Period, Color, and Luminosity for Cepheid Variables

Cecilia H. Payne-Gaposchkin

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ABSTRACT

Payne-Gaposchkin, Cecilia H. Period, Color, and Luminosity for Cepheid Variables. *Smithsonian Contributions to Astrophysics*, number 17, 10 pages, 8 tables, 1974.—The Cepheids of the Galaxy and the Magellanic Clouds display a gradation in properties, but when amplitude as well as period is taken into account, their intrinsic colors are sensibly similar. The period-luminosity relations in the three systems are probably parallel, but not necessarily coincident.

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The Period-Color Relation

The well-known relationship between period and color for classical Cepheids has usually been expressed in terms of a mean color. Magnitude at mean intensity is rightly considered an appropriate measure of luminosity. It is certainly more significant than mean magnitude, though it must differ from true (bolometric) luminosity by terms dependent on effective wavelength and intrinsic color. An extension of this principle has led to the use of mean colors, either $\langle B \rangle - \langle V \rangle$ where both magnitudes have been reduced to mean intensity, or mean $(B-V)$ expressed in magnitudes. These quantities are of course not synonymous, though their numerical values do not differ greatly. But the mean colors thus expressed have no obvious physical relation to the light curve: they are not characteristic of a particular phase, or indeed of the same phase for stars with differing light curves.

It therefore seems worthwhile to consider separately the colors $(B-V)$ at maximum and at minimum. These colors are associated with physically recognizable stages of the star's variation and are the same for all stars. We shall choose maximum and minimum in V light as reference points. By considering the colors that correspond to the extremes of light variation, we include the effects of light amplitude on brightness and color, significant parts of the data that are rejected by the use of mean colors.

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Oosterhoff (1960) has shown that the ranges in V and $(B-V)$ are correlated, and that thus maximal or minimal color (or both) must be functions of light amplitude as well as of period. We therefore evaluate a relation of the form

$$(B-V) = x + y \log P + zA, \qquad (1)$$

where $(B-V)$ is the intrinsic color at maximum or minimum, P the period, and A the light amplitude in magnitudes. We select the amplitude in V light, Δ_V, but the results could be converted to an expression involving Δ_B, the amplitude in B light, if desired.

To evaluate the constants, we make use of 60 Cepheids whose color excesses have been determined spectroscopically by Kraft (1961) and by Bahner, Hiltner, and Kraft (1962), which were standardized by means of 5 known Cepheids that are members of galactic clusters.

The photoelectric observations of Magellanic Cepheids by Gascoigne and Kron (1965) and Gascoigne (1969) have been similarly treated. The maximal and minimal colors are those tabulated by Gascoigne, corrected for reddening by $0.^m05$ for the Large Cloud and by $0.^m02$ for the Small Cloud, in accordance with his determinations. These colors are more uncertain than those of galactic Cepheids on account of the smaller number of observations for each star. Gascoigne's colors have been used as they stand; although slightly different values might be deduced from his light curves, the effect on the final results would be small. Two solutions were made for each Cloud, one using all the published data (except those for the highly reddened HV 2749 in the Large Cloud), and another excluding stars whose minima are not adequately covered by the observations (HV 2432, 2447, and 883 in the Large Cloud; HV 1425, 1328, and 1342 in the Small).

The results of these least-squares solutions are summarized in Table 1; successive columns give the following: the system; the number of stars used; the values of the constants x, y, and z in equation (1) for maximal color; and the values of the corresponding constants x′, y′, and z′ for minimal color.

The number of stars discussed for the Galaxy is large enough to justify a least-squares solution, but there are too few stars for either Cloud to give much weight to the results. The values of z and z′ do not seem to differ sensibly in the three systems. The agreement of y and y′ is less good. The values of x and x′ are the least concordant and suggest a difference in the zero points of the colors such that the Cepheids of the Large Cloud are slightly redder, and those of the Small Cloud slightly bluer, than are those of the Galaxy. We note that the means of the determinations for the two Clouds ($x = 0.364$, $y = 0.223$, $z = -0.238$, $x' = 0.340$, $y' = 0.352$, $z' = 0.122$) do not differ sensibly from the solution for the Galaxy. Although the accidental error of the zero point of the colors may be greater for the Clouds, their high galactic latitudes make the colors less vulnerable to systematic error than are those of galactic Cepheids, all of which are appreciably reddened. The zero point of the colors for galactic Cepheids depends on the intrinsic colors of the stars in the clusters that contain the five calibrating Cepheids. These considerations are of fundamental importance in examining the justification for using a period-luminosity relation deduced from Magellanic Cepheids in discussing galactic Cepheids.

Fernie (1970) has carried out a similar discussion for 27 galactic Cepheids, of which all but 2 (U Aql and α UMi) were included in our own list. He obtains relationships of similar form to those just presented, but with different numerical coefficients:

$$(B-V)_{max} = 0.297 + 0.307 \log P - 0.194\, A_v,$$
$$(B-V)_{min} = 0.238 + 0.373 \log P + 0.373\, A_v.$$

Table 2 contains the photoelectrically observed galactic Cepheids for which color excesses have been determined spectroscopically by Kraft (1961) and by Bahner et al. (1962), with multicolor photometry by Kron and Svolopoulos (1959) and by Williams (1966). Successive columns give the name of the star; the logarithm of its period; the amplitude A_V; the color excesses published by Kraft (K), by Bahner, Hiltner, and Kraft (BHK), by Kron and Svolopoulos (KS), and by Williams (W); color excesses at maximum and minimum predicted by means of the coefficients given in Table 1 for galactic Cepheids; color excesses predicted by Fernie from his relations quoted above; and Fernie's adopted color excess. The data for EU Tau are taken from Guinan (1972); the observed color excess given in the column headed K,BHK and marked with an asterisk is his value.

In comparing the observed and the predicted color excesses of Table 2, we recall that the coefficients of Table 1 were derived from the K and BHK color excesses, so that the agreement of the individual values is a measure of the closeness with which the observations have been represented. The average algebraic residuals are −0.002 and −0.005 at maximum and minimum; the average arithmetic residual is 0.06 for both. We conclude that the representation is satisfactory.

