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THE METEORIC HEAD ECHO

by ALLAN F. COOK AND GERALD S. HAWKINS



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FRED L. WHIPPLE, *Director,*
Astrophysical Observatory,
Smithsonian Institution.

Cambridge, Mass.

The Meteoric Head Echo¹

By ALLAN F. COOK² AND GERALD S. HAWKINS³

Radar observers have in certain instances detected an echo associated with the head of a meteor. The head echo has been observed at a frequency of 30 mc by McKinley and Millman (1949) and at a frequency of 60 mc by Hey, Parsons, and Stewart (1947). Apart from a general description of the phenomenon very little information has been published on head echoes and no statistical analysis is available that describes their general properties. We may surmise that head echoes are a relatively rare phenomenon and that they are associated with bright meteors. The target that reflects the echo is almost spherical in shape because it usually gives no marked increase in signal strength as it crosses the line of sight. The target cross section is large—many hundreds of meters—whereas the solid meteor itself cannot be more than a few centimeters in diameter. From this we are led to postulate that a coma of ionization surrounding the meteor particles produces the reflection.

An alternative explanation was put forward by Browne and Kaiser (1953), who suggested that the head echo was produced by the discontinuity at the end of the extending cylinder of ionization which the meteoroid was generating. This hypothesis predicts an increase in echo strength of the order of 1000 as the meteoroid passes the minimum range position. Such an increase does not occur, however, and the theory is therefore disproved (McKinley, 1955).

McKinley and Millman (1949) have suggested a hypothesis to explain the coma of ionization: that the meteoroid acts as a source of intense ultraviolet radiation which ionizes an extensive region surrounding the meteoroid. The approximately spherical shape of the coma

is maintained by recombination of the ionized particles after passage of the meteoroid. The present paper is an attempt to examine the ultraviolet-light hypothesis on a quantitative basis.

Observational material

McKinley (1955, table 1) has estimated the strength of the head echo for two meteors, both of which occurred during the daytime.

Meteor I was observed from a single station only and was assumed to be a Perseid with a radiant at R.A. 46°, dec +58°. Its maximum visual magnitude is inferred from the duration of the echo by use of the correlation between brightness and duration determined by Millman (1950). McKinley showed that the difference in heights between the beginning and end points was 53 km. Comparison with photographic measurements makes it reasonable to assume that the end height was 70 km and the beginning height was 123 km. On this scale the head echo reached maximum intensity at a height of 77 km. Meteor II was observed simultaneously at three separate stations, so that its trajectory could be accurately determined.

The magnitudes of the meteors were calculated from the following equations (Greenhow, 1952; Hawkins, 1955):

$$q = 4\pi D n_e t, \quad (1)$$

$$M = 39.8 - 2.5 \log_{10} q, \quad (2)$$

where q is the electron line density in the trail, D is the diffusion coefficient of the atmosphere, n_e is the critical electron density for the wavelength of the radar, t is the duration of the backscatter echo from the meteor trail, and M is the visual magnitude of the meteor.

The parameters of the radar equipment used by Millman and McKinley at Ottawa are as follows:

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² Harvard College Observatory, Cambridge, Mass.

³ Boston University, Boston, Mass., and Harvard College Observatory.

G : Antenna gain over an isotropic radiator, 3.0.

ϵ : Noise level in the receiver, 2×10^{-14} watts.

P : Transmitter power, 3×10^5 watts.

λ : Radar wavelength, 9.2 meters.

n_c : Critical electron density, $1.28 \times 10^{13} \text{m}^{-3}$

The echo power p given by a sphere of unit reflection coefficient and radius a at a range R is given by the radar formula:

$$p = \frac{G^2 P \lambda^2 \pi a^2}{64 \pi^3 R^4}. \quad (3)$$

Using McKinley's published estimates of echo strength and the parameters given above, we can calculate the radius of the ionized sphere from equation (3), as shown in figure 1.

