.2564$ days, or 213.07 days. It is therefore interesting to note that, quite independently, my friend Dr. F. P. Marshall found a period of 212 days in a graph of 3 years of daily basal pulse rates. Dr. Marshall also found more than a dozen shorter periods, all aliquot parts of 212 days. Several of these periods I now find, as just said, to be also solar. It would seem to be interesting to physiologists to know that basal pulse rates are found to present regular periodic variations agreeing, within less than 1 percent, with periods found in solar variation. As my studies of weather elements show,[3] the family of periods that relate integrally to 273 months is also active in the control of temperature and precipitation. It may therefore well be that solar variation affects weather, and weather affects basal pulse rate.

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[3] See Smithsonian Misc. Coll., vol. 128, No. 3, Publ. 4211, Apr. 28, 1955.

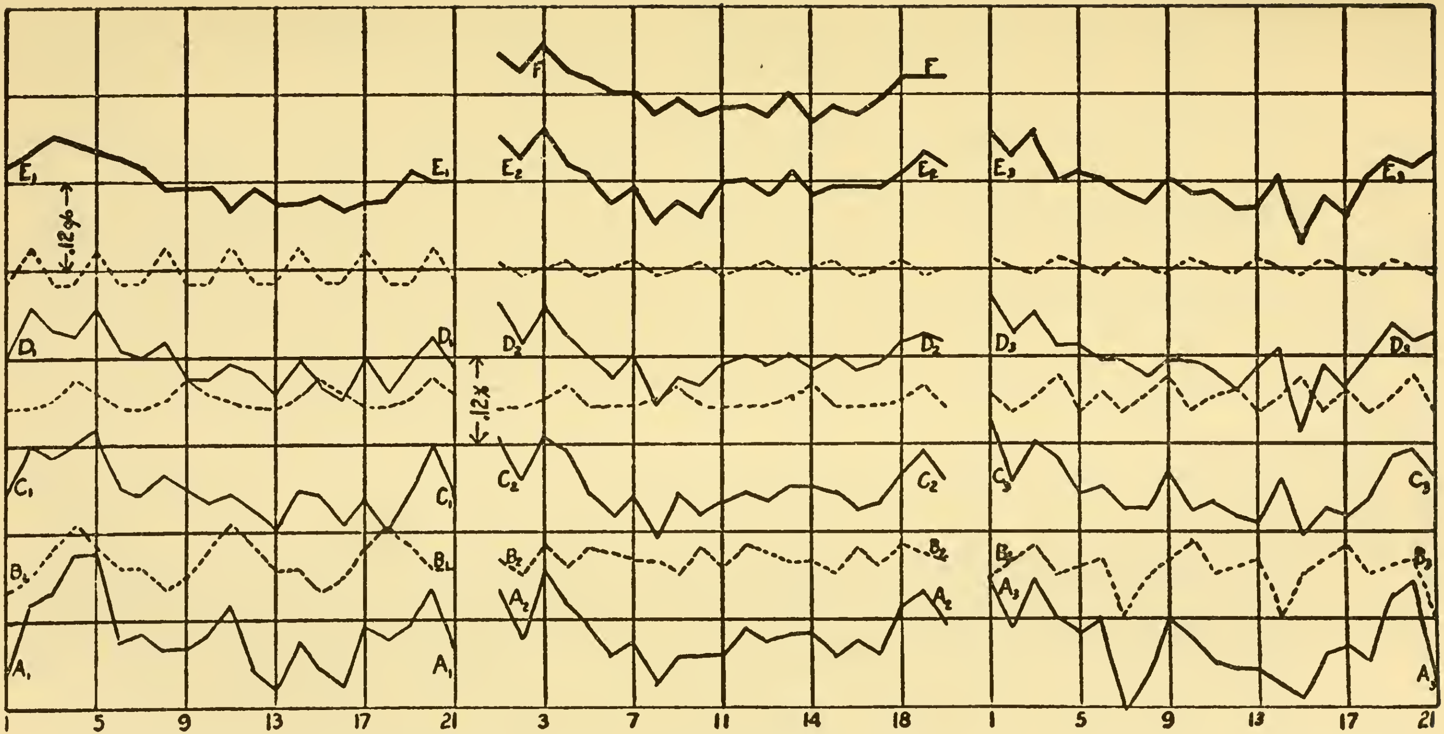

FIG. 1.—Three 10-year successive determinations of the 7.0-month period in solar variation, as cleared of subordinate interfering integrally related periods.