Since the color excesses of Kron and Svolopoulos and of Williams were not used in deriving

TABLE 1.—*Coefficients of equation* (1)

System	No.	x	y	z	x′	y′	z′
Galaxy	60	0.357 ± 0.029	0.292 ± 0.034	−0.282 ± 0.038	0.374 ± 0.027	0.327 ± 0.033	0.190 ± 0.036
LMC, all	20	0.455	0.163	−0.249	0.373	0.268	0.231
LMC, sel.	17	0.447 ± 0.060	0.170 ± 0.053	−0.264 ± 0.083	0.441 ± 0.083	0.292 ± 0.075	0.156 ± 0.115
SMC, all	27	0.261	0.264	−0.194	0.186	0.404	0.142
SMC, sel.	24	0.280 ± 0.083	0.276 ± 0.073	−0.213 ± 0.115	0.240 ± 0.116	0.412 ± 0.042	0.087 ± 0.116
All, sel.	101	0.353	0.269	−0.263	0.353	0.342	0.161

TABLE 2.—*Galactic Cepheids: Observed and predicted color excess*

Star	log P	A_V	Observed K, BHK	Color KS	Excess W	Predicted Color Excess Max.	Min.	Ad.	Fernie Pred.	Fernie Ad.
SU Cas	0.290	0.39	0.34			0.31	0.32	0.315	0.31	0.30
EU Tau	0.323	0.32	0.23*			0.21	0.19	0.20		
DT Cyg	0.398	0.36		0.09		0.11	0.06	0.085	0.13	0.14
AY Cas	0.458	0.68	0.79			0.81	0.83	0.82		
EV Sct	0.490	0.30	0.58		0.61	0.65	0.63	0.64		
SZ Tau	0.498	0.36		0.33	0.36	0.36	0.38	0.37	0.39	0.37
BY Cas	0.508	0.39	0.95			0.80	0.78	0.79		
DW Per	0.562	0.71	0.65			0.69	0.65	0.67		
SS Sct	0.565	0.49	0.33			0.44	0.44	0.44		
RT Aur	0.571	0.76		0.14	0.08	0.09	0.10	0.095	0.07	0.13
DF Cas	0.583	0.59	0.615			0.63	0.64	0.635		
SU Cyg	0.585	0.72		0.20	0.17	0.08	0.01	0.045		
ST Tau	0.602	0.82			0.32	0.39	0.34	0.365		
SY Cas	0.610	0.81	0.53			0.48	0.47	0.475		
Y Lac	0.636	0.69	0.135	0.26		0.21	0.19	0.20		
CG Cas	0.640	0.84	0.76			0.67	0.70	0.685		
T Vul	0.647	0.61		0.15	0.09	0.10	0.18	0.14	0.10	0.15
FF Aql	0.650	0.36	0.34	0.27	0.31	0.25	0.20	0.226	0.24	0.31
DF Lac	0.651	0.63	0.695		0.063	0.66	0.64	0.65		
CE Cas b	0.651	0.68			(0.54)	0.59	0.59	0.59		
XY Cas	0.653	0.58	0.43			0.60	0.59	0.595		
V482 Sco	0.656	0.63	0.43			0.45	0.43	0.44		
UX Per	0.667	0.95	0.51		0.73	0.50	0.44	0.47		
CF Cas	0.688	0.55	0.555		0.54	0.64	0.64	0.64	0.64	0.52
V Lac	0.698	1.00	0.385			0.38	0.35	0.365		
DW Cas	0.699	0.57	0.84			0.90	0.88	0.89		
AP Sgr	0.704	0.83	0.29			0.26	0.27	0.265		
CE Cas a	0.711	0.52			(0.54)	0.61	0.65	0.63		
V386 Cyg	0.720	0.69	1.02			1.03	0.99	1.01		
CR Ser	0.724	0.80			1.28	1.08		1.08		
TV Cam	0.724	0.97	0.60:		0.76	0.64	0.57	0.605		
BG Lac	0.727	0.63	0.33:	0.34		0.43	0.41	0.42		
UY Per	0.730	0.91	0.985			1.03	1.00	1.015		
δ Cep	0.730	0.83	0.11	0.14	0.11	0.10	0.12	0.11	0.08	0.14
SW Cas	0.736	0.67	0.55			0.56	0.55	0.555		
X Lac	0.736	0.41	0.40			0.37	0.36	0.365		
VY Per	0.743	0. .9	1.06			1.12:	1.01	1.05		
CZ Cas	0.753	0.83	0.785			0.89	0.84	0.865		
Y Sgr	0.761	0.72		0.23	0.17	0.27	0.32	0.295	0.27	0.26
AB Cam	0.762	1.01	0.69		0.97	0.67	0.65	0.66		
FM Cas	0.764	0.58	0.32			0.45	0.47	0.46		
VW Cas	0.778	0.70	0.63			0.66	0.61	0.635		
RV Sco	0.783	0.81	0.40			0.41	0.42	0.415		
VV Cas	0.793	0.85	0.54			0.56	0.55	0.555		
CR Cep	0.795	0.39	0.78			0.87	0.85	0.86		
RS Cas	0.799	0.76	0.78			0.89	0.91	0.90		
RR Lac	0.807	0.83	0.31			0.36	0.34	0.35	0.30	0.29
U Sgr	0.829	0.78	0.55	0.40	0.46	0.53	0.52	0.625	0.48	0.48
AP Cas	0.836	0.64	0.915			0.81	0.81	0.81		
X Sgr	0.846	0.65			0.14	0.18	0.16	0.17	0.16	0.20
η Aql	0.856	0.84	0.18	0.20	0.15	0.22	0.23	0.225	0.20	0.21
AK Cep	0.859	0.67	0.84			0.73	0.74	0.735		
CK Sct	0.870	0.52			0.94	1.01	0.98	0.995		
W Sgr	0.881	0.84			0.12	0.16	0.19	0.175	0.14	0.17
CD Cas	0.892	0.80	0.83			0.86	0.89	0.685		
W Gem	0.898	0.81	0.31			0.31:	(0.36)	0.31:		
VY Cyg	0.898	0.81	0.69			0.65	0.66	0.655		
RX Cam	0.898	0.66	0.62		0.61	0.66	0.64	0.66	0.62	0.57
DL Cas	0.903	0.60	0.50		0.56	0.59	0.69	0.59	0.58	0.57
S Sge	0.923	0.82		0.13	0.17	0.20	0.23	0.215	0.17	0.20
V500 Sco	0.969	0.76			0.62	0.72	0.68	0.70		
FN Aql	0.977	0.70			0.64	0.65	0.61	0.63	0.58	0.60
S Nor	0.989	0.66	0.205			0.35	0.35	0.35		
DD Cas	0.992	0.59	0.56			0.58	0.65	0.615		
ζ Gem	1.006	0.49	0.19	0.12	0.04	0.18	0.18	0.16	0.18	0.15
BZ Cyg	1.006	0.55	1.00			0.98	0.95	0.965		
Y Sct	1.018	0.83			0.87	0.94	0.98	0.96		
Z Lac	1.037	0.95	0.49			0.51	0.56	0.545	0.46	0.42
TY Sct	1.043	0.86			1.28	1.09	1.06	1.085		
AA Gem	1.053	0.65			0.60	0.42	0.46	0.44		
RX Aur	1.065	0.71			0.40	0.33	0.28	0.315		
RY Cas	1.084	1.03	0.74		0.73	0.71	0.69	0.70		
Z Sct	1.110	1.02			0.68	0.69	0.71	0.70		
SZ Cas	1.134	0.40	0.88		1.02	0.79	0.81	0.80		
TT Aql	1.138	1.13		0.43	0.59	0.60	0.67	0.635	0.54	0.53
CY Cas	1.158	1.10	1.11		1.13	1.04	1.06	1.06		
TX Cyg	1.167	1.15	1.23		1.49	1.18	1.24	1.21		
UZ Sct	1.169	0.87			1.16	1.18	1.21	1.195		
RW Cas	1.170	1.18	0.53		0.58	0.48	0.60	0.54		
SZ Cyg	1.179	0.89			0.59	0.80	0.87	0.835		
SV Mon	1.183	1.09			0.30	0.35	0.43	0.39		
X Cyg	1.214	1.06		0.28	0.36	0.43	0.54	0.485	0.39	0.36
RW Cam	1.221	1.07	0.74		1.01	0.75	0.61	0.68		
CD Cyg	1.233	1.21	0.68		0.58	0.58	0.67	0.625		
Y Oph	1.234	0.54	0.65	0.58	0.74	0.69	0.66	0.675	0.67	0.74
SZ Aql	1.234	1.28			0.71	0.68	0.80	0.74	0.59	0.58
CP Cep	1.252	0.72	1.08		0.71	0.92	1.03	0.975		
YZ Aur	1.260	0.76			0.76	0.63	0.68	0.655		
RU Sct	1.294	1.10			1.22	0.97	0.99	0.98		
VX Cyg	1.304	1.05			0.89	1.00	1.01	1.005		
WZ Sgr	1.339	1.03		0.56	0.53	0.58	0.72	0.65		
T Mon	1.432	1.06	0.41	0.27	0.32	0.41	0.46	0.435	0.33	0.37
AQ Pup	1.477	1.16	0.66			0.51	0.68	0.595		
SV Vul	1.654	1.02			0.70	0.56	0.66	0.61		

