

Designing Nelder wheel plots for tree density experiments

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Abstract The Nelder (Biometrics 18:283–307, 1962) wheel design allows a researcher to test multiple tree densities in a single plot. Because spatial relationships among planted trees are fundamental to a Nelder wheel, a researcher needs a specific set of layout parameters to establish a Nelder plot. While Nelder (Biometrics 18:283–307, 1962) provides calculus-based equations for determining the required layout parameters, the presentation focuses on derivation of these equations and not their application to forestry research. Other authors have outlined the design of Nelder plots for forestry research, but have done so using trigonometry-based equations. Existence of two layout methodologies in the literature is a source of confusion. In this paper, we present a straightforward means to determine the design parameters critical to the establishment of Nelder plots used within tree density research. The layout equations presented are expressed in terms that allow applied forestry researchers to easily answer the following question. Given the number and range of tree densities I want to evaluate, what are the required Nelder wheel layout parameters? Finally, we provide a step-by-step example of the design and installation of a Nelder plot for a scenario familiar to tree density research and discuss analysis of Nelder wheel experiments.

Keywords Experimental design · Tree spacing · Stand density · Tree growth

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Introduction

Nelder (1962) presented an experimental design that can be used to test multiple tree spacings in a single plot. The Nelder wheel design is a circular plot containing concentric rings radiating outward with spokes connecting the center with the furthest ring. At the intersections of spokes and arcs, a tree is planted (Fig. 1). This creates variable tree densities across the length of the spokes within a single plot and eliminates the need for separate experimental plots for each tree density.

Nelder (1962) introduced his design as a way to overcome the plant material and space restrictions of replicated, full factorial spacing experiments. Nelder (1962) presented four systematic circular designs that differ in the change of shape and potential growing space for trees within and between each spoke. For this paper, we will address the design that maintains a fixed shape of space between trees that increases with radius length (i.e., Design Ia). This design is easiest to establish and has received the most attention in the literature.

One fundamental difference between a replicated, full factorial design and Nelder wheel design is the level of the experimental unit and replication. The experimental unit of a full factorial design is typically a plot representing a given tree density, while the experimental unit in a Nelder design is an individual tree. As a result, researchers use regression techniques to analyze Nelder wheel studies (Imada et al. 1997; Mabvurira and Miina 2002; Aphalo and Rikala 2006). Regression models typically relate the potential growing area (i.e., amount of ground area between trees) of each tree to growth rates or tree architecture.

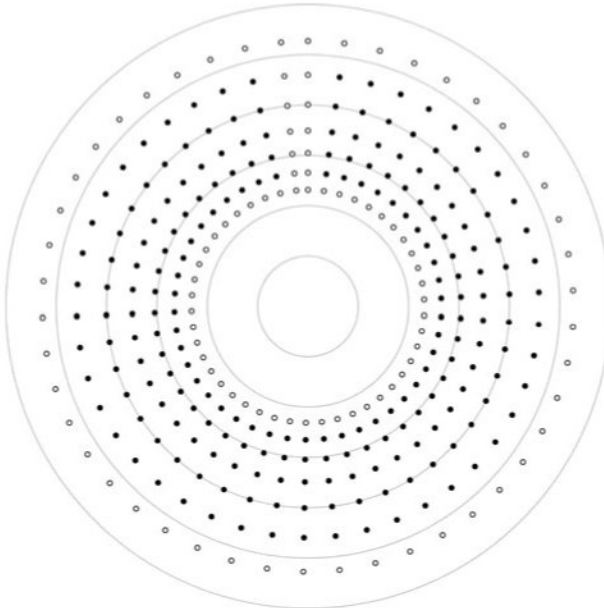


Fig. 1 Depiction of a Nelder wheel plot design that maintains a fixed shape of space between plantings that increases with radial length. Plot layout is based upon an example that includes five different tree densities between 1,000 and 3,000 trees ha^{-1} . *Black circles* represent trees within the experimental arcs, while *white circles* represent trees within border arcs and spokes. Concentric *circles* illustrate the radial distance from plot center in 5 m intervals and are included to show the scale of the figure

Given the spatially explicit nature of Nelder designs, regression analysis should utilize statistical models that account for spatial autocorrelation (Affleck 2001; Fox et al. 2001).