For Meteor II, observations on echo strength were made from two stations, at Ottawa and at Arnprior. The peaks of the curve for Meteor II, at 47.25 and 48.75 seconds, correspond to the times at which the meteor was passing the

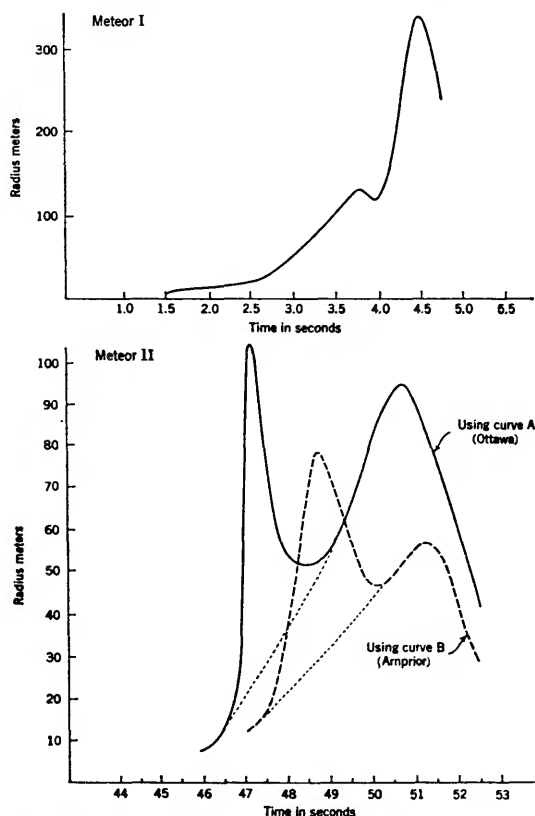


FIGURE 1.—The radius of the ultraviolet coma as a function of time.

minimum range position for the two stations. These peaks are probably due to an elongation of the surface of critical electron density in the direction of the meteor. The discrepancy in the diameters as measured from Ottawa and Arnprior is probably due to an error in the estimated sensitivities of the equipments.

Ultraviolet light hypothesis

We will assume that the reflection of radio energy occurs at the surface where the electron density is equal to the critical density n_c for the radar. The problem is then to determine the shape of this surface of constant density, or isosurface. The equation of radiative transfer in an elementary volume may be written in terms of distance, ρ , from the center of the meteoric coma as follows:

$$\frac{\partial I_r}{\partial \rho} = -\sigma_r n(I_r - S_r), \quad (4)$$

where $I_r d\nu d\omega$ is the intensity of the radiation through the solid angle $d\omega$ between frequency ν and $\nu + d\nu$; σ_r is the absorption cross section; n is the number of molecules per unit volume; and S_r is the source function (Chandrasekhar, 1950). We assume that the oxygen molecule is the one ionized.

Let us choose a coordinate system at rest with respect to the atmosphere. The kinetic equation for the problem may be written in the form

$$\frac{\partial n_e}{\partial t} = n \oint \int_{\nu_0}^{\infty} \frac{\sigma_r I_r}{h\nu} d\nu d\omega - \alpha n_e^2 \quad (5)$$

where n_e is the number of electrons (or ions) per unit volume, t is time, h is Planck's constant, ν is the frequency of the radiation, ν_0 is the threshold frequency for ionization, $d\omega$ is an elementary solid angle, α is the volume recombination coefficient of the ions, and the symbol \oint denotes integration over all directions. The term under the integral sign indicates that within a given solid angle, $d\omega$, the intensity of photons (number per cm^2) produces ionizations at a rate $n(\sigma_r I_r / h\nu)$ over the range of frequency, $d\nu$.

Finally, if n_0 is the initial number of oxygen molecules per unit volume before passage of

the meteor, the conservation of particles requires that

$$n = n_0 - n_e - \frac{1}{2} n'_0 \quad (6)$$

where n'_0 is the number of oxygen atoms released per unit volume by dissociative recombination.