*Guinan (1972).

the coefficients of Table 1, they furnish an independent check. The comparison with the KS color excesses shows systematic trends; the KS color excesses tend to be larger than the K,BHK color excesses for short periods and smaller for long periods. Thus, the KS system differs systematically from the K,BHK system. The W color excesses show no evident systematic trend, but a much larger scatter. The comparison with Fernie's adopted color excesses shows no sensible difference for periods under 10 days, but for longer periods his adopted color excesses are systematically smaller than ours. A comparison between our formulas and his shows that the predicted color indices (in the sense of ours minus Fernie's) will differ at maximum by

$$0.060-0.015 \log P-0.088\, A_v,$$

and at minimum by

$$0.136-0.046 \log P-0.183\, A_v.$$

The greatest differences therefore occur for long periods and large V ranges (which are also associated with the longest period). The difference between our formulation and Fernie's must be ascribed to the difference in our selection of data. Our own was based on the system of K,BHK, which has the merit of being based on many stars and is quite homogeneous. A discussion based on the KS system would lead to results close to those obtained by Fernie.

The color excesses of Kraft and his collaborators were based on mean colors, and in using them to calibrate our conversions, we have tacitly assumed that the color excess of a Cepheid will be the same at maximum and minimum. The entries of Table 2 show in fact that color excess is not independent of intrinsic color. At longer periods, the predicted color excesses tend to be too large at minimum. The effect has been pointed out and correctly analyzed by Wildey (1963). The effect is to render the color excesses (assumed independent of intrinsic color and based on the O and B stars) too small for redder stars, and the intrinsic colors derived from them are accordingly too red. At maximum, the excess redness for galactic Cepheids is about 0.03; at minimum, it ranges from 0.06 to 0.12.

We are now in a position to attack the basic problem: Are the Magellanic Cepheids identical in properties with galactic Cepheids? The legitimacy of adopting a Magellanic period-luminosity relation in discussing galactic Cepheids hinges on the answer to this question.

Observable photometric properties that can be used in comparing galactic and Magellanic Cepheids are the following:

1. Light curves in V and B (yielding properties such as amplitude, asymmetry, interval from minimum to maximum, and interval between maxima where two are observed).

2. Data obtained by differencing light curves in V and B (yielding $(B-V)_{max}$ and $(B-V)_{min}$ as defined above).

3. Data obtained by integrating light curves at mean intensity (yielding $\langle V \rangle$ and $\langle B \rangle$ and also $\langle B \rangle - \langle V \rangle$).