TABLE 2.—*The 7-month period*

From 10-day means

No.	1	2	3	4	5	6	7	8
1	225	226	−1	−1	0	−2	+2	+5
2	234	228	+6	−1	+7	+3	+4	+3
3	236	232	+4	0	+4	−2	+6	+7
4	241	235	+6	+3	+3	−2	+5	+3
5	240	232	+8	+1	+7	+3	+4	+2
6	229	229	0	−1	+1	−2	+3	0
7	230	229	+1	−1	0	−2	+2	0
8	228	226	+2	0	+2	+3	−1	−3
9	228	228	0	+3	−3	−2	−1	−1
10	230	232	−2	+1	−3	−2	−1	−3
11	234	235	−1	0	−1	+3	−4	−2
12	225	232	−7	−1	−2	−2	−1	−2
13	223	229	−6	−1	−5	−2	−3	−3
14	229	229	0	0	0	+3	−3	0
15	225	226	−1	+3	−4	−2	−2	−4
16	223	228	−5	+1	−6	−2	−4	−2
17	231	232	−1	−1	0	+3	−3	−3
18	229	235	−6	−1	−5	−2	−3	−1
19	231	232	−1	0	−1	−2	+1	+2
20	235	229	+6	+3	+3	+3	0	+2
21	228	229	−1	+1	−2	+2	0	+2
22	246	240	+6	−1	+7	+1	+6	..
23	239	238	+1	−1	+2	−1	+3	..
24	249	242	+7	0	+7	0	+7	..
25	244	239	+5	+2	+3	+1	+2	..
26	241	242	−1	−1	0	−1	+1	..
27	237	241	−4	−1	−3	0	−3	..
28	239	240	−1	−1	0	+1	−1	..
29	233	240	−7	0	−7	−1	−6	..
30	237	238	−1	+2	−3	0	−3	..
31	237	242	−5	−1	−4	+1	−5	..
32	237	239	−2	−1	−1	−1	0	..
33	241	242	−1	−1	0	0	0	..
34	239	241	−2	−1	−1	+1	−2	..
35	240	240	0	0	0	−1	+1	..
36	240	240	0	+2	−2	0	−2	..
37	237	238	−1	−1	0	+1	−1	..
38	239	242	−3	−1	−2	−1	−1	..
39	237	239	−2	−1	−1	0	−1	..
40	244	242	+2	0	+2	+1	+1	..
41	246	241	+5	+2	+3	−1	+4	..
42	241	240	+1	−1	+2	0	+2	..
43	247	238	+9	+1	+8	+1	+7	..
44	241	240	+1	−2	+3	0	+3	..
45	248	242	+6	0	+6	−1	+7	..
46	242	238	+4	+3	+1	+1	0	..

TABLE 2.—Continued

47	238	239	−1	−2	+1	0	+1	..
48	240	240	0	+1	−1	−1	0	..
49	229	232	−3	−2	−1	+1	−2	..
50	235	238	−3	0	−3	0	−3	..
51	242	240	+2	+3	−1	−1	0	..
52	239	242	−3	−2	−1	+1	−2	..
53	236	238	−2	0	−2	0	−2	..
54	235	239	−4	+1	−5	−1	−4	..
55	235	240	−5	−2	−3	+1	−4	..
56	233	232	+1	0	+1	0	+1	..
57	231	238	−7	+3	−10	−1	−9	..
58	237	240	−3	−2	−1	+1	−2	..
59	238	242	−4	+1	−5	0	−5	..
60	236	238	−2	−2	0	−1	+1	..
61	243	239	+4	0	+4	+1	+3	..
62	245	240	+5	+3	+2	0	+2	..
63	233	232	+1	−2	+3	−1	+4	..

Turning to the 68¼-month period, instead of 51 repetitions of it, as in the tabulation of the 7-month period, there are only 5. Yet, as monthly means are used instead of 10-day means, the individual values are probably more precise in proportion to $\sqrt{3}$. Combining these two considerations, the probable error of the mean values in the 68¼-month tabulation is $\sqrt{\frac{51}{5\times 3}}=1.8$ times as great as of those in curve *F* of figure 1. Hence we are not to expect as smooth a curve for 68¼ months as curve *F* of the 7-month period. But on the other hand, as we shall see, the range of the 68¼-month solar period is nearly three times as great as that of the 7-month period. There is therefore no difficulty in recognizing the 68¼ period as well evidenced.