The following approximations may reasonably be made and should be quite accurate:

$$n \simeq n_0, S_r = 0. \quad (7)$$

The first expression is justified because $n_e \simeq n_e \ll n_0$; that is to say, a small fraction of the oxygen molecules is ionized. The source function, S_r , may be neglected because the main recombination process of the ionized molecule involves the dissociation of the molecule which does not produce ultraviolet radiation. These approximations permit the integration of equation (4), to give the expression

$$I_r = I_r^* \exp(-\sigma_r n_0 \rho), \quad (8)$$

where $I_r^* d\nu d\omega$ is the intensity along the direction of integration in the absence of absorption. The average intensity, J_r , is given by the integration

$$J_r = \frac{1}{4\pi} \oint I_r d\omega = \frac{J_r^*}{\rho^2} \exp(-\sigma_r n_0 \rho), \quad (9)$$

where J_r^* denotes the average intensity at unit distance from the meteor in the absence of absorption. We have assumed that ρ is large when compared with the radiating coma of the meteor. Thus equation (9) in effect corresponds to the case of a point source. Substitution of this expression and $n = n_0$ into equation (5) yields the equation,

$$\frac{\partial n_e}{\partial t} = 4\pi \frac{n_0}{\rho^2} \int_{r_0}^{\infty} \frac{\sigma_r J_r}{h\nu} \exp(-\sigma_r n_0 \rho) \cdot d\nu - \alpha n_e^2. \quad (10)$$

We will now introduce a mean absorption cross section $\bar{\sigma}$ defined as follows:

$$\int_{r_0}^{\infty} \frac{\sigma_r J_r^* \exp(-\sigma_r n_0 \rho)}{h\nu} \cdot d\nu \simeq \bar{\sigma} J^* \exp(-\bar{\sigma} n_0 \rho) \quad (11)$$

where J^* is the average photon intensity at unit distance from the meteor in the absence of absorption:

$$J^* = \int_{r_0}^{\infty} \frac{J_r^* d\nu}{h\nu}. \quad (12)$$

This is often called the "gray" approximation, and is valid when the medium is optically thin.

In addition to substituting approximation (11) into the kinetic equation (10), we shall find it convenient to use a frame of reference that moves with the meteor. The time derivative in this new frame of reference is given by the equation,

$$\frac{Dn_e}{Dt} = \frac{\partial n_e}{\partial t} - V \frac{\partial n_e}{\partial z} \quad (13)$$

where cylindrical coordinates are used; r is the distance from the meteor path, V is the velocity of the meteoroid, and z is the distance along the path, taken positive in front of the meteor. It is reasonable to assume that the steady-state approximation holds, that is to say $\frac{Dn_e}{Dt} = 0$. The kinetic equation then becomes

$$\frac{\partial n_e}{\partial z} = -4\pi \frac{\bar{\sigma} n_0 J^*}{V(r^2 + z^2)} \exp[-\bar{\sigma} n_0 \sqrt{r^2 + z^2}] + \frac{\alpha}{V} n_e^2 \quad (14)$$

Solution of the kinetic equation

A general solution of the kinetic equation (14) is not known. The best that can be done is to treat various limiting cases; in particular, those in which one of the three terms is small compared to the other two.

Case I: Nearly steady state.

$$\frac{\partial n_e}{\partial z} \ll -4\pi [\bar{\sigma} n_0 J^* / V(r^2 + z^2)] \times \exp[-\bar{\sigma} n_0 \sqrt{r^2 + z^2}] \approx (\alpha/V) n_e^2.$$

Neglecting the term, $\partial n_e / \partial z$, we may write the lowest order solution as:

$$n_e^{(0)}(r, z) = 2 \left(\frac{\pi \bar{\sigma} n_0 J^*}{\alpha} \right)^{\frac{1}{2}} \frac{\exp \left[-\frac{1}{2} \bar{\sigma} n_0 \sqrt{r^2 + z^2} \right]}{(r^2 + z^2)^{\frac{1}{2}}}. \quad (15)$$