4. Data obtained by integrating the $(B-V)$ color curve (yielding $\langle B-V \rangle$).

A comparison of the properties enumerated under 1 has been published elsewhere by the writer (1969). When considered as functions of period, most of them differ between the Galaxy, the LMC, and the SMC; only the interval between two maxima is related to period in the same way in all three systems.

The properties enumerated under 2 are summarized in Table 1. Here again, the coefficients of x, y, z, x′, y′, z′ differ in the three systems. But it is to be noted that in each system the relationship is of the same form: x, y, x′, y′, z′ are always positive, z always negative; the same is true of Fernie's formulation.

The numerical values, however, strongly suggest a real difference. The zero point of the intrinsic colors (expressed by the coefficients x and x′) shows that stars of similar period and amplitude in the Large Cloud are slightly redder, and in the Small Cloud, slightly bluer, than in the Galaxy. All studies of the Small Cloud have emphasized this blueness of the Cepheids in the Small Cloud.

The small number of stars on which the results for the Magellanic Clouds are based makes the actual values of the coefficients uncertain, and in order to examine the difference between the galactic and Magellanic colors, we therefore use the galactic relations (equations (2) and (3)) to predict colors for the Magellanic Cepheids from the published periods and V ranges. The comparison of observed colors with those thus predicted is given in Tables 3 and 4.

TABLE 3.—*Large Magellanic Cloud: Observed and predicted colors*

Star	log P	A_V	$(B-V)_{max}$ Obs.	$(B-V)_{max}$ Pred.	$(B-V)_{min}$ Obs.	$(B-V)_{min}$ Pred.
W 44	0.408	0.80	0.30	0.25	0.95	0.66
HV 5541 W 24	0.429	1.20	0.30	0.14	0.67	0.74
HV 12225 W 33	0.478	0.50	0.53	0.36	0.66	0.62
W 25	0.529	0.52	0.30	0.36	0.48	0.65
HV 12747 W 10	0.556	0.37	0.40	0.42	0.72	0.63
HV 12226 W 29	0.569	0.54	0.44	0.37	0.47	0.66
W 22	0.669	0.82	0.32	0.32	0.95	0.75
(HV 2432)	1.038	0.74	0.50	0.45	0.49	0.85
HV 886	1.380	1.28	0.33	0.40	1.13	1.07
HV 1003	1.387	1.05	0.33	0.47	1.02	1.03
HV 902	1.421	1.27	0.25	0.41	1.08	1.08
HV 2251	1.447	1.25	0.45	0.43	0.90	1.09
HV 1002	1.483	1.46	0.33	0.38	0.97	1.14
HV 2294	1.564	1.10	0.40	0.50	1.10	1.09
HV 909	1.575	0.92	0.43	0.56	0.96	1.06
HV 900	1.677	1.02	0.58	0.56	1.03	1.12
HV 953	1.680	1.00	0.50	0.57	1.10	1.11
HV 2369	1.684	1.08	0.52	0.54	1.29	1.13
(HV 2447)	2.074	0.64	0.85	0.78	1.43	1.17
(HV 883)	2.127	1.25	1.02	0.63	1.31	1.33

TABLE 4.—*Small Magellanic Cloud: Observed and predicted colors*

Star	log P	A_V	$(B-V)_{max}$ Obs.	$(B-V)_{max}$ Pred.	$(B-V)_{min}$ Obs.	$(B-V)_{min}$ Pred.
HV 1897	0.094	0.54	0.20	0.23	0.36	0.51
HV 1871	0.114	0.80	0.18	0.16	0.43	0.56
HV 1907	0.216	1.33	0.08	0.04	0.40	0.70
HV 1779	0.251	0.60	0.25	0.26	0.38	0.57
HV 1869	0.391	0.64	0.32	0.29	0.56	0.62
HV 11114	0.433	1.22	0.24	0.14	0.47	0.75
HV 2015	0.458	0.82	0.23	0.26	0.58	0.68
HV 1906	0.486	1.07	0.20	0.20	0.67	0.74
HV 11216	0.494	1.03	0.20	0.21	0.53	0.73
HV 11113	0.517	0.96	0.19	0.23	0.48	0.72
HV 212	0.591	0.93	0.18	0.27	0.55	0.74
HV 214	0.624	1.08	0.18	0.23	0.38	0.78
(HV 1425)	0.658	0.91	0.22	0.29	0.63	0.76
HV 1492	0.799	0.87	0.12	0.34	0.55	0.80
HV 1400	0.823	0.84	0.40	0.36	0.64	0.80
HV 11112	0.826	0.71	0.33	0.30	0.62	0.78
HV 827	1.129	0.96	0.32	0.42	0.74	0.92
(HV 1328)	1.200	0.68	0.36	0.60	0.63	0.90
(HV 1342)	1.254	0.78	0.35	0.50	0.57	0.93
HV 817	1.276	0.86	0.36	0.49	0.82	0.95
HV 823	1.514	1.22	0.48	0.45	1.06	1.10
HV 2195	1.621	1.06	0.40	0.53	1.14	1.11
HV 837	1.629	1.05	0.64	0.54	1.13	1.11
HV 824	1.818	0.97	0.49	0.61	1.12	1.15
HV 834	1.866	0.66	0.58	0.72	0.87	1.11
HV 829	1.942	0.73	0.70	0.72	0.97	1.15
HV 821	2.104	0.58	0.85	0.81	1.29	1.17

The comparison between the observed colors of Magellanic Cepheids and the colors predicted with the aid of equation (1) shows what appear to be real differences of intrinsic color. At maximum, the average residual (O−C) is +0.017 for the Large Cloud and −0.029 for the Small. It is especially noteworthy that our "galactic" relationship predicts maximal colors for Small Cloud Cepheids that are far bluer than any observed in the Galaxy, a consequence of the small periods and large amplitudes. At minimum, the average residuals are −0.062 for the Large Cloud and −0.112 for the Small. However, as pointed out in a preceding paragraph, the color excesses have been underestimated, and the intrinsic colors implied by equation (1) for galactic Cepheids are too red—by about 0.03 at maximum and by 0.06 to 0.12 at minimum. These differences are within the uncertainties of the quantities measured, and there is no longer any reason to insist on real physical differences between Magellanic and galactic Cepheids as a consequence of their observed colors. A similar conclusion has been reached by Sandage and Tammann (1971).