In figure 2, *AA* is the direct mean of five repetitions of the monthly values composing a period of 68¼ months. The curve *AA* is very rough, but shows some indication of a superrider of 68¼ ÷ 3, or 22¾ months. Extracting this short period, curve *BB* results. This curve discloses a period of ½ × 68¼ or 34⅛ months. Extracting it, curve *CC* results. This shows very plainly a period of 1/7 × 68¼ months. Extracting it, curve *DD* results. Here, somewhat less distinctly, a period of 1/11 of 68¼ months is seen. Extracting it, curve *EE* results. Here, by careful analysis, a period of 1/19 × 68¼ months is found. Removing it, there remains curve *FF*, which, though it may contain shorter subperiods, is smooth enough to plainly exhibit the trend of the 68¼-month period, originally so much obscured in curve *AA*. The smooth curve *GG* has a range of 0.18 percent of the solar constant.

Having shown by these two figures examples of the smoothing of the solar-radiation periodicities by removing overriding shorter periodicities integrally related, I add that these two examples are by no means exceptional. Twenty others as convincing might be similarly presented. My ignorance of hydrodynamics prevents me from suggesting a theory to explain why this large family of integrally related periods occurs in solar radiation. Surely, however, it is a problem that deserves the attention of experts in stellar theory, for the sun is a convenient laboratory for stellar investigations.

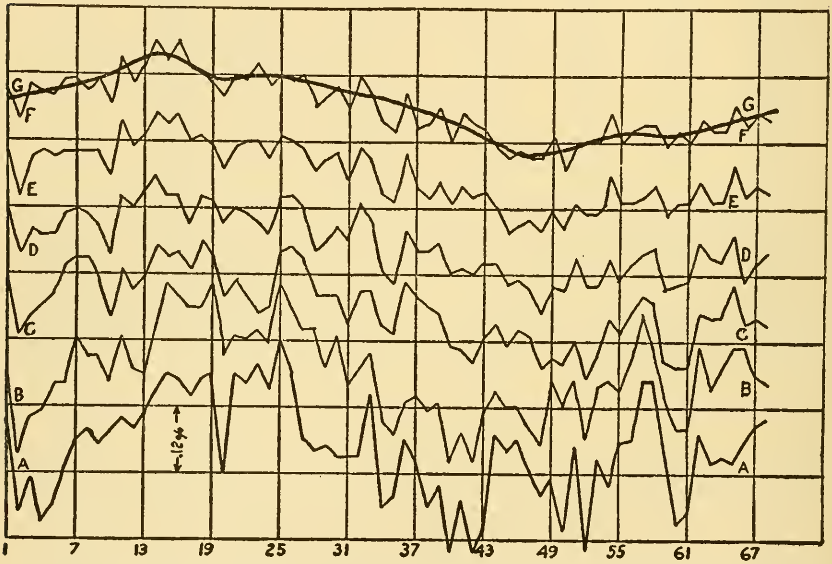

FIG. 2.—The 68¼-month period in solar variation as cleared of subordinate interfering integrally related periods.

Some critics, incredulous of the causation of weather changes by such small percentages in solar radiation, may be inclined to turn the phenomena about. They may suggest that some *terrestrial* atmospheric condition sets up the family of periods first in weather elements. Then they may suggest that these atmospheric changes produce, by our inability to entirely eliminate such influences, the observed tiny percentage periodic fluctuations in solar-constant measures. Such an explanation seems untenable to me. For the solar-constant periodicities go on with perfect regularity of phases over long periods of years, as table 2 and figure 1 show. This is so, despite the measurements being made in two hemispheres. The terrestrial periodicities, on the other hand, are altered in phases (a) with the season of the years; (b) with the prevalence of sunspots; (c) with the locality; and (d) with changes in the occupation of the land surface. Such phase-alterable agencies in the atmosphere could not, it seems to me, possibly produce periods of *unalterable* phases in solar-constant measures.

The fact remains, as yet unexplained, that both the sun's radiation and the elements of the weather are subject to variations in identical families of numerous periods. All are to within 1 percent exact aliquot parts of 273 months. It would be incredible if they are not associated in their cause. Furthermore, though over 20 of the solar variations range only from 0.05 to 0.21 percent of the solar constant, the corresponding periods in precipitation at St. Louis, Peoria, and Albany range from 5 to 25 percent of normal precipitation. So the percentage range in precipitation is a hundred times the range in solar variation. Here is indeed a paradox worthy of explanation.

I now give in table 3 a summary of all the periods discovered thus far in solar-constant measures, their ranges, and the range of corresponding periods in precipitation at St. Louis. Over 20 of the solar periods, identical with those I have used in weather studies, have been cleared of superriding periods, as shown among the total of 26 assembled in figure 3, and are well determined to be real. The other solar periods given in table 3 are the superriding periods which have been cleared away from the forms of periods shown in figure 3. They also are real, but their ranges and forms are only roughly estimated from the tabulations. In figure 3 the numbers attached indicate how many times the period shown will repeat in 273 months.