Note that equation (15) involves only the distance from the meteoroid ρ . A solution of the form

$$n_e^{(2)}(r, z) = n_e^{(0)}(r, z)[1 + \phi_1(r, z) + \phi_2(r, z)] \quad (16)$$

is now sought. The following result is obtained:

$$\beta \equiv 2 \left(\frac{\pi \bar{\sigma} n_0 \mathcal{J}^*}{\alpha} \right)^{\frac{1}{2}}, \gamma \equiv \frac{V}{2} \left(\frac{1}{4\pi \bar{\sigma} n_0 \alpha \mathcal{J}^*} \right)^{\frac{1}{2}}, \quad (17)$$

$$n_e^{(2)}(r, z) = \frac{\beta}{\rho} \exp \left(-\frac{1}{2} \bar{\sigma} n_0 \rho \right) \left\{ 1 - \gamma \left[1 + \frac{1}{2} \bar{\sigma} n_0 \rho \right] \times \right. \\ \left. \cos \theta \exp \left(\frac{1}{2} \bar{\sigma} n_0 \rho \right) + \frac{3}{2} \gamma^2 \left[1 - \frac{1}{12} \bar{\sigma}^2 n_0^2 \rho^2 \right] \times \right. \\ \left. \cos^2 \theta \exp \left(\bar{\sigma} n_0 \rho \right) \right\}, \quad (18)$$

where a change has been made to polar coordinates ρ and θ ,

$$\rho = \sqrt{r^2 + z^2}, \quad \cos \theta = \frac{z}{(r^2 + z^2)^{\frac{1}{2}}}. \quad (19)$$

So long as γ is small, the term with γ as a factor indicates that the surfaces of equal density, n_e , are spheres with centers along the z axis. The third term with γ^2 as a factor in these surfaces introduces an elongation. As γ becomes very small these surfaces can be represented as ellipsoids. We may now evaluate the distances $\rho(0)$, $\rho\left(\frac{\pi}{2}\right)$ and $\rho(\pi)$ at which $n_e = n_e$.

At $\theta = 0$,

$$n_e = \beta \frac{\exp \left[-\frac{1}{2} \bar{\sigma} n_0 \rho(0) \right] \times \left\{ 1 - \gamma \left[1 + \frac{1}{2} \bar{\sigma} n_0 \rho(0) \right] \exp \left[\frac{1}{2} \bar{\sigma} n_0 \rho(0) \right] + \right. \\ \left. \frac{3}{2} \gamma^2 \left[1 - \frac{1}{12} \bar{\sigma}^2 n_0^2 \rho^2(0) \right] \exp \left[\bar{\sigma} n_0 \rho(0) \right] \right\}}{\rho(0)}. \quad (20)$$

At $\theta = \frac{\pi}{2}$,

$$n_e = \beta \frac{\exp \left[-\frac{1}{2} \bar{\sigma} n_0 \rho \left(\frac{\pi}{2} \right) \right]}{\rho \left(\frac{\pi}{2} \right)}. \quad (21)$$

At $\theta = \pi$,

$$n_e = \beta \frac{\exp \left[-\frac{1}{2} \bar{\sigma} n_0 \rho(\pi) \right] \times \left\{ 1 + \gamma \left[1 + \frac{1}{2} \bar{\sigma} n_0 \rho(0) \right] \exp \left[\frac{1}{2} \bar{\sigma} n_0 \rho(\pi) \right] + \right. \\ \left. \frac{3}{2} \gamma^2 \left[1 - \frac{1}{12} \bar{\sigma}^2 n_0^2 \rho^2(0) \right] \exp \left[\bar{\sigma} n_0 \rho(\pi) \right] \right\}}{\rho(\pi)}. \quad (22)$$

