SMITHSONIAN CONTRIBUTIONS to ASTROPHYSICS

VOLUME 2

SMITHSONIAN
INSTITUTION

WASHINGTON
D.C.

Contents of Volume 2

Smithsonian

Contributions to Astrophysics

VOLUME 2, NUMBER 1

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by SYDNEY CHAPMAN

with a

SUPPLEMENTARY NOTE

by HAROLD ZIRIN

SMITHSONIAN INSTITUTION

Washington, D. C.

1957

Publications of the Astrophysical Observatory

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Cambridge, Mass.
February 1957.

For sale by the Superintendent of Documents, U. S. Government Printing Office
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Notes on the Solar Corona and the Terrestrial Ionosphere

By Sydney Chapman [1]

Outward thermal conduction from the solar corona

We shall consider in the simplest possible way some properties that would characterize a model solar corona, static and spherically symmetrical (while recognizing that the actual corona is dynamic and asymmetric). The problem has already been discussed by many writers (Woolley and Stibbs, 1953). Van de Hulst (1953) has critically reviewed much recent work. The problem deals partly with the escape of coronal gas from the sun's gravitational field, e. g., as considered by Pikelner (1948, 1950) whose estimate of the rate of escape is judged by van de Hulst to be excessive. Rosseland (1933), Biermann (1948, 1951), and others have discussed the outward streaming of gas from or through the corona. These matters need further discussion, but I hope the present simple limited treatment is worth consideration.

Let the model corona consist entirely of protons and electrons, with equal number densities n. This and the temperature T are functions only of R, the distance from the center of the sun. Without inquiring how this comes to be, let us suppose that T has a maximum (T_0) at a radius R_0 which is taken to be 7.36 $\times 10^{10}$ cm, namely 4×10^9 cm greater than $R_\odot$, the "radius of the sun," 6.96×10^{10} cm (Woolley and Stibbs, 1953). Initially no account is taken of magnetic fields; their influence is briefly considered in the fourth section.

The thermal conductivity K of a proton-electron gas can be calculated fairly exactly (Chapman, 1954). It is 0.324 K_e, where K_e denotes the thermal conductivity, for the same n and T, of a gas composed of electrons alone, half positive, half negative, supposed permanent, without recombination.

Let k denote Boltzmann's constant, e the electrostatic unit of charge, m the electron mass. Then, allowing for a factor 4/3 to correct the first approximation to the formula for K (Chapman, 1954, p. 155), we obtain

$$K_e=\frac{25}{2A_2(2)}\frac{k^{7/2}T^{5/2}}{(\pi m)^{1/2}e^4},$$

where

$$A_2(2)=2\ \ln(1+x^2)-2x^2/(1+x^2);$$

and

$$x=\frac{4kT}{e^2}\left(\frac{kT}{8\pi e^2 n}\right)^{1/2}$$

by use of the Debye cutoff distance (Spitzer, 1956) instead of the cutoff distance that I proposed in 1922 (Chapman, 1922); the adoption of a fixed cutoff distance is a weakness in the calculation, but the resulting value of K is probably correct within 10 percent. With values of T and n appropriate to the corona, x is large, and approximately

$$A_2(2)=35.5+13.8\ \log\ (T/n^{1/3}).$$

Thus, if $T=10^6$ and $n=10^9$, $A_2(2)=76.9$ (the use of my cutoff distance would give 55.8). If $T=10^6$ and $n=10^6$, $A_2(2)=90.7$. If $T=2\times 10^5$ and $n=10^3$, $A_2(2)=94.9$. The dependence of $A_2(2)$ on T is not great, and on n is decidedly small. Here $A_2(2)$ will be taken to have the value 85, independent of T and n.

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Then for proton-electron gas, at temperature T,

$$K=A\ T^{5/2}\ \text{erg/cm sec°K} \qquad (1)$$

where $A=5.2\times10^{-7}$; Woolley and Allen (1950) adopted 5×10^{-7} as the value of A (Woolley and Stibbs, 1953, p. 234), taking $A_2(2)$ as 50, and not using the factors 0.324 and 4/3, not then published (Chapman, 1954). The present estimate of A may be uncertain to 10 percent.

It is convenient to rewrite equation (1) as

$$K=K_0(T/T_0)^s, \qquad (2)$$

so that K_0 denotes the thermal conductivity at T_0, and $s=5/2$.