There remain the striking differences in period frequency between the Galaxy and the Clouds, which have been long known and are undoubtedly real. No doubt they provide a clue to evolutionary history, in which the timing of active star formation, the factors that have governed the mass frequency of the stars formed, and the condition and composition of the prestellar matter must all be taken into account. Are we justified in using the Magellanic Cepheids to establish the form of the period-luminosity relation and then applying this period-luminosity relation to the study of galactic Cepheids and of Cepheids in other galaxies? As an introduction to this problem, we now reexamine the period-luminosity relation for the Magellanic Clouds.

The Period-Luminosity Relation for the Magellanic Clouds

It has long been known that (excluding stars with sinusoidal light curves) the Cepheids of largest amplitude tend to lie above the average period-luminosity curve. It should therefore be possible to express the period-luminosity relation in the form

$$M = x + y \log P + zA, \qquad (2)$$

where M is the absolute magnitude, P the period in days, and A the amplitude. In what follows, we shall use the V magnitudes and amplitudes in applying this relation. As has already been shown, the stars of large amplitude are also the bluest for their periods, so that there will also be a relationship of the form

$$M = x' + y' \log P + z' (B-V)_0. \qquad (3)$$

Equation (3) requires a knowledge of intrinsic colors; equation (2) does not, and is applicable if A has been determined.

Least-squares solutions for the selected stars in the two Clouds lead to the following results (apparent magnitudes, m, are taken from the data published by Gascoigne (1969) to evaluate the coefficients for the apparent period-luminosity relations).

Large Cloud:
at maximum:

$$m_V = 17.588 - 2.852 \log P - 0.733A, \qquad (4)$$

at minimum:

$$m = 17.590 - 2.852 \log P - 0.224A, \qquad (5)$$

at mean intensity:

$\langle V \rangle = 17.640 - 3.003 \log P - 0.266A$; (6)

Small Cloud:

at maximum:

$m_v = 17.903 - 2.828 \log P - 0.763A$, (7)

at minimum:

$m = 17.931 - 2.838 \log P - 0.221A$, (8)

at mean intensity:

$\langle V \rangle = 17.982 - 2.858 \log P - 0.317A$. (9)

The coefficients of A are so similar in equations (4) and (7), (5) and (8), and (6) and (9) that we feel justified in adopting mean values. The coefficients of log P in equations (4) and (5) and in (6) and (7), that here again we adopt a mean. The first term expresses the zero point, which differs for the two Clouds. We adopt means from equations (4) and (5) for the Large Cloud and (7) and (8) for the Small. We thus obtain the following:

$m_{max} = 17.589 - 2.840 \log P - 0.773A$ (Large Cloud)
$17.916 - 2.840 \log P - 0.773A$ (Small Cloud), (10)

$m_{min} = 17.589 - 2.840 \log P + 0.223A$ (Large Cloud)
$17.916 - 2.840 \log P + 0.222A$ (Small Cloud), (11)

$\langle V \rangle = 17.640 - 2.930 \log P - 0.292A$ (Large Cloud)
$17.982 - 2.930 \log P - 0.292A$ (Small Cloud), (12)

The difference in zero point (SMC−LMC) is $0.^m327$ for the maximum-minimum relation and $0.^m342$ for the mean-intensity relation. The mean of these values, $0.^m334$, is adopted to represent the difference in true modulus. A value of $0.^m27$ was derived by the writer (1969) from direct comparison of the photographic period-luminosity curves. The present value, based on more accurate (though less) material and a more sophisticated approach, is to be preferred.

Tables 5 and 6 compare the observed maximal, minimal, and mean-intensity magnitudes with the predictions from equations (10), (11), and (12). Two final columns give the magnitudes predicted from the relations given by Gascoigne:

Large Cloud: $\langle V \rangle = 17.58 - 3.072 \log P$, (13)
Small Cloud: $\langle V \rangle = 17.73 - 2.879 \log P$, (14)

and the corresponding residuals, (O−C), which have been calculated with an allowance for absorption of $0.^m15$ in the Large Cloud and $0.^m06$ in the Small, in accordance with the color excesses derived by Gascoigne ($0.^m05$ and $0.^m02$) for the two systems.

Our predicted magnitudes in Tables 5 and 6 were derived by using the same factors for slope and amplitude in the two Clouds, whose predicted magnitudes thus differ only in zero point. Gascoigne (equations (13) and (14)) determined period-luminosity relations separately for the two Clouds, and these differ in slope as well as