In table 3, line 1 gives the number of times periods of length given in line 2 will repeat in 273 months. In line 3 the values, called S, give the ranges of these periods in percentage of 1.946 calories per square centimeter per minute. Line 4, called L, gives the ranges of identical periods in percentages of normal precipitation at St. Louis, Mo. Line 5 and the following lines give the ratio numbers and approximate percentages of 1.946 calories of superriding periodicities found associated with periods in line 2.

Details regarding the period of 273 months are omitted in table 3 for lack of repeating data. I tested every submultiple of it up to 21, then kept on with only those periods that were used in weather studies. In all, 31 periods were investigated. Those marked with asterisks were found to differ slightly from exact integral submultiples of 273 months, when their lengths were adjusted to best fit the observations of the solar constant. The periods with ratio values, 13, 16, 17, 19, 20, and 26 proved to have such small amplitudes as to be negligible.

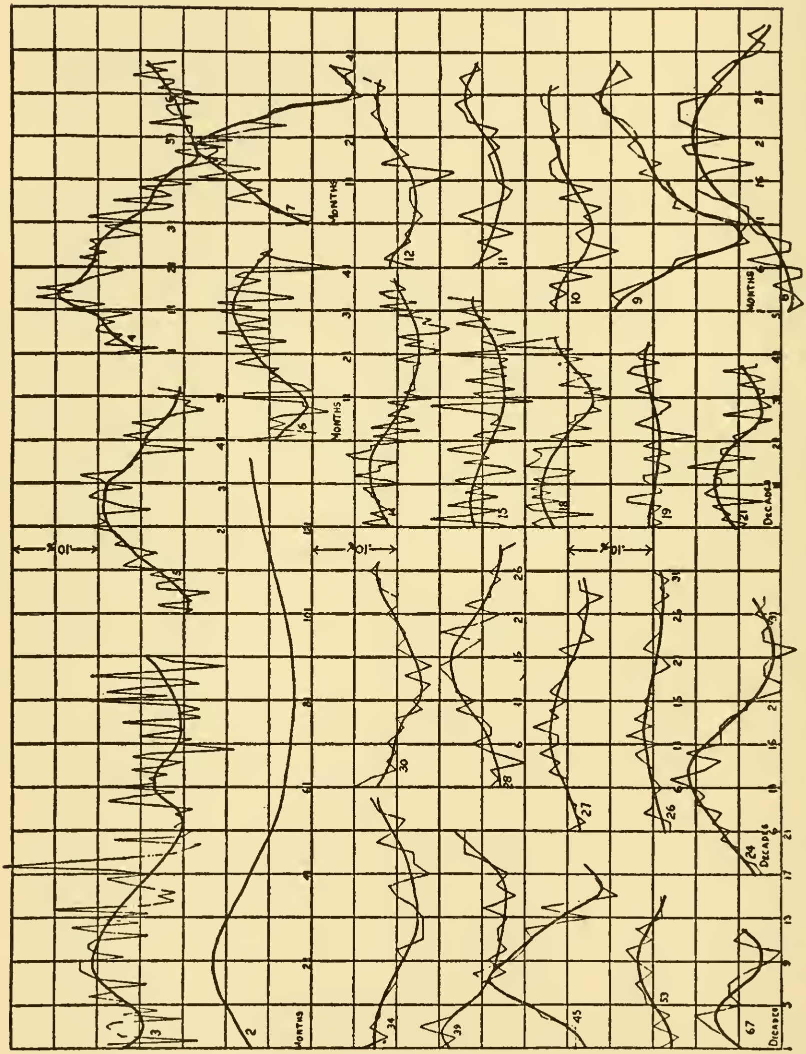

FIG. 3.—Twenty-six periods in solar variation, ranging from 4.08 months to 136½ months, as cleared of subordinate interfering integrally related periods.

TABLE 3.—*Summary of solar periods*

Ratio:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Period:	273	136.5	91.0	68.25	54.6	45.5	39.0	34.12	30.33	27.3	24.82	22.75	21.0	19.5	18.2	17.06
Range S:	...	0.09	0.11	0.18	0.09	0.09	0.21	0.12	0.17	0.06	0.05	0.05	0.03	0.07	0.05	0.0
Range L:	...	.....	13	13	0	8	23	21	15	19	18	17	....	....	....	....
Riders and Ranges S:	... ...	4 0.18 8 0.15	9 0.20 18 0.04 21 0.04 24 0.09 33 0.04 39 0.07 66 0.04	8 0.14 12 0.11 28 0.08 44 0.05 76 0.02	10 0.08 15 0.06 20 0.02 35 0.04 45 0.11	12 0.09 18 0.05	21 0.08 28 0.05 35 0.03	24 0.09 40 0.02 48 0.02	36 0.04 45 0.05 54 0.06 63 0.06	30 0.04	22 0.06 33 0.03 55 0.02	24 0.08	26 0.03 39 0.10 52 0.05 65 0.03	28 0.02 42 0.06 112 0.02 154 0.02	30 0.03 45 0.06	32 0.12