The model corona will extend to infinity, where we suppose that $T=0$. The conditions being steady, there will be a constant outward flow of heat by thermal conduction; the flux F over the sphere of radius R will be independent of R, and is given by the equation,

$$F=-4\pi R^2KdT/dR. \qquad (3)$$

The gas being supposed everywhere fully ionized, so that equation (2) is valid at all distances, equation (3) may be rewritten as

$$dT^{s+1}=Cd(1/R) \qquad (4)$$

where

$$C=(s+1)F\ T_0^s/4\pi K_0. \qquad (5)$$

As $T=T_0$ at $R=R_0$, the integration of equation (4) gives

$$T_0^{s+1}-T^{s+1}=C/R_0-C/R. \qquad (6)$$

As $T=0$ at infinity,

$$C=R_0T_0^{s+1},$$

and therefore

$$F=4\pi R_0K_0T_0/(s+1). \qquad (7)$$

Hence also

$$T=T_0(R_0/R)^{1/(s+1)}, \qquad (8)$$

which gives T at any radius R. As $s=\frac{5}{2}$, T varies as $1/R^{2/7}$, decreasing only slowly from the sun.

In table 1 are given the mean distances x of the planets from the sun in astronomical units, and the corresponding values of $10^6T/T_0$ according to equation (8); if T_0 is 10^6, column c gives the inferred values of the temperature of the coronal gas at the distance of each planet. If T_0 is greater or less than 10^6, the values in column c must be increased or decreased proportionately.

Owing to the eccentricities of the orbits, especially of Mercury, Mars, and Pluto, the temperature of the gas encountered in their orbits will vary; the variations of $10^6\ T/T_0$ are as follows for these three planets:

Mercury	270, 000° to 308, 000°
Mars	185, 000° to 195, 000°
Pluto	72, 000° to 83, 000°

The variation of T near the sun, and at greater distances up to the earth ($R_E/R_\odot=215$) and beyond, are illustrated in table 2 by the ratio T/T_0. (The distance 1.06 $R_\odot$ represents R_0; more exactly the chosen value of R_0 is 1.058 $R_\odot$.) Table 2 also illustrates the distribution of the (negative) radial temperature gradient, in degrees per km; it refers to $T_0=10^6$, and for other values of T_0 the values in table 2 need to be multiplied by $T_0/10^6$, because from equation (8) it follows that

$$-\frac{dT}{dR}=\frac{2}{7}\frac{T}{R}=\frac{2}{7}\frac{T_0}{R_0}\left(\frac{R_0}{R}\right)^{9/7}.$$

The table well shows how small is the temperature gradient, as compared with that in the earth's own atmosphere, especially for distances greater than 3 R_0.

TABLE 1.—*Properties of corona model in neighborhood of planets.*

a	b	c	d	e	f	g
	x	$10^6\ T/T_0$	E(ev)	v_+	$10^{-3}v_e$	v_P
Mercury	0. 387	288, 000	37. 2	106	4. 56	48
Venus	0. 723	240, 000	31. 0	77	3. 30	32
Earth and Moon	1	219, 000	28. 2	74	3. 16	30
Mars	1. 524	190, 000	24. 5	69	2. 94	24
Jupiter	5. 503	135, 000	17. 4	58	2. 48	13
Saturn	9. 539	115, 000	14. 8	53	2. 29	10
Uranus	19. 191	94, 000	12. 1	48	2. 07	7
Neptune	30. 07	83, 000	10. 7	45	1. 94	5
Pluto	39. 6	77, 000	9. 9	44	1. 87	5

TABLE 2.—*Coronal temperature gradients.*

$R/R_\odot$	T/T_0	$-dT/dR$ in degrees/km for $T_0=10^6$	$R/R_\odot$	T/T_0	$-dT/dR$ in degrees/km for $T_0=10^6$
1.06	1.000	.388	5	.642	.0527
1.1	.989	.369	6	.609	.0417
1.2	.964	.330	10	.526	.0216
1.3	.943	.298	15	.469	.0128
1.5	.905	.248	50	.332	.00273
1.7	.873	.211	100	.273	.00112
2.0	.833	.171	150	.243	.00066
2.6	.773	.122	200	.224	.00046
3.0	.742	.102	215	.219	.00042
4.0	.733	.072	250	.210	.00034

