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ON TWO PARAMETERS USED IN THE PHYSICAL THEORY OF METEORS

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On Two Parameters Used in the Physical Theory of Meteors[1]

by Luigi G. Jacchia[2]

The fragmentation index

The "fragmentation index" χ was introduced by the author in an earlier paper (Jacchia, 1955a) as a convenient measure of the progressive departure of the observed meteor deceleration from single-body theory. Accelerations $\dot{v}_T$ were computed from the drag equation

$$\dot{v}_T = K_1 \rho_T m^{-1/3} v^2, \tag{1}$$

with atmospheric densities ρ_T as given by the Rocket Panel Atmosphere (Rocket Panel, 1952) and masses, m, obtained from the integration of the light-intensity curve. When these theoretical accelerations were compared with the observed accelerations $\dot{v}_{obs}$, it was found that the ratio $\dot{v}_{obs}/\dot{v}_T$ increased for most meteors in the course of the trajectory; in the section of the meteor trajectory which yielded usable decelerations, the increase was nearly proportional to $(m_\infty - m)/m$; i. e., the ratio of the meteor's loss in mass from its original value m_∞, to its instantaneous value m. We therefore introduced a mass-loss parameter s defined as

$$s = \log_{10}\left(\frac{m_\infty}{m} - 1\right), \tag{2}$$

and defined the "fragmentation index" as

$$\chi = \frac{d}{ds} \log_{10} \frac{\dot{v}_{obs}}{\dot{v}_T}. \tag{3}$$

Strictly speaking, the instantaneous mass at the time t should be proportional to the integral, from t to ∞, of $Iv^{-n}dt$, where I is the instantaneous brightness of the meteor, and n is the power of the velocity, which may be different for meteors of different luminosity groups. In practice v varies only slightly, in percent, in the course of the trajectory, and we can replace the masses by the corresponding integrals of the luminosity curves. We can thus write

$$E = \int_t^\infty I dt; \; E_\infty = \int_{-\infty}^\infty I dt; \; s = \log_{10}\left(\frac{E_\infty}{E} - 1\right). \tag{4}$$

In this paper, I will represent the photographic intensity of the meteor, in units of the intensity of a zero-magnitude star.

A correlation between χ and meteor brightness was pointed out in an earlier paper (Jacchia, 1955a); later the question (Jacchia, 1955b) arose whether a maximum was observable for χ within the range of brightness of the Super-Schmidt cameras. As a measure of meteor brightness a parameter ϵ was used, defined as

$$\epsilon = \log_{10}\left(\frac{2}{\tau} E_\infty\right) = \log_{10} E_\infty + 18.49, \tag{5}$$

where τ is the adopted value of the coefficient of luminous efficiency for meteors. If a conversion to actual meteor magnitudes is desired, use can be made of the following empirical formula, which was derived from photographic meteors with smooth light curves:

$$E_\infty = 1.0 \times 10^6 \frac{I_{pm}^{\;1.093}}{v \cos Z_R}. \tag{6}$$

[1] Work carried out in part under U. S. Army Office of Ordnance Research Contract DA-19-020-ORD-2556, with Harvard University.
[2] Smithsonian Astrophysical Observatory; Harvard College Observatory.

Here I_{pm} is the photographic intensity of the meteor at maximum light, expressed in units of a zero-magnitude star, and Z_R is the zenith distance of the meteor radiant. The velocity v is expressed in cm/sec. The exponent of I_{pm} is larger than unity, which means that fainter meteors have relatively steeper light curves than bright meteors—a fact which was adduced by Jacchia (1955a) as further evidence of fragmentation. For meteors exhibiting flares the value of E_∞ is, of course, somewhat smaller than that computed from (6) if the peak magnitude reached in a flare is taken as I_{pm}.

Values of χ computed from decelerations are tabulated as a function of E in table 1 for a total of 420 meteors, 369 photographed with the Super-Schmidt cameras and 51 with small cameras. The data are plotted in figure 1.