TABLE 3.—Continued

Ratio:	17	18	19	20	21*	24*	26	27	28	30	34*	39	45	53	67
Period:	16.06	15.07	14.37	13.65	13.1	11.39	10.5	10.11	9.75	9.1	8.1	7.0	6.07	5.15	4.07
Range S:	0.00	0.08	0.02	0.00	0.07	0.11	0.03	0.05	0.07	0.06	0.06	0.08	0.13	0.04	0.08
Range L:	....	26	....	18	10	15	12	10	23	18	12	9	8	7	4
Riders and Ranges S:	....	54 0.07 72 0.03 180 0.03	38 0.02 57 0.05 76 0.05 95 0.03 209 0.02	....	42 0.08 105 0.03 147 0.04	72 0.05 168 0.03	52 0.08 78 0.03 130 0.03 234 0.04	54 0.08 81 0.03 105 0.04	84 0.03 112 0.03	60 0.06 90 0.04	102 0.05 136 0.03 170 0.04	117 0.08 156 0.04 273 0.02	....	106 0.03	134 0.04

Only two of these, ratio values 20 and 26, have ever been included in my weather investigations.

Besides the 25 periods well evidenced among the 31 especially sought for in solar variation, the following 14 with ranges of 0.05 percent or more were found in association with them:

Ratio	15	22	32	42	44	52	54	57
Range	0.06	0.08	0.08	0.08	0.05	0.08	0.08	0.05
Ratio	60	63	72	76	102	117		
Range	0.06	0.06	0.05	0.05	0.05	0.06		

In addition to all these, the following 27 periods, with ranges of 0.04 percent or less, were removed from the 30 periods specially investigated, being superriders:

Ratios 26, 33, 35, 40, 48, 65, 66, 78, 81, 84, 90, 95, 105, 106, 112,
Ratios .... 120, 134, 136, 144, 154, 156, 168, 170, 180, 209, 234, 273.

Thus (omitting duplicates) in all 64 periods, integral submultiples of 273 months were found with various strengths of evidence in the variation of the solar-constant measures. The most convincing evidences of the reality of these periodic variations in solar radiation are found: First, in the smooth runs of the curves in figures 1, 2, 3, resulting from the elimination of the 41 periods just enumerated. Second, the cumulative effect upon the mind of finding in solar variation 31 smooth periodicities, all aliquot parts of 273 months, to within 1 percent. Third, in the fact that 20 of the especially sought and discovered periods in solar variations are found identically, in large percentage ranges, in weather features. In my future weather tabulations, the lengths of the periods found in solar variation will be followed exactly, not merely approximately, as heretofore. The fairly large solar periodicities, 14, 15, 22, 32, 42, 52, and 54, not heretofore used in weather, will be tested as possible weather periods. Numbers 20 and 26, hitherto used, will perhaps be omitted. Number 10 is sometimes found strong in weather features, and sometimes only its half, 27.3 months, appears.

In figure 3, the curves are designated by the ratio numbers given in table 3. All the curves in figure 3 are drawn to the same scale of ordinates, and that scale is indicated by three scales of 0.10 percent each. Three scales of abscissae are used, and these are indicated in figure 3. The first group of periods, 34 to 67, has the most open scale. The two groups 24 to 30 and 14 to 21 have identical scales, less open than used in the group 34 to 67. All the longer periods, 2 to 12 inclusive, are given in months instead of decades. Of these, the group 8 to 12 employs for months the same scale that is used for the decades in groups 24 to 30 and 14 to 21. All the remaining curves, 2 to 7, are plotted on a closer scale in abscissae as indicated in figure 3.

All the curves in figure 3 are based on September 1923 as zero. The decade 10-day means begin with September 5, and the monthly means, used in plots 2 to 12, begin with September 15, 1923. Inasmuch as all the curves in figure 3 depend for their forms and ranges on *all* the Smithsonian solar-constant work from September 1923 to December 1952, a synthesis of them all should represent the total monthly variation of the sun during that epoch, if the monthly solar variation is altogether composed of the family of periodicities integrally related to 273 months.[4]

I have made such a synthesis from September 1923 to December 1935, and present it in figure 4. Except for periods 2 and 3, I used ragged curves of figure 3, not the smooth ones. The dotted curve in figure 4 is my synthesis, and the heavy full curve represents the monthly solar-constant means observed. Except in 1924 and 1925, there is as fair an agreement as the probable error of observation would lead to expect.