The thermal flux *inward*, from the corona to the chromosphere, has been estimated by Alfvén (1941), Giovanelli (1949), and Woolley and Allen (1950) (see also Woolley and Stibbs, 1953, p. 234) with different results, depending on the different estimates of K and dT/dR. The value given by Woolley and Allen is 1.8×10^4 ergs/cm^2 sec, or, over the whole corona,

$$\text{inward heat flux}=1.1\times10^{27}\ \text{ergs/sec.}$$

From equation (7), for $T_0=10^6$, it is calculated that

$$\text{outward flux}=1.38\times10^{26}\ \text{ergs/sec.}$$

This is naturally decidedly less than the inward flux, because the estimated inward temperature gradient (12.5°/km) so much exceeds the outward gradient.

If T_0 differs from 10^6, this result must be multiplied by $(T_0/10^6)^{7/2}$, a factor here given for a few other values of T_0:

T_0	500,000	1,500,000	2×10^6	3×10^6
$(T_0/10^6)^{7/2}$	0.088	4.13	11.3	46.8

This shows how considerably the outward conductive loss of heat from the corona to outer space would alter if T_0 should change from a half million to 1.5 million degrees.

Woolley and Allen (1950) estimated the radiative loss of energy from the corona to outer space to be 2.7×10^{26} ergs/sec, or about twice the outward conductive flux here given. This radiative loss of energy proceeds mainly from the layers of the corona near the sun, where metal ions are most abundant and the density is greatest. Beyond a few solar radii the radiative loss of energy must be a small fraction of the conductive loss.[1] Their estimate of the total power needed to maintain the corona at 10^6 degrees is little altered by the inclusion of the outward conductive flux: at 1.5×10^{27} ergs/sec it is about 4×10^{-7} of the total solar output.

Column *d* of table 1 gives the energy E per proton or electron, in electron volts, for a gas whose T equals the number shown in column *c*; columns *e* and *f* give the corresponding root mean square speeds v_+, v_e of the protons and electrons, in km/sec. Column *g* gives the mean orbital speed v_P of the planet, in km/sec. The speed v_+ exceeds the speed of sound V in the ratio $(3/\gamma)^{1/2}$, where γ denotes the ratio of the specific heats: for a proton-electron gas, $\gamma=5/3$, hence $(3/\gamma)^{1/2}=1.34$. Comparison of columns *e* and *g* shows that if $T_0=10^6$ the planetary motion is decidedly subsonic in the coronal gas, and the Mach ratio v_P/V decreases from about 1/3 for Mercury to about 1/12 for Pluto.

Column *d* shows that if $T_0=10^6$ the mean particle energy exceeds 13.6 ev, which is the ionization potential of atomic hydrogen, up to and beyond the orbit of Saturn. As E decreases, the proportion of neutral hydrogen atoms will slowly increase. This will modify the radial variation of the thermal conductivity K, and of the temperature gradient and density.

Statical equilibrium of the model corona

The equation of statical equilibrium of the model corona is

$$dp=-\rho g\ dR$$

or

$$d\ln p=-(dR)/H, \tag{9}$$

where

$$H=kT/Mg. \tag{10}$$

Here p and ρ denote pressure and density, H the scale height, M the mean molecular mass; for a proton-electron gas, M is 8.37×10^{-25}, half the mass of a hydrogen atom. Also

$$g=g_0(R_0/R)^2, \tag{11}$$

where g and g_0 denote the value of solar gravity at

[1] See the accompanying Supplementary Note by H. Zirin dealing with this question.

R and R_0; as the surface gravity on the sun, at $R_\odot=6.96\times10^{10}$, is 2.74×10^4, the value of g_0 is 2.45×10^4. If $T_0=10^6$, the value (H_0) of H at R_0 is 6.73×10^9 cm. Let

$$y=R/R_0,\ u=7R_0/5H_0;$$

for $T_0=10^6$, $u=15.31$.

Substitution in equation (10) for T from equation (8), and for g from equation (11), gives

$$H=H_0y^{12/7}, \tag{12}$$

and so equation (9) leads to the solution

$$\ln(p/p_0)=-y(1-y^{-5/7}). \tag{13}$$

Hence p decreases steadily outwards, not to zero but to the finite limit p_0e^{-u}; if $T_0=10^6$, the factor e^{-u} is 2.24×10^{-7}.