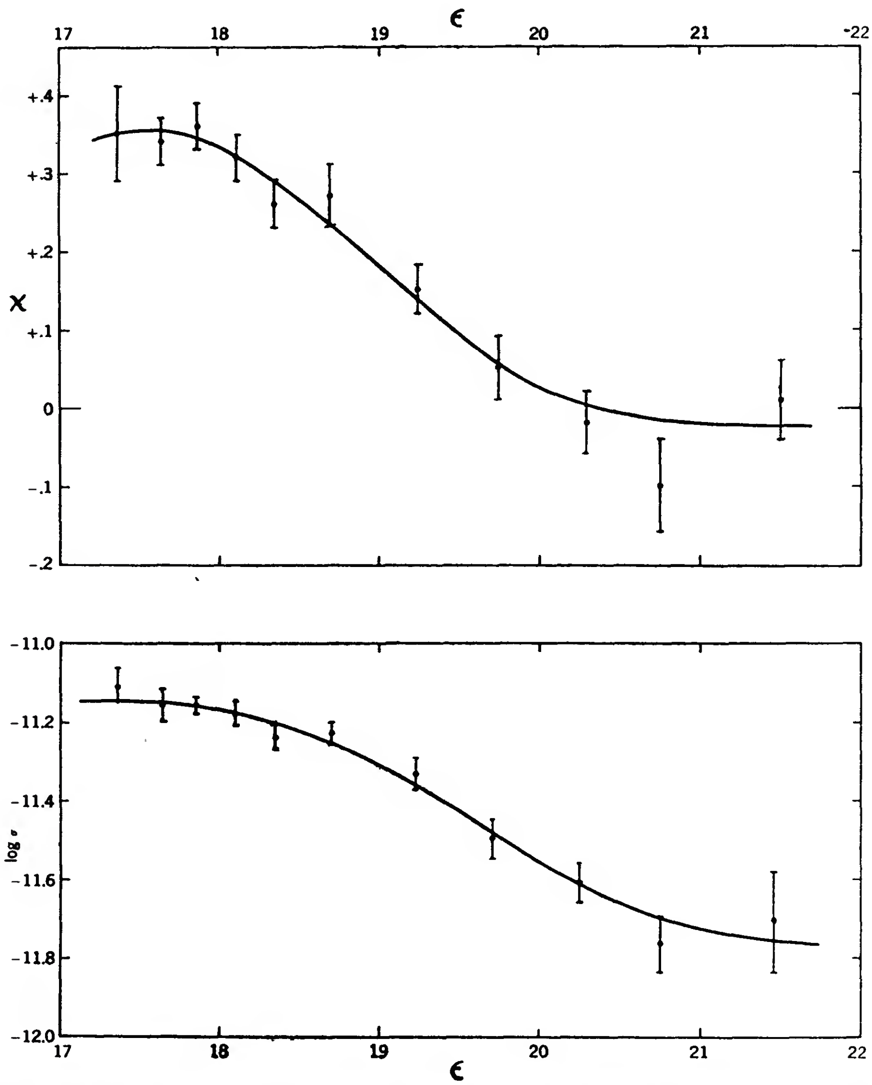

FIGURE 1. Mean values of χ and σ as a function of meteor brightness. In abscissae the parameter $\epsilon=\log_{10}E_\infty+18.49$, where E^∞ is the integrated photographic intensity of the meteor in units of a zero-magnitude star. The confidence marks represent standard deviations of the means.

TABLE 1.—*The fragmentation index χ as a function of the integrated meteor brightness ϵ.*