Regarding 1924 and 1925, in chapter 3, pages 33 to 42, volume 6 of the Annals of the Astrophysical Observatory, there are detailed the difficulties found in preserving a sound and constant scale of solar-constant measurements. On pages 40 to 42 is told in detail how the results of other cooperating observatories were made to fit with the scale of results of Montezuma, which we considered the best. Several changes of the scale of measurement at Harqua Hala were made in 1924 and 1925 to harmonize with Montezuma. Besides these more or less arbitrary changes, it is related on pages 33 to 35 that a defective process of reduction of observations was introduced in 1923. Its erroneous character was not discovered till 1936. A new and correct method was then devised. In order to correct the results obtained from 1923 to 1936, the observers at the field stations were required to remeasure the bolographic plates of that interval in order to obtain the data needed in the new method devised in 1936. But parts of the earlier years 1924 and 1925 could not be thus salvaged. Some results for those years were therefore recomputed from the observations in the best way possible with data available, but they may not be as good as the work preceding and following. Accordingly I do not regard the heavy curve in 1924 and 1925 as being of

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[4] It is, of course, well known that day-to-day short ups and downs occur in solar variation, and are associated with weather, and with ionospheric fluctuations.

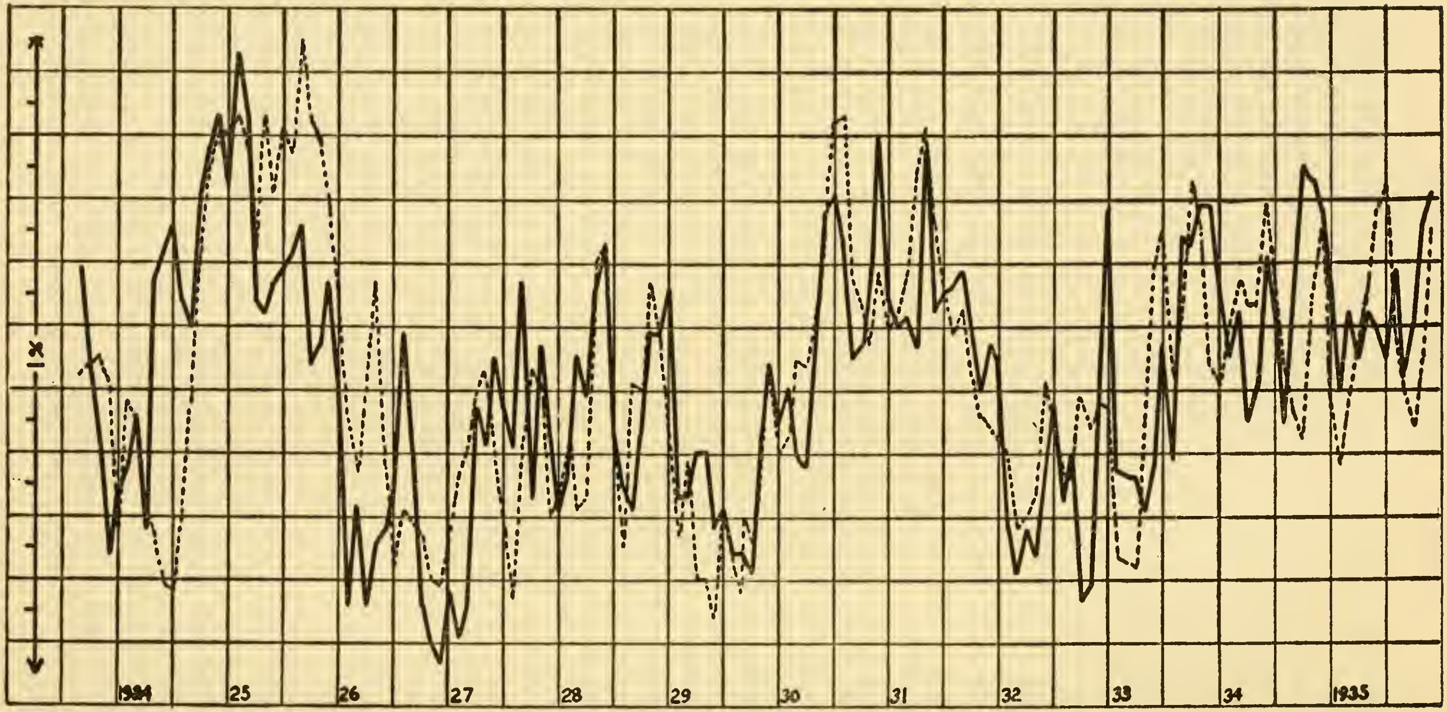

Fig. 4.—Synthetic combination of 31 periods in solar variation, as shown unsmoothed in figure 3, and compared with observed monthly solar-constant values of years 1923 to 1935. Full line, observed; dotted line, synthetic.

much weight. Indeed I regard the dotted one as best for those years, since the dotted curve fits so well elsewhere.