For the number density $n(=p/kT)$,

$$\frac{n}{n_0}=\frac{T_0}{T}\frac{p}{p_0}=y^{2/7}\exp\{-u(1-y^{-5/7})\}; \tag{14}$$

the factor T_0/T increases outwards from the sun, the factor p/p_0 decreases outwards; n has a minimum value at R_m, where

$$\frac{R_m}{R_0}=\left(\frac{7}{2}\frac{R_0}{H_0}\right)^{7/5}=\left(\frac{5u}{2}\right)^{7/5}. \tag{15}$$

The minimum number density n_m is given by

$$\frac{n_m}{n_0}=\left(\frac{5u}{2}\right)^{2/5}e^{-u+0.4} \tag{16}$$

For $T_0=10^6$ the minimum n is at 1.21×10^{13} cm or 0.809 astronomical unit; and n_m/n_0 is 1.435×10^{-6}. The corresponding value of n/n_0 at the earth's distance is only slightly higher, by less than .5 percent; at ten astronomical units n/n_0 is 2.11×10^{-6}.

The distance R_m and ratio n/n_0 is rather sensitive to a change in T_0. For a few values of T_0 table 3 gives values of R_m (expressed in astronomical units as x_m), n_m/n_0, and also of n_E/n_0, where n_E signifies the number density at the earth's distance.

Table 3 indicates a considerable variation of the distance of minimum n as T ranges from 8×10^5 to 1.5×10^6; for the lowest of these temperatures, the minimum is attained just beyond the earth's orbit; for the highest of these temperatures, it is attained about half way between the sun and the earth. Over this considerable range of T and of x_m, however, the variation of n is very slight, near and beyond the distance of minimum n, and at the earth's distance n has almost the same value as at the minimum.

Table 3 shows also that at the earth's distance, n/n_0 increases by a factor 620 as T_0 increases from 8×10^5 to 1.5×10^6. Van de Hulst (1950) finds that n_0 decreases from sunspot maximum to sunspot minimum; for R-1.06 solar radii (approximately the same as R_0) his sunspot maximum value of n_0 is 2.35×10^8, the same in all solar latitudes; at sunspot minimum his polar value of n_0, 8.7×10^7, is notably less than the equatorial value, 1.3×10^8. If T_0 also is greater at sunspot maximum than at sunspot minimum, the disparity of n_0 at the two phases will make the number density n at the earth vary by an extra factor of about two as compared with what table 2 would indicate, if n_0 were assumed *constant.*

TABLE 3.—*Minimum density and density at 1 A. U. for coronal models.*

(In this table, numbers of the form a^v signify $a\times10^v$)

$10^{-6}T_0$	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
x_m	1.011	0.899	0.809	0.735	0.674	0.622	0.578	0.539
n_m/n_0	3.35^{-7}	2.73^{-7}	1.44^{-6}	5.57^{-6}	1.71^{-5}	4.45^{-5}	9.98^{-5}	2.01^{-4}
$n_E/n_0(x=1)$	3.43^{-7}	2.74^{-7}	1.44^{-6}	5.64^{-6}	1.75^{-5}	4.57^{-5}	1.04^{-4}	2.12^{-4}

Behr and Siedentopf (1953) have estimated that near the earth n ($=n_E$) is about 600. If n_0 is taken to be 2×10^8, their value gives 3×10^{-6} for n_E/n_0 at the earth's distance. According to table 2, this would correspond to a value of T_0 between 10^6 and 1.1×10^6. If the actual corona varies with T_0 at all like the model corona here discussed, it would suggest that the average value of T_0 over the sun undergoes only moderate changes.

The changing radial distribution of the proton-electron gas (if T_0 changes) involves an outward movement of gas if T_0 increases; if T_0 decreases, the gas must draw back towards the sun. Changes propagated with the speed of sound would take about two months to reach the earth's orbit. The actual corona seems to show a nonuniform changing distribution of T_0. The irregularities of T_0 over the corona will influence the proton-electron gas at a distance from the sun, but in a smoothed-out manner.