It is now necessary to substitute the expressions

$$\rho(0) = \rho \left(\frac{\pi}{2} \right) - \Delta_1 \rho + \Delta_2 \rho, \quad (23)$$

$$\rho(\pi) = \rho \left(\frac{\pi}{2} \right) + \Delta_1 \rho + \Delta_2 \rho,$$

into equations (20), (21), and (22) and solve for $\Delta_1 \rho$ and $\Delta_2 \rho$. We neglect $\Delta_2 \rho$ in the terms involving γ and both $\Delta_1 \rho$ and $\Delta_2 \rho$ in terms involving γ^2 . The solutions are:

$$\Delta_1 \rho = \gamma \rho \left(\frac{\pi}{2} \right) \exp \left[\frac{1}{2} \bar{\sigma} n_0 \rho \left(\frac{\pi}{2} \right) \right], \quad (24)$$

$$\Delta_2 \rho = \frac{3}{2} \gamma^2 \rho \left(\frac{\pi}{2} \right) \exp \left[\bar{\sigma} n_0 \rho \left(\frac{\pi}{2} \right) \right] \times \frac{\left[1 - \frac{1}{3} \bar{\sigma} n_0 \rho \left(\frac{\pi}{2} \right) + \frac{1}{3} \bar{\sigma}^2 n_0^2 \rho^2 \left(\frac{\pi}{2} \right) \right]}{1 + \frac{1}{2} \bar{\sigma} n_0 \rho \left(\frac{\pi}{2} \right)}. \quad (25)$$

The corresponding surface where $n_e = n_e$ is a prolate ellipsoid for small values of γ with a ratio of major to minor axes of $1 + \Delta_2 \rho / \rho(\pi)$. The ratio of power reflected laterally to that reflected axially will exceed unity by an amount we shall call E . Then it follows that

$$1 + \frac{1}{2} [\Delta_2 \rho / \rho(\pi)] \equiv 1 + E.$$

Case II: Small recombination rate.

$$(\alpha/V) n_e^2 \ll \partial n_e / \partial z \approx -4\pi (\bar{\sigma} n_0 \mathcal{J}^* / V \rho^2) \exp (\bar{\sigma} n_0 \rho).$$

If we neglect the term $\left(\frac{\alpha}{V} \right) n_e^2$, the lowest order solution is given by the equation,

$$n_e^{(0)}(r, z) = 4\pi \frac{\bar{\sigma} n_0 \mathcal{J}^*}{V} \int_z^\infty \frac{\exp [-\bar{\sigma} n_0 \sqrt{r^2 + z^2}]}{r^2 + z^2} dz, \quad (26)$$

where we have applied the boundary condition that $n_e^{(0)}(r, z)$ vanishes as z becomes positively infinite. Solutions in closed form can be derived only in the limits, as the optical distance from the meteor goes either to zero or to infinity. The latter case is of no current interest, so we shall consider only the former, for which we have the solution:

$$n_e^{(0)}(r, z) = 4\pi \frac{\bar{\sigma} n_0 \mathcal{J}^*}{V r} \operatorname{arccot} \left(\frac{z}{r} \right); \quad (27)$$

or, in terms of ρ and θ ,

$$n_e^{(0)}(\rho, \theta) = 4\pi \frac{\bar{\sigma} n_0 \mathcal{J}^*}{V} \frac{\theta}{\rho \sin \theta}. \quad (28)$$

The corresponding solution for the surface where the electron density equals the critical density for the radar is:

$$\rho^{(0)}(\theta) = 4\pi \frac{\bar{\sigma} n_0 \mathcal{J}^*}{V n_c} \frac{\theta}{\sin \theta}. \quad (29)$$

The first approximation can be derived in the usual way by substitution of solution (27) into the neglected term, whence by integration we obtain the equation,

$$n_e^{(1)}(r, z) = 4\pi \frac{\bar{\sigma} n_0 \mathcal{J}^*}{V r} \left[\operatorname{arccot} \left(\frac{z}{r} \right) - \frac{\alpha}{V} 4\pi \frac{\bar{\sigma} n_0 \mathcal{J}^*}{V} \int_{z/r}^{\infty} \operatorname{arccot}^2 u du \right]. \quad (30)$$

The maximum value of n_e occurs at z_m , where

$$\left[1 + \left(\frac{z_m}{r} \right)^2 \right] \operatorname{arccot} \left(\frac{z_m}{r} \right) = \frac{V}{\alpha} \frac{V}{4\pi \bar{\sigma} n_0 \mathcal{J}^*}, \quad (31)$$

at which point the approximation is breaking down. The corresponding value of θ is θ_m where

$$\left(\frac{\sin \theta_m}{\theta_m} \right) = \frac{4\pi \alpha \bar{\sigma} n_0 \mathcal{J}^*}{V^2}. \quad (32)$$

This equation shows that all contours of constant electron density have a maximum width at an angle $\theta = \theta_m$ as indicated in figure 2.