One primary drawback of the Nelder wheel design is the sensitivity of the analysis to tree mortality. Hall (1994) recommends that seedling survival should be a primary concern when establishing Nelder plantings. Managers should consider double plantings, intensive weed control, and other site preparation methods to increase survival. It is also suggested that trees adjacent to dead stems should be removed from the analysis (Mark 1983; Stape and Binkley 2010).

Because spatial relationships among trees are fundamental to the Nelder wheel, a specific set of layout parameters are needed to test a given range of tree densities. While information needed to create a Nelder wheel was given by Nelder (1962), the presentation focused on the derivation of the calculus-based layout equations and not on their practical application in the design of tree density trials. Bleasdale (1967) presented a practical application of Nelder's (1962) calculus-based equations in the context of row crop research. Other authors have discussed the layout and application of Nelder wheels in forestry research, but have done so using trigonometry-based equations (Namkoong 1966). In a theoretical review of the applications of the Nelder designs in tree improvement, Namkoong (1966) offered numerous tables with pre-fabricated designs. However, the equations provided for custom designs lacked clarity and were constructed using trigonometry-based equations that differed from Nelder's original calculus-based equations. Later, Mark (1983) provided a separate trigonometry-based approach to Nelder wheel design.

The existence of differing derivations of the layout equations is a source of confusion. Several papers report experimental results and detailed descriptions of their planting densities, number of arcs, planting intervals, and angle between spokes, but do not present the means by which they derived the layout parameters (Imada et al. 1997; Schlönvoigt and Beer 2001; Redmond et al. 2005; Geyer 2006; Ritchie et al. 2007; Waghorn et al. 2007). The uncertainty as to which equations were used to construct a given design makes charts describing predesigned Nelder wheel plots such as those presented in Redmond et al. (2005) difficult to utilize as a basis to develop one's own design.

Since few studies have offered detailed information on plot layout parameters and even fewer have provided the methods used to design the wheels, there is a need in the current literature to present a practical methodology for developing Nelder designs and an efficient approach for establishing Nelder plots with minimal labor and costs. The intent of this paper is to: (1) translate Nelder's (1962) design equations for practical field application in forestry, (2) present an approach for designing Nelder wheel plots that is accessible to applied forestry researchers, (3) provide an annotated example of all the steps involved in Nelder wheel layout and field installation, and (4) discuss analysis techniques appropriate given the spatially correlated nature of data within Nelder designs. While two forms of Nelder wheel design equations (e.g., calculus-based versus trigonometry-based) are present in the literature, we chose to utilize the original calculus-based equations (Nelder 1962) in this paper. Trigonometry-based equations assume that the growing space shape within a Nelder wheel is trapezoidal in nature when in fact the inner and outer borders are arcs. Thus, from a mathematical perspective, calculus-based equations more accurately represent the shape of growing space in Nelder designs. One could debate whether the difference between the calculus-based and trigonometry-based methods is significant enough to affect the outcome of tree density experiments using Nelder plots. While the effect of this difference could be determined using a field experiment to compare the two design methods, such work is outside the scope of our paper.

Methodology

Precise spatial distribution of trees in a Nelder wheel allows multiple tree densities in a single plot (Fig. 1). Specific design parameters are required to achieve the correct tree arrangement. These parameters include: (1) minimum and maximum tree densities to be tested, (2) number of densities to be tested including the minimum and maximum density, (3) “rectangularity” proportion, (4) rate of change along planting spokes (radii), (5) angle between spokes, and (6) distance to first planting arc. Prior to designing a Nelder wheel experiment, the researcher must define the range of densities to be tested and the number of densities within this range. Due to the typical analysis used for a Nelder wheel study, the hypothesized “ideal” density should fall somewhere in the middle of the chosen range. The researcher must also specify a desired proportional relationship between the arc length between spokes and the radial length between arcs where the numerator represents the arc length and the denominator represents radial distance (Nelder 1962). This proportion has been referred to as “rectangularity” in the historical literature and it remains constant throughout the design. Given that the inner and outer borders of the growing space shape surrounding a tree in a Nelder design are arcs and that the shape is not truly rectangular or trapezoidal in nature, the term “rectangularity” may be confusing. However, we retain the use of this term in our manuscript to be consistent with terminology usage in other work on Nelder designs. The remaining parameters are then calculated using layout equations that utilize the aforementioned factors.