Integrated brightness, ϵ (logarithmic scale)	Super-Schmidt cameras				Small cameras			All cameras				
	Mean ϵ	Mean χ	Median χ	No. obs.	Mean ϵ	Mean χ	No. obs.	Mean ϵ	Mean χ	Standard deviation of mean	Standard deviation of 1 obs.	No. obs.
17.11 to 17.49	17.36	+.35	+.32	20	-	-	-	17.36	+.35	±.06	±.26	20
17.50 17.74	17.64	+.34	+.34	64	-	-	-	17.64	+.34	.03	.23	64
17.75 17.99	17.87	+.36	+.31	97	-	-	-	17.87	+.36	.03	.30	97
18.00 18.24	18.11	+.31	+.24	71	-	-	-	18.11	+.32	.03	.23	71
18.25 18.49	18.35	+.26	+.24	51	-	-	-	18.35	+.26	.03	.23	51
18.50 18.99	18.68	+.28	+.24	45	18.80	+.08	2	18.69	+.27	.04	.24	47
19.00 19.49	19.18	+.15	(+.14)	18	19.35	+.16	9	19.24	+.15	.03	.14	27
19.50 19.99	-	-	-	2	19.74	+.05	12	19.74	+.05	.04	.14	14
20.00 20.49	-	-	-	-	20.29	−.02	14	20.29	−.02	.04	.16	15
20.50 20.99	-	-	-	-	20.75	−.10	8	20.75	−.10	.06	.17	8
21.00 21.83	-	-	-	-	21.50	+.01	6	21.50	+.01	.05	.11	6

Although it appears quite clear that an upper limit for χ is reached at $\epsilon=17.5$, it is not possible to decide whether this corresponds to a peak or to a plateau; probably this question must remain unsolved until means are found to obtain accurate decelerations for much fainter meteors. The standard deviation of χ is considerably larger for fainter meteors than for brighter meteors, reflecting the fact that the relative amount of fragmentation varies greatly among fainter meteors.

Tables 2 and 3 give mean values of χ for groups of v_∞ and $\cos Z_R$. A clear correlation exists between χ and $\cos Z_R$, which can be neatly expressed by the linear equation

$$\text{mean } \chi=+0.50-0.27 \cos Z_R. \qquad (7)$$

This correlation is reasonable if we consider that the lifetime of individual fragments is longer when $\cos Z_R$ is smaller, so that fragments become scattered over a greater length on the trail (this is clearly shown by the fact that terminal blending increases as $\cos Z_R$ decreases).

The correlation between χ and meteor velocity is not so clear-cut. From an inspection of the data it is obvious that high-velocity meteors have, on the average, higher values of χ than slow meteors, but there is little evidence of a smooth correlation.

The evidence even seems to suggest that meteors with v_∞ <40 km/sec have a uniform average value of χ near +0.30, while meteors with v_∞ >40 km/sec have an average χ close to +0.38. Since the means of individual velocity groups are likely to be strongly influenced by major showers, and it has been shown by Jacchia (1955b, 1956) that the fragmentation index differs greatly for different showers, another set of means was prepared for χ in

TABLE 2.—*The fragmentation index χ as a function of meteor velocity v_∞.*

Meteor velocity outside atmosphere, v_∞ (km/sec)	All meteors		Sporadic meteors	
	Mean χ	No. obs.	Mean χ	No. obs.
11.5 to 20	+.30	60	+.30	60
20 to 25	+.31	58	+.31	52
25 to 30	+.31	61	+.30	49
30 to 35	+.22	46	+.25	30
35 to 40	+.29	37	+.29	13
40 to 50	+.41	38	+.38	22
50 to 60	+.36	24	+.36	20
60 to 72.8	+.39	45	+.40	32

TABLE 3.—*Fragmentation index χ as a function of zenith distance of meteor radiant, $\cos Z_R$.*

Cos Z_R	Mean χ	No. obs.
0.1 to 0.2	+ 46	4
0.2 to 0.3	+ 38	8
0.3 to 0.4	+ 47	28
0.4 to 0.5	+ 44	35
0.5 to 0.6	+.28	30
0.6 to 0.7	+.32	78
0.7 to 0.8	+.30	72
0.8 to 0.9	+.26	70
0.9 to 1.0	+.25	44

function of v_∞, for sporadic meteors only (table 2). Although the number of meteors in some of the velocity groups is then greatly changed, the picture regarding χ remains essentially unaltered. Since the apparent velocity of 40 km/sec corresponds very closely to the gap between the meteors of the Jupiter family and the long-period meteors, the possibility is not excluded that there is some difference in fragmentability between the two groups of meteors. A similar difference in behavior between the two groups has been found by the writer while examining the beginning heights of meteors, and will form the subject of another paper.