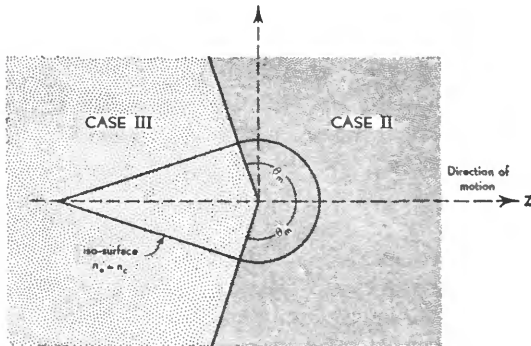


FIGURE 2.—Coordinate system (ρ, θ) and regions of solution for Case II and Case III.

Case III: Small photo-ionization rate.

$$-4\pi [\bar{\sigma} n_0 \mathcal{J}^* / V(r^2 + z^2)] \exp(-\bar{\sigma} n_0 \sqrt{r^2 + z^2}) \\ \ll \partial n_e / \partial z \simeq (\alpha/V) n_e^2.$$

Integration is straightforward and exact, and yields the equation:

$$n_e^{(0)}(r, z) = \frac{1}{F(r) - \frac{\alpha}{V} z} \quad (33)$$

where $F(r)$ is a function of z to be determined from the initial conditions. This solution applies to the tail of the reflecting surface as shown in figure 2. If we ignore a transition zone and use a boundary at $\theta = \theta_m$ then the initial conditions are given by Case II, equation (27). These assumptions determine $F(r)$, and the density, n_e , is then given by the expression

$$n_e(r, z) = \frac{4\pi \bar{\sigma} n_0 \mathcal{J}^*}{V} \frac{\theta_m^2}{r \theta_m + \left(\frac{\sin \theta_m}{\theta_m} \right)^2 (z_m - z)}, \quad (34)$$

where $z_m = r \cot \theta_m$.

Combination of Cases II and III: An iso-surface of electron density is found in the range $0 \leq \theta \leq \theta_m$ from equation (29). From equation (34) it follows that in the range $\theta_m \leq \theta \leq \pi$ the isosurface is given by the equation:

$$\theta_m \leq \theta \leq \pi, \rho^{(0)}(\theta) = 4\pi \frac{\bar{\sigma} n_0 \mathcal{J}^*}{V n_c} \times \frac{\theta_m^2}{\theta_m + \left(\frac{\sin \theta_m}{\theta_m} \right)^2 (\cot \theta_m - \cot \theta)} \quad (35)$$

Equations (29), (32), and (35) apply only in the limit as $\bar{\sigma} n_0 \rho$ goes to zero.

Application to the observations

Meteor I appears to be best treated as an example of Case I, where the shape of the target is a prolate ellipsoid, elongated in the direction of motion of the meteor. We have, from equation (21), for the radius of curvature of the ends of the ellipsoid, the expression:

$$\rho = \frac{\beta}{n_c} \exp \left[-\frac{1}{2} \bar{\sigma} n_0 \rho \right]. \quad (36)$$

The ratio of axial to lateral power reflected from the meteor head has been defined in Case I as $(1+E)$ where from equation (25) we see that

$$E = \frac{1}{2} \frac{\Delta_2 \rho}{\rho} = \frac{3}{4} \gamma^2 \frac{1 - \frac{1}{3} \bar{\sigma} n_0 \rho + \frac{1}{3} \bar{\sigma}^2 n_0^2 \rho^2}{1 + \frac{1}{2} \bar{\sigma} n_0 \rho} \exp[\bar{\sigma} n_0 \rho]. \quad (37)$$