Table 4.—*Empirical and model densities in corona.*
(In this table, numbers expressed as a^y signify $a\times 10^y$)

$R/R_\odot$	n (van de Hulst: Sunspot max.)	n (calculated)		
		$T_0=10^6$	$T_0=1.1\times 10^6$	$T_0=1.2\times 10^6$
1.06	2.35^8	(2.35^8)	(2.35^8)	(2.35^8)
1.1	1.60^8	1.54^8	1.61^8	1.67^8
1.2	7.08^7	6.59^7	7.31^7	8.11^7
1.3	3.76^7	3.08^7	3.70^7	4.34^7
1.5	1.48^7	8.97^6	1.20^7	1.55^7
1.7	7.11^6	3.33^6	4.91^6	6.89^6
2.0	2.81^6	1.04^6	1.70^6	2.66^6
2.6	6.65^5	2.16^5	4.18^5	7.26^5
3.0	3.13^5	1.02^5	2.11^5	3.89^5
4.0	9.0^4	2.9^4	6.7^4	1.4^5
5.0	4.4^4	1.3^4	3.3^4	7.1^4
6.0	2.9^4	7.2^3	2.0^4	4.4^4
10.0	——	2.2^3	6.6^3	1.7^4
15.0	——	1.1^3	3.7^3	9.8^3
50.0	——	4.2^2	1.5^3	4.6^3
100.0	——	3.5^2	1.3^3	4.1^3
150.0	——	3.4^2	1.3^3	4.0^3
200.0	——	3.4^2	1.3^3	4.1^3
215.0	——	3.4^2	1.3^3	4.1^3
250.0	——	3.4^2	1.3^3	4.2^3

Baumbach (1937) and van de Hulst (1950), using eclipse coronal data, have given estimated distributions of n as a function of R up to a few solar radii. The extension of the solar corona has also been studied by observing how the radio waves from the radio star in Taurus vary in intensity when they traverse the outer corona (Hewish, 1955). The radial distribution of n estimated by van de Hulst (1950) for sunspot maximum is compared in table 4 with the distributions given by equation (14) by taking three different values of T_0 and adopting van de Hulst's value of n_0, namely, 2.35×10^8. None of the three distributions exactly fits van de Hulst's series of estimates, but the "fit" of the calculated values, depending on only one chosen parameter, T_0 (n_0 having been adopted to agree with the eclipse estimate at $R_\odot$), seems satisfactory, for $T_0=1.1\times 10^6$ or for some value between this and $T_0=1.2\times 10^6$. This value has the order of magnitude indicated by coronal temperature measurements; it gives a value of n at the earth's distance of the same order as that indicated by the zodiacal light observations; and at the intermediate distances, 5 to 15 times $R_\odot$, to which the Taurus data refer (Hewish, 1955), the calculated values of n seem to be consistent with the radio star observations, though these only set lower limits to n.

In some recent discussions of the far extension of the corona it has been supposed that n varies as some inverse power of R; there seems to have been no suggestion that n might have a minimum value. The formula in equation (14) suggests a different kind of distribution. Table 3 shows that near the earth's distance the calculated radial variation of n is very slight for the three values of T_0 there considered; this is also implied by the low values of dT/dR given in table 2. In this respect the present calculations differ from the results given by Behr and Siedentopf (1953); it is difficult to see how a rapid outward decline of n near the earth could be produced.

The mean free path, collision frequency and collision interval for the electrons

The thermal conductivity in the proton-electron gas is mainly due to the electrons (Chapman, 1954); it is reduced by the presence of the protons. The collision frequency ν for an electron (of mass m) is given by

$$\nu = 2n(\pi kT/m)^{1/2}(2\sigma^2 + 2^{1/2}\sigma_1^2), \qquad (17)$$

where σ, σ_1 are the mean collision distances for electron-electron and electron-proton collisions respectively. These may be evaluated as follows. Let D and K_e denote the first approximations to the coefficient of diffusion in a proton-electron gas, and to the coefficient of thermal conductivity in an all-electron gas (see first section). Formulae for D and K_e (Chapman and Cowling, 1952)[1] are calculated (a) by regarding the particles as rigid elastic spheres, and (b) by considering them to be centers of Coulomb forces; D depends only on σ_1, and K_e only on σ. By comparing the two forms for D and the two forms for K_e, we obtain expressions for σ and σ_1. They are found to be almost the same (Chapman and Cowling, 1952, pp. 218, 235):

$$\sigma^2 = \tfrac{1}{2} A_2(2)(e^2/2kT)^2, \qquad (18)$$

and (Chapman and Cowling, 1952, p. 245),

$$\sigma_1^2 = A_1(2)(e^2/2kT)^2, \qquad (19)$$

where (Chapman and Cowling, 1952, p. 179) in the notation in the first section,

$$A_1(2) = \ln(1+x^2). \qquad (20)$$

Thus $A_1(2)$ differs from $\tfrac{1}{2} A_2(2)$, when x is large (as here), only by 1. For $A_2(2)=85$, it is sufficiently accurate to take

$$\sigma^2 = \sigma_1^2 = 3.0\times10^{-6}/T^2. \qquad (21)$$

Hence

$$\nu = 14.14\ n/T^{3/2}, \qquad (22)$$

and the mean collision interval t is $1/\nu$.