The coefficient σ

As was previously shown (Jacchia, 1955b), the coefficient σ of the mass equation

$$\frac{\dot{m}}{m} = -\frac{I}{E} = \sigma v \dot{v} \tag{8}$$

shows a strong dependence on meteor brightness. The cause should be clear: I/E is a photometric quantity and refers to the sum of all the fragments, while $v\dot{v}$ is a dynamic quantity and refers to the larger fragments only.

On the whole σ is a little easier to determine than χ, because it involves only the first time derivative of the velocity, while χ involves the second derivative. The determination of σ from equation (8) has two weak points: 1) it involves the instantaneous meteor brightness I and therefore is affected by the vagaries of the light curve. A two-magnitude flare will cause a six-fold local increase in the value of σ; 2) I is the instantaneous brightness, while $\dot{v}$ is a value of the deceleration obtained from observations over a long portion of the meteor trajectory, often over all of it, so the two quantities do not, strictly speaking, correspond to each other. Whenever possible, σ should be determined from the integral of (8), i. e., from the equation

$$\ln \frac{E_1}{E_2} = \frac{2}{\sigma}(v_1^2 - v_2^2), \tag{8a}$$

with the use of two separate points (subscripts 1 and 2) on the meteor trajectory.

For all Super-Schmidt meteors, "instantaneous" values of σ were determined for each observed deceleration; in addition, equation (8a) was used whenever more than one deceleration was available on the meteor trajectory. A mean value of σ was derived for each meteor on the basis of all these determinations; those derived from equation (8a) were given greater weight in calculating the mean.

The observational results relative to σ as a function of ϵ are collected in table 4 and shown graphically in figure 1. It appears that σ follows the same general trend as χ, although the plateau corresponding to fainter meteors seems to be a little broader. It is interesting to note, however, that the standard deviation of σ does not seem to increase with decreasing ϵ, as is the case with χ. In other words, for faint meteors we observe a large range in χ, from the largest values all the way down to zero, while we observe only large values of σ,

TABLE 4.—*The coefficient σ[1] as a function of the integrated brightness, ϵ.*

Integrated brightness, ϵ (logarithmic scale)	Super-Schmidt cameras: Mean ϵ	Super-Schmidt cameras: Mean log σ	Super-Schmidt cameras: No. obs.	Small cameras: Mean ϵ	Small cameras: Mean log σ	Small cameras: No. obs.	All cameras: Mean ϵ	All cameras: Mean log σ	All cameras: Standard deviation of mean	All cameras: Standard deviation of 1 obs.	All cameras: No. obs.
17.11 to 17.49	17.36	−11.11	21	-----	-------	--	17.36	−11.11	±.05	±.21	21
17.50 17.74	17.64	−11.16	59	-----	-------	--	17.64	−11.16	.04	.28	59
17.75 17.99	17.86	−11.16	92	17.87	−11.05	1	17.86	−11.16	.02	.22	93
18.00 18.24	18.11	−11.18	66	-----	-------	--	18.11	−11.18	.03	.22	66
18.25 18.49	18.36	−11.24	50	-----	-------	--	18.36	−11.24	.03	.20	50
18.50 18.99	18.68	−11.22	43	18.82	−11.28	11	18.71	−11.23	.03	.23	54
19.00 19.49	19.18	−11.33	18	19.30	−11.34	16	19.24	−11.33	.04	.25	34
19.50 19.99	19.59	−11.57	2	19.72	−11.49	22	19.71	−11.50	.05	.24	24
20.00 20.49	20.41	−11.88	1	20.25	−11.60	17	20.26	−11.61	.05	.23	18
20.50 20.99				20.75	−11.77	11	20.75	−11.77	.07	.24	11
21.00 21.83				21.46	−11.71	8	21.46	−11.71	.13	.37	8

[1] See equation (8), above.