TABLE 5.—*Large Magellanic Cloud: Observed and predicted magnitudes*

Star	log P	A_V	Observed			Predicted			Residuals (O−C)			Predicted	
			M	m	V	M	m	V	M	m	V		(O−C)
W 44	0.408	0.80	15.75	16.55	16.21	15.78	16.57	16.21	−0.03	−0.02	0.00	16.33	+0.03
HV 5541 = W 24	0.429	1.20	15.40	16.60	16.06	15.41	16.60	16.03	−0.01	0.00	+0.03	16.26	−0.05
HV 12225 = W 33	0.478	0.50	15.89	16.39	16.06	15.81	16.31	16.09	+0.08	+0.08	−0.03	16.11	+0.10
W 25	0.529	0.52	15.55	16.07	15.88	15.65	16.17	15.94	−0.10	−0.10	−0.06	15.95	+0.08
HV 12747 = W 10	0.566	0.37	15.33	15.70	15.61	15.69	16.06	15.90	−0.36	−0.36	−0.29	15.87	−0.11
HV 12226 = W 29	0.569	0.54	15.41	15.95	15.65	15.52	16.06	15.82	−0.11	−0.11	−0.17	15.83	−0.03
W 22	0.669	0.82	14.98	15.80	15.40	15.02	15.84	15.44	−0.04	−0.04	−0.04	15.52	+0.03
(HV 2432)	1.038	0.74	13.72	14.46	14.08	14.04	14.77	14.38	−0.32	−0.31	−0.30	14.39	−0.16
HV 886	1.380	1.28	12.47	13.75	13.15	12.65	13.92	13.22	−0.18	−0.17	−0.07	13.34	−0.04
HV 1003	1.387	1.05	12.55	13.60	13.10	12.81	13.85	13.27	−0.26	−0.25	−0.17	13.32	−0.07
HV 902	1.421	1.27	12.40	13.67	13.07	12.54	13.80	13.10	−0.14	−0.13	−0.03	13.22	0.00
HV 2251	1.447	1.25	12.45	13.70	12.95	12.49	13.72	13.04	−0.04	−0.02	−0.09	13.13	−0.03
HV 1002	1.483	1.46	12.07	13.53	12.77	12.22	13.67	12.87	−0.15	−0.14	−0.10	13.02	−0.10
(HV 2294)	1.564	1.10	11.95	13.05	12.54	12.27	13.36	12.74	−0.32	−0.32	−0.20	12.78	−0.09
(HV 909)	1.575	0.96	12.07	12.99	12.56	12.37	13.29	12.76	−0.30	−0.30	−0.20	12.74	−0.03
HV 900	1.677	1.02	12.17	13.19	12.61	12.01	13.02	12.43	+0.16	+0.17	+0.18	12.43	+0.33
HV 953	1.680	1.00	11.60	12.60	12.12	12.02	13.00	12.43	−0.42	−0.40	−0.31	12.42	−0.15
HV 2369	1.684	1.08	11.98	13.06	12.42	11.94	13.01	12.39	+0.04	+0.04	+0.03	12.41	+0.16
HV 2447	2.074	0.64	11.55	12.19	11.82	11.17	11.81	11.38	+0.38	+0.38	+0.44	11.21	+0.76
HV 883	2.127	1.25	11.47	12.72	11.97	10.55	11.79	11.04	+0.92	+0.93	+0.93	11.05	+1.05

Table 6.—*Small Magellanic Cloud: Observed and predicted magnitudes*

Star	log P	A_V	Observed			Predicted			Residuals (O-C)			Predicted	
			M	m	V	M	m	V	M	m	V		(O-C)
HV 1897	0.094	0.54	16.69	17.54	16.71	17.24	17.77	17.55	-0.55	-0.34	-0.84	17.46	-0.69
HV 1871	0.114	0.80	16.70	17.57	17.10	16.98	17.77	17.41	-0.28	-0.20	-0.31	17.40	-0.26
HV 1907	0.216	1.33	16.17	17.50	16.86	16.28	17.80	18.96	-0.11	-0.10	-0.10	17.11	-0.19
HV 1779	0.251	0.60	15.80	16.63	16.04	16 74	17.34	17.11	-0.94	-0.71	-1.07	17.01	-0.91
HV 1869	0.391	0.64	16.69	17.34	16.98	16.32	16.95	16.64	+0.37	+0.39	+0.34	16.60	+0.44
HV 11114	0.433	1.22	15.86	17.00	16.44	15.75	16.96	16.35	+0.11	+0.04	+0.09	16.48	+0.02
HV 2015	0.458	0.82	16.00	16.85	16.37	15.99	16.80	16.40	+0.01	+0.05	-0.03	16.41	+0.02
HV 1906	0.486	1.07	15.66	16.80	16.21	15.72	16.78	16.24	-0.06	+0.02	-0.02	16.33	-0.06
HV 11216	0.494	1.03	15.72	16.74	16.21	15.72	16.74	16.23	0.00	0.00	-0.02	18.31	-0.04
HV 11113	0.507	0.96	15.94	16.89	16.46	15.74	16.69	16.21	+0.20	+0.20	+0.25	16.27	+0.25
HV 212	0.591	0.93	15.33	16.31	15.79	15.52	16.44	15.97	-0.19	-0.13	-0.18	16.03	-0.18
HV 214	0.624	1.08	15.03	16.07	15.61	15.32	16.38	15.83	-0.29	-0.31	-0.22	15.93	-0.26
(HV 1425)	0.658	0.91	15.38	16.29	15.87	15.35	16.25	15.78	+0.03	+0.04	+0.09	15.84	+0.09
HV 1492	0.799	0.87	14.86	15.64	15.21	14.98	15.84	15.38	-0.12	-0.20	-0.17	15.43	-0.16
HV 1400	0.823	0.84	15.17	16.00	15.46	14.94	15.77	15.32	+0.23	+0.23	+0.14	15.36	+0.16
HV 11112	0.826	0.71	15.30	16.00	15.59	15.03	15.73	15.35	+0.27	+0.27	+0.24	15.35	+0.30
HV 827	1.129	0 96	13.98	14.88	14.39	13.97	14.92	14.39	+0.01	-0.04	0.00	14.48	-0.03
(HV 1328)	1.200	0.68	13.70	14.38	14.07	13.99	14.66	14.26	-0.29	-0.29	-0.19	14.28	-0.15
(HV 1342)	1.254	0.78	13.93	14.71	14.27	13.76	14.53	14.07	+0.17	+0.18	+0.20	14.12	+0.21
HV 817	1.276	0.86	13.30	14.10	13.76	13.63	14.48	13.99	-0.33	-0.38	-0.23	14.06	-0.24
HV 823	1.504	1.22	13.19	14.39	13.83	12.71	13.92	13.21	+0.35	+0.47	+0.62	13.40	+0.43
HV 2195	1.621	1.06	12.46	13.51	12.92	12.50	13.55	12.92	-0.04	-0.04	0.00	13.06	-0.08
HV 837	1.629	1.05	12.86	13.70	13.22	12.48	13.52	12.90	+0.35	+0.18	+0.32	13.04	+0.24
HV 824	1.818	0.97	11.97	12.94	12.29	12.01	12.97	12.37	-0.04	-0.03	-0.08	12.50	-0.15
HV 834	1.866	0.66	11.83		12.11	12.11		12.32	-0.27		-0.21	12.36	-0.19
HV 829	1.942	0.73	11.63	12.32	11.91	11.84	12.56	12.07	-0.21	-0.24	-0.16	12.14	-0.17
HV 821	2.104	0.58	11.53	12.11	11.86	11.50	12.07	11.64	+0.03	+0.04	+0.22	11.67	-0.27

in zero point. The average residual for our predictions in V, without regard to sign, is $0.^m18$ for each of the Clouds. The average residuals from Gascoigne's formulas are $0.^m17$ for each of the Clouds. Thus, we have represented the data very nearly as well by using a single period-amplitude-magnitude relation for both Clouds as Gascoigne has done by using two separately determined period-magnitude relations.