The original design equations presented by Nelder (1962) were expressed in terms of available growing area. However, when considering a methodology for applied forestry research, it seemed practical to express the design equations in terms of a variable commonly used in the description of forest management practices such as tree density. Thus, the initial step to constructing an intuitive methodology for design of Nelder wheel plots was to put the equations in terms of minimum and maximum tree densities to be tested. In addition, we present the equations in a way that follows the logical order of designing an experiment. We chose to work with the original equations presented in Nelder (1962).

The first design step determines the rate of change in density along the spokes. To calculate this rate of change (α), we expressed the original Nelder (1962) equation in terms of tree density. The resulting equation allows for the quick insertion of desired minimum and maximum tree densities and the number of densities to observe within this range to calculate the rate of change along the spokes (α) as follows:

$$\alpha = \exp\left(\frac{\log D_1 - \log D_N}{2N-2}\right) \quad (1)$$

where D_1 = tree density at first experimental arc (trees ha^{-1}) (i.e., upper extreme of experimental tree density range), D_N = tree density at the last experimental arc (trees ha^{-1}) (i.e., lower extreme of experimental tree density range), N = number of experimental arcs (i.e., number of densities within the specified range to be tested), \exp is the exponential function, and \log is the natural logarithm. Nelder (1962) points out that planting locations along arcs are not in the exact center of the potential growing space available to a plant and that rapid increases in the space between arcs can result in significantly disproportionate distribution of space around a plant. The measure of this deviation from the center point is termed non-centrality and its calculation was presented in Affleck (2001) as follows:

$$C_o = \frac{2 - (\alpha + \alpha^{-1})}{2(\alpha - \alpha^{-1})} \times 100\% \quad (2)$$

where C_o = non-centrality percentage and α = rate of change along spokes. Nelder (1962) suggests that when dealing with row crops, this value should not exceed 5%, which restricts α to a limit of 1.1. However, as Namkoong (1966) suggests, issues regarding non-centrality and high rate of change (α) values are reduced in tree spacing trials since trees have more time to grow and adjust to available space than do row crops. Namkoong (1966) sets maximum C_o at 25% in his Nelder wheel templates; however, the allowable amount of non-centrality is at the discretion of the experimenter and should be based on growth patterns of species and the plant characteristics under investigation.

Once the rate of change parameter (α) had been calculated, the required angle between the Nelder wheel spokes (θ) could be determined. The interspoke angle was determined using Eq. 2, which required the rate of change parameter (α) and the desired “rectangularity” proportion (τ).

$$\theta = \tau \left(\alpha^{\frac{1}{2}} - \alpha^{-\frac{1}{2}} \right) \quad (3)$$

where θ = angle between spokes in radians, τ = rectangularity of growing area (arc length/radial length), and α = rate of change along spokes. An alternative to calculating angle between spokes (θ) using the rate of change (α) parameter and a specified “rectangularity” is to develop a design by assigning an angle between spokes (θ) that will evenly fit into a complete circle. While this will eliminate the need for border rows, it leaves the “rectangularity” out of the control of the experimenter and could lead to extreme “rectangularity” values. Extreme values of “rectangularity” can cause bias by creating an unreasonably asymmetric arrangement of space around trees, similar to an extreme rate of change parameter. Like with potential bias associated with non-centrality and rate of change, issues with “rectangularity” are a smaller concern for trees than for row crops (Namkoong 1966). Existing tree plantation research suggests rectangularity of spacing up to 2–3 times has only minimal influence on tree growth patterns (West 2006). Ultimately, it is up to the experimenter to maintain an appropriate “rectangularity” based on growth patterns of tree species studied and practical plantation arrangements.