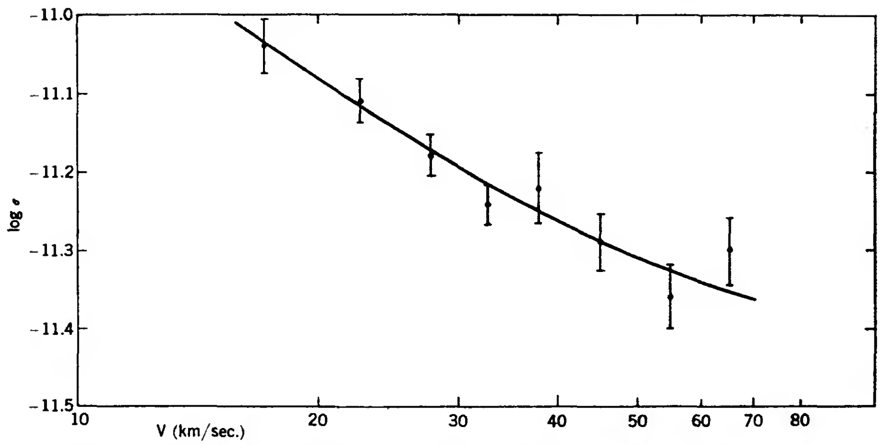

FIGURE 2. Mean values of log σ in function of meteor velocity. The confidence marks represent standard deviations of the means. Super-Schmidt meteors only.

which do not overlap with the smaller values characteristic of bright meteors. This shows that the variation of σ is due not only to meteor fragmentation, but must also have a true systematic component connected with meteor size.

Mean values of log σ taken for 8 velocity groups (table 5, figure 2), for Super-Schmidt meteors only, show a clear dependence on velocity. The variation is nearly linear with log v_∞; the slowest meteors ($v_\infty<20$ km/sec) yield values of σ which are nearly twice as large as those of the fastest meteors ($v_\infty>60$ km/sec).

In the course of the meteor trajectory σ changes but little, although there is a general tendency for it to decrease a little from the earlier to later portions of the meteor trajectory.

Values of d log σ/ds were determined for 111 meteors which had yielded multiple decelerations. The mean of all these determinations is mean $\frac{d \log \sigma}{ds}=-0.079\pm0.016$ (m. e). The standard deviation of a single determination was ±0.17. Thirty-nine meteors gave a positive value of d log σ/ds, 69 gave a negative value, and three gave a value of zero.

To examine this trend more closely, all the 1115 determinations of log σ from Super-Schmidt meteors (an average of a little over 3 determinations per meteor) were arranged in increasing order of s and means were taken within equidistant groups. The results, given in table 6 and in figure 3, show that the mean observed range in log σ in the reliable part of

TABLE 5.—*Super-Schmidt meteors: the coefficient σ*[1] *in function of meteor velocity v.*

Velocity, v (km/sec)	Mean v	Mean log σ	Standard deviation of mean	Standard deviation of 1 obs.	No. obs.
11. 5 to 20	17. 1	−11. 04	±. 03	±. 19	55
20 to 25	22. 5	−11. 11	. 03	. 20	56
25 to 30	27. 3	−11. 18	. 03	. 21	56
30 to 35	31. 8	−11. 24	. 03	. 19	46
35 to 40	36. 7	−11. 22	. 04	. 26	37
40 to 50	44. 3	−11. 29	. 04	. 24	40
50 to 60	56. 3	−11. 36	. 04	. 19	22
60 to 72. 8	65. 3	−11. 30	. 04	. 27	43

[1] See page 184, equation (8).

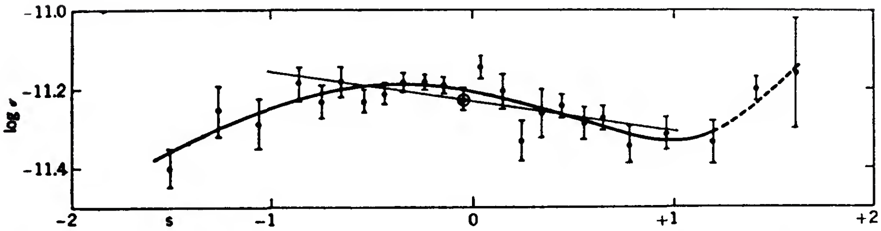

FIGURE 3. Mean variation of σ in the course of the meteor trajectory, derived from Super-Schmidt meteors only. In abscissae the mass-loss parameter s. The open circle at $s=-0.055$ represents the weighted mean of all 1115 observed values of σ; the slanting straight line through it has a slope equal to the mean value of $d\log\sigma/ds$ determined directly from individual meteors. The confidence marks represent standard deviation of the means.