At R_0, taking $n_0 = 2.35\times10^8$, $T_0 = 10^6$, gives

$$\nu_0 = 3.32/\text{sec},\ t_0 = 0.30\ \text{sec},$$

and at R_E, the distance of the earth,

$$\nu_E = 4.7\times10^{-5}/\text{sec},\ t_E = 2.1\times10^4\ \text{sec} = 5.9\ \text{hours}.$$

[1] Second edition. Pagination cited herein also applies to first edition' 1939.

The mean free path L is given by the expression,

$$L = (8kT/\pi m)^{1/2} t = 4.5\times10^4 T^2/n.$$

Hence at R_0, taking T_0 and n_0 as before,

$$L_0 = 1900\ \text{km},$$

and at R_E,

$$L_E = 6.4\times10^7\ \text{km}.$$

Thus the mean free path of an electron at the distance of the earth is given as rather less than half the distance to the sun. As table 4 indicates, over this distance the number density of the coronal gas varies only very slightly; also the temperature gradient is small, this being ensured by the long electron paths. The proton mean free paths are of similar magnitude.

The question may be raised: Is the thermal conductive flux rightly given by equation (3) when the number density is so small as 10^3, or even 10^8? A question of this kind has been considered by Cowling and myself (Chapman and Cowling, 1952, p. 260) for a simple gas. The value r of the conductive term in $\Phi^{(1)}$, where $f^{(0)}$ and $f^{(0)}(1+\Phi^{(1)})$ are the first and second approximations to the velocity distribution function f, is there given as of order

$$\frac{45}{32}\left(\frac{2}{\pi}\right)^{1/2} \frac{1}{n\sigma^2 T}\frac{dT}{dR},$$

wherein use has been made of formula (1) on page 218 of Chapman and Cowling (1952). Substituting for σ^2 and dT/dR gives the formula

$$r = 10^5\ T^2/nR.$$

At R_0 this has the value 0.006, showing that in the hottest part of the corona the usual kinetic theory of thermal conduction is adequate. At the earth's distance R_E, taking n_0 and T_0 as before, r is slightly less than 1. Even under these conditions of very low density, the disturbance of the Maxwellian velocity distribution function f_0, though substantial, is moderate; but in view of this value of r it is desirable to consider what contribution is made to the flux F by the next term in f. The third approximation to f is discussed in Chapter 15 of Chapman and Cowling (1952). It is there shown (p. 265) that the flux of conducted heat is unaffected by the further approximation, if the gas is at rest; no new terms are introduced into the heat flux "depending on the second or higher space-derivatives of T, nor on squares or products of derivatives of T." The third approximation to f, however, can supply a thermal flux even when T is uniform, if the gas is in motion. In the present discussion the gas is supposed to be everywhere at rest, although this is clearly an idealization of the actual changing corona. Hence it is concluded that the above calculations of T and n for our model corona are unlikely to be seriously in error even up to and somewhat beyond the earth's orbit.

Possible influence of the solar magnetic field

It is known (Chapman and Cowling, 1952, pp. 325–329) that in a magnetic field the thermal conductivity is reduced for heat flow perpendicular to the field. It is still uncertain whether the sun has a general magnetic field extending into space like the field of a dipole *in vacuo*. If it has, and even if the strength at its poles is no more than 1 gauss, the outward heat flow here calculated must probably be much reduced, except in high heliomagnetic latitudes. Thus the temperature would decrease outwards more rapidly, and the extension of the coronal gas would be more limited. There is order-of-magnitude agreement between the density near the earth as here calculated (if we assume only n_0, T_0 and no solar magnetic field for $R>R_0$) and that inferred from zodiacal light observations. This may be taken as an indication that the sun's general magnetic field (if any) does not pervade the corona. The present calculations would be much less affected by local sunspot fields, or by local magnetic fields convected by solar clouds or streams issuing from the sun. This question of the possible influence of magnetic fields on the heat flow in the corona, and on the far extension of the corona, seems worthy of further consideration, but this will not be attempted here.