The final step was to calculate the distance to the first planting arc. Determining this distance is important as the first arc serves as the inner border row and is the starting point for all successive planting locations within a spoke. The Nelder (1962) design equation for this parameter included an undefined variable. However, Bleasdale (1967) determined that this variable represented the growing area at the first experimental arc. When expressed in terms of tree density, the distance to the initial planting arc (r_0) can be determined using Eq. 4.

$$r_0 = \sqrt{\frac{20,000}{D_1 \theta (\alpha^3 - \alpha)}} \quad (4)$$

where D_1 = tree density at first experimental arc (trees ha⁻¹) (i.e., upper extreme of experimental tree density range), θ = angle between spokes in radians, and α = rate of change along spokes.

Once obtained, the initial radius (r_0) and rate of change along spokes (α) are used to calculate radii of subsequent arcs (r_i).

$$r_i = r_0 \times \alpha^i \quad (5)$$

where r_i = radius of the i th arc in meters, r_0 = distance to the initial planting arc in meters, α^i = rate of change from the initial arc to the i th arc along the Nelder spokes, and i includes arcs 0, 1, 2, ..., N , $N + 1$ where N is number of experimental arcs. It is necessary to calculate radii for one additional arc beyond the last experimental arc since the final arc acts as an outside buffer. Growing space for a tree within a given experimental arc can be calculated as (Nelder 1962):

$$\text{Growing space (m}^2\text{)} = \frac{\theta(r_i)^2(\alpha - \alpha^{-1})}{2} \quad (6)$$

where θ = angle between spokes in radians, r_i = radius of the i th arc in meters, and α = rate of change along spokes. Tree density for any experimental arc can be calculated directly or by using the growing space available at a given arc (Nelder 1962).

$$\begin{aligned} \text{Tree density (trees ha}^{-1}\text{)} &= \frac{20,000}{\theta(r_i)^2(\alpha - \alpha^{-1})} \\ \text{Tree density (trees ha}^{-1}\text{)} &= \frac{10,000}{\text{Growing space (m}^2\text{)}} \end{aligned} \quad (7)$$

where θ = angle between spokes in radians, r_i = radius of the i th arc in meters, and α = rate of change along spokes. The land area required for a full Nelder wheel plot and the maximum number of planting spokes possible was calculated using the equations shown below.

$$\text{Nelder plot area (ha)} = \frac{\pi(r_{N+1})^2}{10,000} \quad (8)$$

$$\text{Number of spokes possible in Nelder plot} = \left\lceil \frac{2\pi}{\theta} \right\rceil \quad (9)$$

where r_{N+1} = radial length of outer border arc in meters and θ = angle between spokes in radians. The number of spokes dictates the number of replicates for each tree density within a plot; however, the first and last spoke are used as border rows and are not be included as an experimental replicate.

Example implementation

To assist in the application of the practical methodology presented in this paper, we provide an example of how to design and layout a Nelder wheel plot. Our example includes five different densities within a range of 1,000–3,000 trees ha⁻¹. By applying Eq. 1, we find that the rate of change along the planting spokes (α) for the example is 1.1472. In the example, we used a 1:1 ratio of the radial length and the arc distance between spokes and, therefore, a “rectangularity” of 1. Using Eq. 3, the angle between spokes (θ) was found to be 0.1374 radians (7.87°). Equation 4 was used to establish that the first radius (r_0) was 11.57 m. Table 1 summarizes the calculation steps for the Nelder wheel layout parameters in this example.

With the layout parameters determined, we then calculated the subsequent six radii (including the radius for the outside border arc) using Eq. 5 and tree densities for the arcs

Table 1 Summary of steps required to calculate layout parameters for an example Nelder wheel plot

Determining Nelder wheel layout parameters: an example

Preliminary step: determine number and range of tree densities to test and the degree of “rectangularity” (τ) to use

For the example:

- Number of densities tested (N) = 5
- Minimum density (D_N) = 1000 trees ha⁻¹
- Maximum density (D_1) = 3000 trees ha⁻¹
- “Rectangularity” = 1