Obtaining differences between synthesis and observation, the following are mean values of deviations in percentages of the solar-constant result:

Epoch	1923 Sept. to 1927 Apr.	1927 May to 1929 Nov.	1923 Sept. to 1929 Nov.
Mean percent ...	±0.18	±0.11	±0.15
Epoch	1929 Dec. to 1932 Sept.	1932 Oct. to 1935 Dec.	1929 Dec. to 1935 Dec.
Mean percent ...	±0.09	±0.15	±0.12

To reach this close agreement between synthesis and observation, not only are the main trends in solar variation followed in unison, but most of the short-period variable features of observation are seen also in the synthesis. Such good agreement over a span of 12 years seems to warrant the belief that a synthesis over the 23-year span, 1900 to 1923, will give a trustworthy view of the solar variation during that interval. Accordingly I have made such a synthesis, and give it graphically in figure 5.

Some critics might be inclined to say that the agreement of synthesis with observation from 1923 to 1935 is no warrant for the value of a synthesis from 1900 to 1923, for, they may say, the features of the 31 periods used were based in considerable part on the observations of 1923 to 1935. Hence the synthesis within those years *should* agree with those observations. But if one takes any three years, as July 1929 to July 1932, the observations of those years could have had but 10 percent influence on the features of the 31 periods derived from 30 years of observation. Yet the average difference between synthesis and event in that interval is but 0.062 percent of the solar constant. The coefficient of correlation between synthesis and event during that interval is 90±5 percent. Is not this satisfactory indication that a synthesis of the 31 periodicities at any epoch is apt to be nearly true to fact?

Holding this conviction, I offer figure 5 as probably a good representation of the variation of the solar radiation for most of the first two decades of the twentieth century. Of course, if there occurred some other extraordinary variations of the sun, such as depressed the solar-constant measures by 5 percent in 1922, there may have been other changes outside of those represented in figure 5.

In figure 4 the solar-constant values began with September 1923. Counting back 273 months brings us to December 1900. All the periods being exact submultiples of 273 months, it follows that, so far as these periods comprise all the elements of change in solar radia-