the trajectory is only 0.14. A decrease in σ is observed from $s=-0.2$ to $s=+1.0$, which accounts for the negative mean of all the values of $d\log\sigma/ds$. The lower values of σ for large negative values of s may be accounted for by the fact that only for long, bright meteors, which have smaller values of σ (fig. 1), can decelerations be determined so early in the trajectory. The rise in σ for $s>+1.0$ is extremely uncertain, being based on very few observations. If real, it could be due to the presence of a residual mass, which the numerical integration of the light-intensity curve fails to take into account.

The larger values of the standard deviation for σ in the interval between $s=0$ and $s=+0.3$ reflect the fact that most of the irregularities and individual peculiarities of meteor light

TABLE 6.—*Super-Schmidt meteors: the coefficient σ in function of the mass-loss parameters.*

Mass-loss parameter s	Mean s	Mean $\log\sigma$	Standard deviation of mean	Standard deviation of 1 obs.[1]	No. obs.
−1.64 to −1.4	−1.51	−11.40	±.05	±.09	4
−1.4 to −1.2	−1.26	−11.25	.06	.20	12
−1.2 to −1.0	−1.06	−11.29	.07	.23	12
−1.0 to −0.8	−0.86	−11.18	.04	.22	25
−0.8 to −0.7	−0.75	−11.23	.07	.22	27
−0.7 to −0.6	−0.65	−11.18	.04	.20	25
−0.6 to −0.5	−0.54	−11.23	.03	.19	51
−0.5 to −0.4	−0.44	−11.21	.03	.23	79
−0.4 to −0.3	−0.34	−11.18	.02	.23	83
−0.3 to −0.2	−0.24	−11.18	.02	.20	139
−0.2 to −0.1	−0.15	−11.19	.02	.23	144
−0.1 to −0.0	−0.05	−11.22	.02	.25	111
0.0 to 0.1	+0.04	−11.14	.03	.27	84
0.1 to 0.2	+0.15	−11.20	.05	.30	43
0.2 to 0.3	+0.24	−11.33	.05	.31	33
0.3 to 0.4	+0.34	−11.26	.06	.36	33
0.4 to 0.5	+0.44	−11.24	.03	.21	49
0.5 to 0.6	+0.54	−11.28	.04	.25	39
0.6 to 0.7	+0.64	−11.27	.03	.21	42
0.7 to 0.9	+0.78	−11.34	.05	.30	41
0.9 to 1.1	+0.96	−11.31	.04	.18	18
1.1 to 1.3	+1.19	−11.34	.06	.18	11
1.3 to 1.5	+1.41	−11.20	.03	.07	7
1.5 to 1.79	+1.61	−11.16	.14	.32	5

[1] Of average weight.

curves occur around maximum light or shortly afterwards.

The author is indebted to Mr. Robert E. Briggs for assembling the observational material on punched cards and preparing it for a statistical analysis.

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Abstract

Observational results are given for two useful meteor parameters. The first is the "fragmentation index" χ, which was introduced by the author in an earlier paper to measure the progressive departure of the observed deceleration of a fragmenting meteor body from the deceleration expected on the basis of single-body theory. The second parameter is the coefficient σ of the "mass equation" $\dot{m}/m=\sigma v\dot{v}$.

The fragmentation index χ is found to vary with the brightness and the angle of incidence of the meteors. A variation with velocity is indicated, but it is probably discontinuous and due to the cosmic distribution of meteors. The coefficient σ of the mass equation shows a variation with brightness which nearly parallels that of χ. The variation of σ with velocity is quite regular. There is but little variation of σ in the course of the meteor trajectory.

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