If these relationships are applicable to galactic Cepheids, we can refine the color excesses and luminosities and rediscuss the galactic distribution. The *practical* advantage of our formulas is that they do not require a knowledge of intrinsic color. The *theoretical* advantage of a period-luminosity-color relation is counterbalanced by the fact that its application requires a knowledge of color excess.

Our success in representing the ⟨V⟩ magnitudes of the Cepheids in the two Clouds by the same period-amplitude relation suggests that, in spite of the differences of period-frequency, amplitude, and color, the period-luminosity relations for the Cepheids in the two Clouds are sensibly the same, at least as regards dependence on period and amplitude. It seems legitimate to consider them parallel; but without an independent check on the moduli, we cannot be sure that they are coincident. When we recollect the period-amplitude-color relations of the two Clouds and the Galaxy, discussed in the previous section, we must admit that this restriction holds with even greater force to any application of a period-luminosity relation derived from Magellanic Cepheids to a discussion of galactic Cepheids. In what follows, we shall *assume* that the period-amplitude-luminosity relation for the Galaxy has the same form as equations (10), (11), and (12). We can then determine its zero point by reference to galactic Cepheids for application to the Galaxy.

For this purpose, we follow the classical procedure of using the color excesses and distance moduli of the open clusters in which Cepheids have been recognized. Two selections of data, which lead to almost identical results, are represented in Tables 7 and 8. Although we do not consider that the four Cepheids assigned by Sandage and Tammann (1968) to the Perseus Association are *temporally* associated with either the double cluster or the Association (and the assumption that they are *spatially* associated rests on the ad hoc assumption that they are at the same distance), their inclusion makes little difference to the final result. We have assumed that the color excess of the Cepheid is the same as that of the cluster in which it is situated. The only star for which our discussion of color excess (under "The Period-Color Relation") might require a modification of this assumption is RS Puppis.

Table 7 contains material for deriving the zero point from the photometric moduli and color excesses given by Johnson et al. (1961) for the relevant clusters. Successive columns give the name of the Cepheid; the logarithm of its period; apparent V magnitude at maximum; ⟨V⟩ averaged for mean intensity; V amplitude; cluster containing the Cepheid; color excess and true modulus of the cluster as given by Johnson et al.; absolute magnitudes at maximum and mean light, derived from columns 3, 4, 7, and 8; values of $-2.840 \log P - 0.773A$ (equation (10)) and of $-2.930 \log P - 0.292A$ (equation (12)); and the arithmetic differences between the entires in columns 9 and 11 and in 10 and 12. The means of these differences should furnish numerical values for the first terms of the galactic equivalents of equations (10) and (12).

Table 8 contains similar material for deriving the zero point from the data used by Sandage and Tammann (1969). Their list contains more stars, and their adopted values for color excess and modulus are not the same as those of Table 7. Despite these differences, the mean values of the entries in columns 13 and 14 are nearly the

TABLE 7.—*Zero point of period-amplitude-luminosity relation (Johnson data)*

1	2	3	4	5	6	7	8	9	10	11	12	13	14
		Apparent V Magnitude				Johnson et al.		Absolute Magnitude				Difference	
Star	log P	Max.	Mean	Amp.	Cluster	E_{B-V}	$(m-M)_0$	Max.	Mean	$-2.840 \log P$ $-0.773 A_V$	$-2.930 \log P$ $-0.292 A_V$	Max.	Mean
EV Sct	0.490	10.00	10.12	0.28	NGC 6664	0.60	11.03	-2.83	-2.71	-1.62	-1.52	1.21	1.19
CE Cas b	0.651	10.61	10.99	0.68	NGC 7790	0.52	12.80	-3.75	-3.37	-2.38	-2.00	1 37	1.37
CF Cas	0.687	10.82	11.12	0.57	NGC 7790	0.52	12.80	-3.54	-3.24	-2.38	-2.17	1.16	1.07
CE Cas a	0.711	10.62	10.92	0.52	NGC 7790	0.52	12.80	-3.73	-3.44	-2.42	-2.15	1.31	1.29
U Sgr	0.828	6.32	6.76	0.78	M 25	0.50	8.9	-4.08	-3.64	-2.96	-2.65	1.12	0.99
DL Cas	0.903	8.64	8.96	0.60	NGC 129	0.58	11.1	-4.20	-3.88	-3.03	-2.84	1.17	1.04
RS Pup*	1.617	6.52	7.08	1.12	Vela III	0.62	11.30	-6.44	-5.98	-5.44	-5.06	1.00	0.92
											Means:	1.19	1.12

*Data from Westerlund (1963).