From equations (36) and (37) we may eliminate the exponential term to obtain:

$$\rho n_c \sqrt{\frac{2}{3}} E = \beta \gamma \sqrt{\frac{1 - \frac{1}{3} \bar{\sigma} n_0 \rho + \frac{1}{3} \bar{\sigma}^2 n_0^2 \rho^2}{1 + \frac{1}{2} \bar{\sigma} n_0 \rho}}, \quad (38)$$

which yields the equation,

$$\alpha = \frac{V}{2 \rho n_c} \sqrt{\frac{3 - \bar{\sigma} n_0 \rho + \bar{\sigma}^2 n_0^2 \rho^2}{2(2 + \bar{\sigma} n_0 \rho)}} \frac{1}{E}. \quad (39)$$

For Meteor I we deduce from figure 1 that at 19^h 8^m 37.5 E.S.T., the radius of curvature of the ends of the ellipsoid $\rho = 10^4$ cm. From the change in signal strength as the meteor crossed the line of sight we deduce $E = 0.3$. We may take $V = 6.0 \times 10^6$ cm sec⁻¹ and $n_0 = 1.5 \times 10^{13}$ cm⁻³. As extreme assumptions we may take $\bar{\sigma} = 0$ or 2.75×10^{-17} cm², the maximum value found by Weissler and Lee (1952). The corresponding limits on the optical distance are $0 \leq \bar{\sigma} n_0 \rho \leq 8.25$ where ρ has been doubled to allow for refraction (Manning, 1953). From equation (39) we find that the recombination coefficient α is approximately 3×10^{-5} cm³ sec⁻¹.

Meteor II appears to be best treated as an example of Cases II and III combined. The curvature of the isosurface parallel to the direction of flight is given by the equation,

$$\frac{1}{r_M} = \frac{d^2 r}{dz^2} = \frac{d}{d\theta} \left(\frac{dr}{dz} \right) \frac{d\theta}{dz} = \frac{\frac{d}{d\theta} \left(\frac{dr/d\theta}{dz/d\theta} \right)}{\frac{(dz/d\theta)(d^2 r/d\theta^2) - (dr/d\theta)(d^2 z/d\theta^2)}{(dz/d\theta)^3}}, \quad (40)$$

while the radius along the axial direction is given by the expression,

$$r_m = \rho \sec \theta_m. \quad (41)$$

A preliminary estimate of the situation is best made from equations (29), (32), and (35). For $\theta_m \leq \theta \leq \pi$, r_M is infinite. To avoid this difficulty we use the value obtained as θ approaches θ_m but is less than θ_m . Then

$$r_M = 4\pi \frac{\bar{\sigma} n_0 \mathcal{J}^* (\theta_m - \sin \theta_m \cos \theta_m)^3}{V n_c \sin 4\theta_m (1 - \sin \theta_m \cos \theta_m)},$$

$$r_m = 4\pi \frac{\bar{\sigma} n_0 \mathcal{J}^*}{V n_0} \theta_m, \quad (42)$$

$$r_M r_m = 4\pi \frac{\bar{\sigma} n_0 \mathcal{J}^*}{V n_c} \left(\frac{\theta_m}{\sin \theta_m} \right)^2 \frac{\left(1 - \frac{\sin \theta_m}{\theta_m} \cos \theta_m \right)^{\frac{3}{2}}}{V_2 (\sin \theta_m - \theta_m \cos \theta_m)^{\frac{1}{2}}}, \quad (43)$$

$$\frac{r_M r_m}{\rho_0} = \left(\frac{\theta_m}{\sin \theta_m} \right)^2 \frac{\left(1 - \frac{\sin \theta_m}{\theta_m} \cos \theta_m \right)}{V_2 (\sin \theta_m - \theta_m \cos \theta_m)^{\frac{1}{2}}}. \quad (44)$$