Step 1: Rate of change along planting spokes (α) (Eq. 1)

$$\alpha = \exp\left(\frac{\log(3000) - \log(1000)}{2(5) - 2}\right)$$

$$\alpha = 1.1472$$

Step 2: Angle between spokes (θ) (Eq. 2)

$$\theta = (1)\left((1.1472)^{\frac{1}{2}} - (1.1472)^{-\frac{1}{2}}\right)$$

$$\theta = 0.1374 \text{ radians}$$

Step 3: Distance to first arc (r_0) (Eq. 3)

$$r_0 = \sqrt{\frac{20000}{(3000)(0.1374)\left((1.1472)^3 - 1.1472\right)}}$$

$$r_0 = 11.57 \text{ m}$$

Table 2 Radial length and associated tree density and potential growing space present along each spoke of the example Nelder wheel plot depicted in Fig. 1 whose layout parameters are given in Table 1

Radius (r_i)	Radial length (m) ^a	Tree density (ha ⁻¹) ^b	Growing space (m ²) ^c
r_0	11.57	– ^d	–
r_1	13.27	3,000.00	3.33
r_2	15.22	2,279.51	4.39
r_3	17.46	1,732.05	5.77
r_4	20.03	1,316.07	7.60
r_5	22.98	1,000.00	10.00
r_6	26.37	–	–

^a Equation 5: $r_i = r_0 \times \alpha^i$

^b Equation 7: Tree density (trees ha⁻¹) = $\frac{20,000}{\theta(r_i)^2(\alpha - \alpha^{-1})}$

^c Equation 6: Growing space (m²) = $\frac{\theta(r_i)^2(\alpha - \alpha^{-1})}{2}$

^d Trees at inner and outer most radii serve as border rows

crossing these radii using Eq. 7 (Table 2). Finally, Eqs. 8 and 9 were used to find that a complete circle of this design would require 0.218 ha and would provide 44 replicates of each density. The 44 replicates is a result of using the first and last spoke as buffer rows since θ does not even fit evenly into a circle; this will ensure the proper spacing for all experimental spokes. Using the calculated parameters, trees would be planted at the intersection of each planting spoke and arc (Fig. 1).

Nelder (1962) suggested using two wires, both with the length of each radius marked, for field implementation of the design. Using a third wire, which would connect the two

marked wires at the outer ends, a triangle would be created with the appropriate angle at the origin when the wires are pulled tight. Planting sites could be marked by leapfrogging this wire triangle around the origin.

While this method may be utilized, modern technology provides tools that can simplify this task. For the setup of our example Nelder wheel, we used a TruPulse 360 laser rangefinder with digital compass (Laser Technology, Inc, Centennial, CO), tripod, 200' fiberglass measuring tape, two stakes, and pin flags. The measuring tape was marked at the lengths of each radius in the design (Table 2). Our method did not require the use or construction of any other tools.

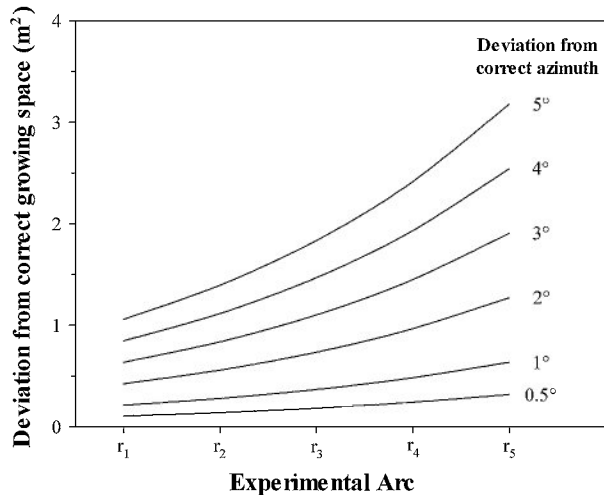
The methodology was simple. A stake was established at plot center and was used to secure the measuring tape. With the tape extended out to the length of the largest radius, the first spoke was laid out at an azimuth of 0°; although, any azimuth would have sufficed. A flag was placed at the end of each calculated radius where a tree would be planted along the spoke. After the first spoke was established, the next spoke was determined by adding the calculated angle between spokes, 7.87°, to the former azimuth. The calculations for each azimuth were performed prior to plot establishment in order to reduce field time. The next spoke was established by rotating the tape to the new azimuth determined with the TruPulse 360 digital compass; the TruPulse 360 was mounted on a leveled tripod to improve accuracy. We repeated this process for each planting spoke around the origin of the wheel for all 46 spokes. In our trial, this design of 322 trees was setup in approximately 53 min.