TABLE 8.—*Zero point of period-amplitude-luminosity relation (Sandage and Tammann data)*

1	2	3	4	5	6	7	8	9	10	11	12	13	14
		Apparent V Magnitude						Absolute Magnitude				Difference	
Star	log P	Max.	Mean	Amp.	Cluster Adopted	E_{B-V}	$(m-M)_0$	Max.	Mean	$-2.840 \log P$ $-0.773 A_V$	$-2.930 \log P$ $-0.292 A_V$	Max.	Mean
SU Cas	0.290	5.76	5.95	0.40		0.33	7.5	-2.75	-2.54	-1.62	-1.46	1.13	1.08
EV Sct	0.490	10.00	10.12	0.28	NGC 6664	0.58	11.03	-2.77	-2.62	-1.62	-1.52	1.15	1.10
CE Cas b*	0.651	10.61	10.99	0.68	NGC 7790	0.555	12.53	-3.585	-3.205	-2.38	-2.00	1.20	1.21
CF Cas	0.687	10.82	11.12	0.57	NGC 7790	0.555	12.53	-3.375	-3.075	-2.38	-2.17	1.00	0.91
CE Cas a*	0.711	10.63	10.92	0.52	NGC 7790	0.555	12.53	-3.565	-3.275	-2.42	-2.15	1.14	1.13
UY Per	0.730	10.83	11.34	0.91	h, χ Per	0.98	11.90	-4.00	-3.54	-2.78	-2.40	1.22	1.14
VY Per	0.743	10.76	11.19	0.79	h, χ Per	1.06	11.90	-4.30	-3.91	-2.72	-2.41	1.58	1.50
U Sgr	0.828	6.32	6.76	0.78	M 25	0.55	8.98	-4.27	-3.92	-2.96	-2.65	1.31	1.28
DL Cas	0.903	8.64	8.96	0.80	NGC 129	0.50	11.28	-4.09	-3.84	-3.03	-2.84	1.06	1.00
S Nor	0.989	6.11	6.46	0.65	NGC 6087	0.23	9.76	-4.34	-4.03	-3.32	-3.08	1.02	0.95
VX Per	1.037	8.97	9.33	0.75	h, χ Per	0.75	11.90	-4.68	-4.34	-3.52	-3.26	1.16	1.08
SZ Cas	1.134	9.60	9.83	0.40	h, χ Per	0.88	11.90	-4.93	-4.71	-3.53	-3.44	1.40	1.27
RS Pup	1.617	6.52	7.08	1.12	Vela III	0.55:	11.30	-6.41	-5.95	-5.44	-5.06	0.99	0.89
											Means:	1.18	1.12

*Data from Sandage and Tammann (1969).

same for both choices of data. The fact that the numbers in each of these columns in Tables 7 and 8 are so nearly the same for all the stars is an admittedly rough verification of our assumption that a period-amplitude-luminosity relation derived from the Clouds only is also applicable to galactic Cepheids.

If this assumption is adopted, we obtain for galactic Cepheids, by combining the results from Tables 7 and 8,

$$M_{\langle V\rangle} = -1.12 - 2.930 \log P - 0.292A, \qquad (15)$$
$$M_{max} = -1.185 - 2.840 \log P - 0.774A. \qquad (16)$$

Equation (15) invites comparison with the period-luminosity-color relation derived by Gascoigne for Magellanic Cepheids:

$$M_{\langle V\rangle} = -2.33 - 3.609 \log P + 2.52\,(B-V). \qquad (17)$$

We arrive at a comparison by means of our relation (equation (12)) and Gascoigne's formulas (equations (18) and (19)) for the two Clouds:

$$\text{Large Cloud: } M_{\langle V\rangle_0} = 17.64 - 2.930 \log P - 0.292A, \qquad (12)$$
$$= 16.40 - 3.609 \log P + 2.52\,(B-V); \qquad (18)$$
$$\text{Small Cloud: } M_{\langle V\rangle_0} = 17.98 - 2.930 \log P - 0.292A, \qquad (12)$$
$$= 17.03 - 3.609 \log P + 2.52\,(B-V). \qquad (19)$$

We can relate our formulations with Gascoigne's by expressing A in terms of (B−V) by means of equation (1):

Maximum:

$$V_0 = \text{const.} - 3.640 \log P + 2.74\,(B-V)_{max}, \qquad (20)$$

Minimum:

$$V_0 = \text{const.} - 3.222 \log P + 1.17\,(B-V)_{min}. \qquad (21)$$

The coefficients of log P and of (B−V) in equation (17) should lie between those of equations (20) and (21), and it is seen that they do so. Our relations are therefore not inconsistent with Gascoigne's.

The period-amplitude-luminosity relation and the period-color-luminosity relation are in fact complementary. The former states that for a given period the stars of greatest amplitude are the brightest; the latter, that for a given period the bluest stars are the brightest. The coefficient of log P in equation (12) expresses a period-luminosity relation for zero amplitude; those of equations (17), (18), and (19) express a period-luminosity relation for zero (B−V). The slope of the period-luminosity relation for ⟨V⟩ light therefore lies between 3.609 and 2.930.

If we consider that the zero point of the period-luminosity relation is the same for the Galaxy and both the Clouds, we can use equations (12) and (15) to determine distance moduli for the latter. Thus, we obtain moduli of 18.77 and 19.10 for the Large and Small Clouds. From equations (17), (18), and (19), Gascoigne has derived the values 18.73 and 19.36. It should be emphasized that there is nothing in our data to justify the assumption that the period-luminosity relations in the three systems are coincident. Because of the similarity of the deduced relationships for the two Clouds (except for the difference of apparent zero point), we have argued that they are probably parallel. But all the differences between the Cepheids in the three systems (period-frequency, amplitude, and color, for example) require us to leave the coincidence of the absolute zero point an open question.

We have determined a true modulus of 18.77 for the Large Cloud, but this value was derived via the galactic Cepheids, on the assumption of coincident period-luminosity relations. An independent method of determining the distance modulus is required. A more direct determination of the moduli of the Clouds, based on the eclipsing stars, has been made by Gaposhkin (1972). His values, 18.36 for the Large Cloud and 19.15 for the Small, when compared with the values 18.77 and 19.10 given above, imply differences in zero point of $0.^m41$ and $0.^m05$ for the two Clouds, the Large Cloud being nearer and the Small Cloud a little more distant than our own moduli imply. It would of course be better to use a direct method of determining the distance, such as that involving the eclipsing stars, than to assume that the period-luminosity relation has everywhere the same zero point, even though the form of the relationship has been shown to be the same everywhere.

In summary, the Cepheids of the Galaxy and the Magellanic Clouds display a gradation in properties, but when amplitude as well as period is taken into account, their intrinsic colors are sensibly similar. The period-luminosity relations in the three systems are probably parallel, but not necessarily coincident.

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