The observations give at 17^h 59^m 47.25, $\rho(0) = 2.4 \times 10^3$ cm, $r_M r_m = 1.03 \times 10^4$ cm, $V = 3.5 \times 10^6$ cm sec⁻¹, $n_0 = 2.5 \times 10^{12}$ cm⁻³. In this case for $\bar{\sigma} = 2.75 \times 10^{-17}$ cm² the corresponding limit on the optical distance is $\bar{\sigma} n_0 \rho \leq 0.165$. Then $(\sin \theta_m / \theta_m) = 0.39$ and $4\pi \bar{\sigma} n_0 \mathcal{J}^* / V = \rho'(0) n_c = 6.14 \times 10^{10}$ cm⁻² where $\rho(0)$ has been increased by a factor of 2 to allow for refraction. From equation (32) we find $\alpha \approx 0.9 \times 10^{-5}$ cm³ sec⁻¹.

Comparison of the two estimates suggests that $\alpha \approx 2 \times 10^{-5}$ cm³ sec⁻¹. This definitely eliminates N_2 as the ionized constituent, since the experiments of Bialecke and Dougal (1958) indicate for N_2 that $\alpha = 9 \times 10^{-7}$ cm³ sec⁻¹, at the height of the meteor. A revised estimate of $(\sin \theta_m / \theta_m)^2 = 0.35$ is indicated by these results, but this corresponds to a value of θ_m only slightly greater than $\pi/2$ so it appears that numerical integrations of the kinetic equation (14) are required to fully interpret the observations of this meteor.

Some conclusions can be drawn concerning the ionizing radiation from Meteor I. By equations (17) and (36),

$$\mathcal{J}^* = \frac{\alpha n_c \rho^2 \exp[\bar{\sigma} n_0 \rho]}{4\pi \bar{\sigma} n_0}. \quad (45)$$

If we now differentiate with respect to $\bar{\sigma}$ and equate the result to zero, we obtain at the minimum value of \mathcal{J}^* the relation

$$\bar{\sigma} n_0 \rho = 1. \quad (46)$$

Thus at maximum radius $\rho = 3.3 \times 10^4 \text{ cm}$, $H = 77 \text{ km}$, $n_0 = 1.4 \times 10^{14} \text{ cm}^{-1}$. For the minimum value of meteor radiation, $\mathcal{J}^*, \bar{\sigma} = 2.2 \times 10^{-19} \text{ cm}^{-2}$ which occurs very close to the ionization limit at 1030 Å (Weissler and Lee, 1952). Then $\mathcal{J}^* = 2.5 \times 10^{22} \text{ sec}^{-1}$. The total emission of ionizing photons from the meteor is at the rate $16\pi^2 \mathcal{J}^*$ and is $\geq 4.0 \times 10^{24} \text{ sec}^{-1}$. The total power of the ionizing radiation is $16\pi^2 \mathcal{J}^* h\nu$ which is $\geq 8 \times 10^{13} \text{ ergs sec}^{-1}$.

Conclusions and discussion

From this work we may conclude that the meteoric head echo can be accounted for by photo-ionization of molecular oxygen caused by radiation from the meteor. The required recombination coefficient is approximately $2 \times 10^{-6} \text{ cm}^3 \text{ sec}^{-1}$. Even if O_2^+ is not formed directly it would appear as the end product of electron exchange with all other ions which could be formed in the meteor region except nitric oxide. Miller (1957) has shown that nitric oxide is present only as a trace constituent and could not produce the head echo because of its low recombination coefficient (Bialecke and Dougal 1958) and other factors.

For the Perseid meteor, with visual magnitude -5.7 , we have been able to estimate the total ultraviolet power required to produce the ionization in the meteor head. It is $\geq 8 \times 10^{13} \text{ ergs sec}^{-1}$.

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Abstract

In this paper the authors examine the hypothesis that the meteoric head echo is due to ultraviolet ionizing radiation from the meteor, and the subsequent dissociative recombination of the molecular ions and electrons produced. It is shown that molecular oxygen is probably the ionized constituent. The hypothesis is examined quantitatively and a recombination coefficient of approximately $2 \times 10^{-6} \text{ cm}^3 \text{ sec}^{-1}$ for molecular oxygen ions is deduced.