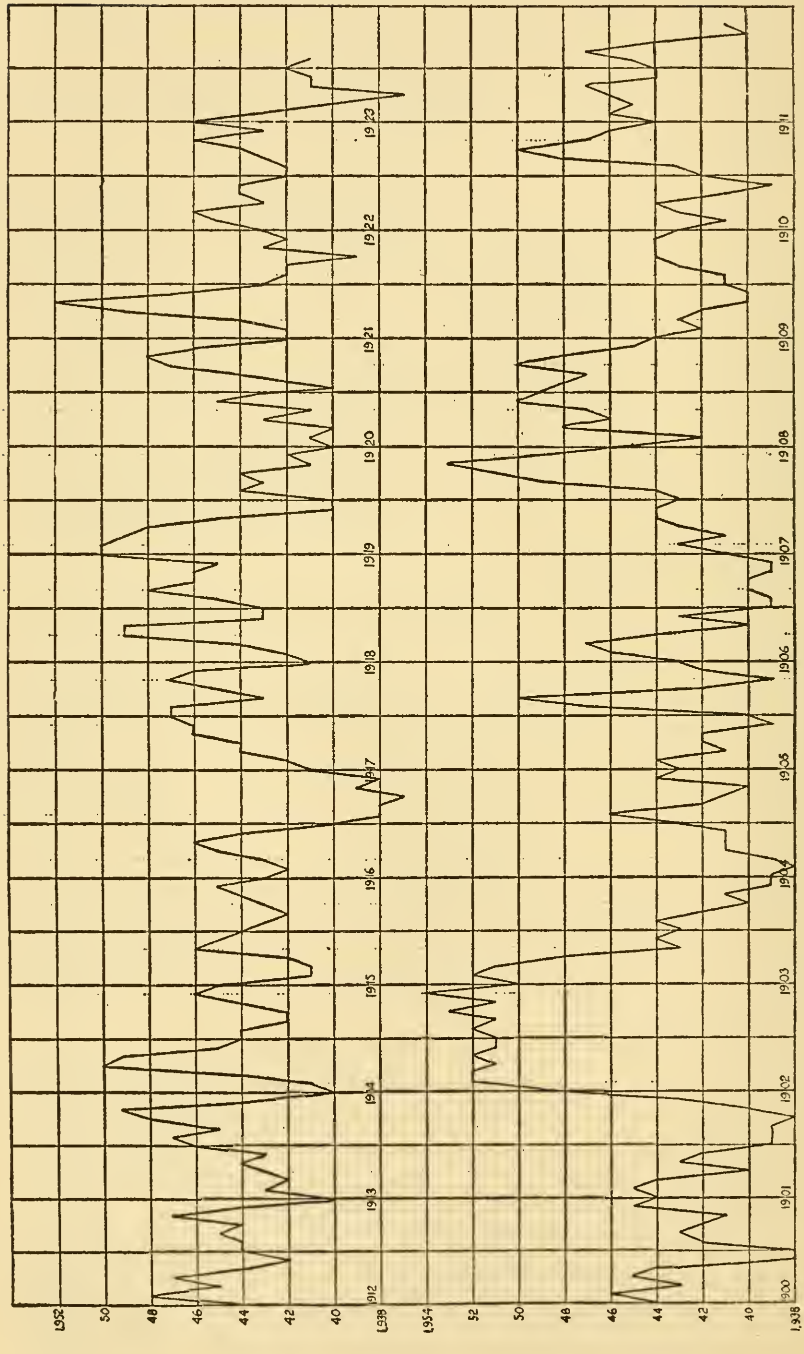

FIG. 5.—Synthetic reproduction of solar variation (unobserved) 1900 to 1923.

tion, the solar-constant values beginning with December 1900 will be the same as those beginning September 1923. In my recent paper on precipitation at St. Louis, Mo. (Smithsonian Misc. Coll., vol. 128, No. 3, 1955), I showed that synthesis by 22 periods, identical with 22 periods in solar variation, represented the precipitation at St. Louis well for 70 out of 100 years. The other 30 years appeared to be disturbed by great volcanic eruptions and great wars.

But it does not follow that, apart from these cataclysms, the weather, though controlled by solar variation, should repeat in 273-month cycles. For the quality of the atmosphere changes (a) with the time of the year; (b) with the prevalence of sunspots; (c) with the change of population, of forestation, and other variable features of the locality. Changes in the quality of the atmosphere alter the phases of weather responses to solar periods. Hence it is that, even apart from cataclysmic events, the weather only roughly repeats itself at intervals of 273 months. For 273 months is not commensurable with 12 months, and sunspot frequency is not sharply periodic at 11⅜ years. Hence two variable agents, (a) and (b) above, somewhat confuse the tendency toward regular weather cycles of 22¾ years. The element (c) above, is quite unpredictable in its effect on the weather.

SUMMARY

Reference is made to 30 years of daily measurements of the solar constant of radiation, at a plurality of stations on arid mountains, 6,000 to 10,000 feet in altitude, and in both hemispheres. Comparison of many pairs of northern and southern daily results give a probable error of ⅛ percent. Tabulations containing 30 to 100 repetitions of 10-day mean results are therefore subject to mean probable error of less than 1/60 percent.

A family of regular periods in solar variation is disclosed, containing 64 members, all, to within 1 percent, exact submultiples of 273 months. Many members of this family are also found in temperature and precipitation at several cities in the United States. In solar variation, ranges from 0.05 to 0.21 percent are found in over 20 of these periods which figure in weather. In the precipitation at St. Louis, Mo., these identical 20 and more periods have ranges from 5 to 25 percent.

Dr. F. P. Marshall found a period of 212 days, and also more than a dozen submultiples of it, in basal pulse rates. Seven months, a solar and weather period, is nearly the same, 213.07 days. A period published by the author in 1949, found in the weather of Washington and

New York as 6.6485 days, is very near 6.6476 days, which is $1/1250 \times 365.2546 \times 22\frac{3}{4}$.

I have presented graphs and tables showing solar periods and how interference by periods aliquotly related to them is removed. With such clearance, the total monthly mean solar constant is synthesized from 31 periods, and compared with mean observation. Close agreement is found, with an average deviation over 3 years of 0.062 percent, and correlation of 90 ± 5 percent.

One graph gives synthesized solar-constant monthly values, 1900 to 1923. It is stated that, apart from the unusual, the solar constant should repeat at intervals of 273 months in the monthly means. I showed 20 years ago that daily trends in the solar constant are closely correlated with weather trends, and later showed them correlated with ionospheric data. Monthly weather features should not closely repeat at intervals of 273 months, because of phase changes of weather periods, due to atmospheric changes.