Statistical analysis of Nelder designs

As the ultimate utility of Nelder plots is in quantifying the relationship between tree density and tree growth, it is important to discuss the analysis of Nelder designs. While the spatially explicit nature of Nelder plots is a strength of the design, it can also create issues with the analysis. Tree mortality can be a crucial concern because if a tree dies this disrupts the spatial distribution of trees within a plot and changes the amount and distribution of growing space available to adjacent trees. Since density around each arc is dependent on the bordering trees, mortality removes the dead tree from the analysis and affects the neighboring trees. Mark (1983) and Stape and Binkley (2010) suggest that if a single tree dies the four bordering trees should be removed from analysis. In our example Nelder plot if trees experienced 20% mortality, it could result in an 80% reduction of the 220 sample trees. However, the exact number of sample trees affected would differ based upon the spatial distribution of mortality. This potential impact of mortality on the Nelder design stresses the need to minimize mortality and the importance of creating replicated Nelder designs when performing tree density trials.

Another factor that can alter the spatial distribution of trees within a Nelder plot is error associated with the layout of the plots in the field. When implementing our example Nelder design in the field, aligning spokes to the appropriate azimuth was the most difficult aspect of the setup process and would be the most likely source of layout error. However, error could also result from incorrect layout of interspoke tree spacing. Using the field methodology detailed in our example implementation, a probable errant scenario would be establishing an incorrect azimuth between two correct azimuths. This type of error in spoke alignment would result in inaccurate estimations of growing space and an altered distribution of space around the trees along the incorrect spoke. Deviation from anticipated area around the trees will increase as arcs move further out the spokes (Fig. 2). In our example,

Fig. 2 Deviation in growing space (m^2) resulting from a single spoke's departure from its correct azimuth by varying degrees in our example Nelder wheel plot



a 3° departure from the intended azimuth would have resulted in a 1.9 m^2 error in growing space at the outer experimental arc (Fig. 2). These errors are not limited to trees along one spoke, as trees on adjacent spokes would also be impacted. While as an isolated incident, this scenario may cause minimal error in the design. However, if multiple spokes deviate from their intended azimuth, these described errors would be compounded and could affect the validity of the planned tree density range for the plot.

The spatially explicit nature of Nelder designs also influence the appropriateness of statistical methods employed. Historical analysis of Nelder wheel experiments has typically used ordinary least squares (OLS) regression to model the relationship between tree density and tree growth (Krinard 1985; Faber 1991; Knowe and Hibbs 1996). However, Affleck (2001) compared the use of OLS regression and a suite of spatial analysis methods in analyzing Nelder wheel experiments. This work found that modeling techniques that accounted for spatial autocorrelation were more valid and efficient than analyses completed using OLS regression. Due to the hierarchical and spatially correlated nature of data from replicated Nelder wheel experiments, the use of mixed-effects models may be more appropriate than OLS regression for analyzing Nelder designs (Fox et al. 2001). Mixed models include fixed-effects that account for the relationships among dependent and independent variables for the population, while random-effects can account for variation associated with a sampling unit (i.e., Nelder plot) (West et al. 2007). Mixed models can also account for spatial correlation of observations within sampling units during parameter estimation (Pinheiro and Bates 2000; West et al. 2007). Therefore, mixed-effect regression simultaneously considers the nesting of observations within a Nelder plot and the exact spatial arrangement of those observations. By accounting for these factors during parameter estimation, the statistical analysis should yield a better understanding of the nature of the relationship between tree density and the growth parameters of interest.

Conclusions

The ambiguous presentation of the original formulas (Nelder 1962) and the variations in calculating parameters for a Nelder wheel design found in the literature (Namkoong 1966; Mark 1983) present a need for an accessible, straightforward methodology for tailoring a Nelder

wheel plot that tests a specific range and number of tree densities. By expressing Nelder wheel layout equations in terms of a tree density rather than area, calculations are based upon a variable common to applied forestry research. The design methodology presented will enable researchers and practitioners to easily develop customized Nelder wheel plots that can be utilized in an array of tree density trials. Finally, this work serves as a clear source regarding the design, layout, and analysis of Nelder wheel plots in applied forestry research.

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