How does the coronal gas affect a nonmagnetic planet?

The mean free path of the protons and electrons of the coronal gas much exceeds the radius of the planets that are immersed in the gas. The mean random speeds of the protons exceed the planetary speed, and the excess is still greater for the electrons. Hence, as in the case of meteors (Herlofson, 1948) traversing the higher layers of our atmosphere, we may infer that no cap of compressed gas will form in front of the planet. Planets without magnetic fields will be subject to the direct impact of the coronal protons and electrons. If the planet has an atmosphere, the coronal particles will penetrate its outermost layers; if there is no atmosphere, as, for example, is supposed to be the case for the moon, the coronal gas will be in direct contact with the planetary surface. It is of interest to consider the impact of the coronal gas, supposing there is no magnetic field.

The flux of particles of number density n and mean random speed v, upon a fixed surface in a gas in equilibrium, is $\frac{1}{4} nv$ (Chapman and Cowling, 1952, p. 100). Now n is the same for the protons and electrons, whereas v_e far exceeds v_+. Hence the flux of electrons upon a planet would much exceed that of the protons. The impinging flux may be balanced for each kind of particle by an equal reflected or scatter flux. Suppose, instead, that the particles are absorbed by the planet or its atmosphere. Then the excess electron flux will rapidly charge the planet negatively, so as to equalize the two fluxes. Let the planet, for the present, be supposed at rest relative to the gas.

The electric field will retard the radial component speed of the electrons, and increase that of the protons. As the protons and electrons must keep together, until they enter the atmosphere or body of a planet, the electrostatic energy gained by the protons equals that lost by the electrons; hence their combined energy is constant. If w_0, w_+ are the radial speeds at a distance, and w their common speed at the earth,

$$\tfrac{1}{2}(m_e w_e^2 + m_+ w_+^2) = \tfrac{1}{2}(m_e + m_+) w^2 = \tfrac{1}{2} m_H w^2, \qquad (23)$$

where m_H denotes the mass of a hydrogen atom. As the mean value of $\frac{1}{2}m_e w_e^2$ and $\frac{1}{2}m_+ w_+^2$ is $\frac{1}{2}kT$, it follows that

$$\overline{w^2}=kT/m_H,$$

where the bar signifies that the mean value of the function below it is to be taken. For $T=219{,}000$, $\overline{w}=42.5$ km/sec. The radial proton speed is increased in the ratio $\sqrt{2}$, and the radial electron speed is reduced to this value.

The energy supplied to the planet will be greater for particles of speed higher than the average; a simple calculation suggests that on this account the average energy brought to the planet per proton-electron pair is not $3kT$ but $4kT$.

If a_P is the radius (from the center of the planet) at which the coronal protons and electrons are absorbed into the planet or its atmosphere, the total flux of energy to the atmosphere (still neglecting the orbital motion of the planet) would be

$$n\overline{w}\,(4kT)\,\pi a_P^2$$

or

$$4\pi n a_P^2\,(kT)^{3/2} m_H^{-1/2}.$$

If we take $T=219{,}000$ (corresponding to $T_0=10^6$), this is

$$1.6\times10^{-3} n a_P^2.$$

The orbital motion of the planet will affect this calculation. The order of magnitude can be estimated, very crudely, as follows. Consider the flux of energy per cm² sec on to the planet at its foremost and rearmost points, where the collisions are respectively head-on and overtaking. The fluxes are

$$\frac{1}{8}\,n m_H\,\overline{(w\pm v_P)\left\{u^2+v^2+(w\pm v_P)^2\right\}},$$

where the mean value indicated by the overbar is to be taken over all molecules with a positive impact speed; u and v denote the lateral component speeds. By making crude approximations, the energy flux averaged over the fore and rear areas is of order (per cm² sec)

$$\frac{1}{8}\,n m_H\,\overline{\left\{w\,(u^2+v^2+w^2)+3\overline{w}v_P^2\right\}},$$

where the mean values are now supposed to be taken as if v_P were zero. The ratio of this to the flux for a stationary planet is of order

$$1+3v_P^2/\overline{w^2}. \quad (24)$$

If $v_P=30$ km/sec and $\overline{w}$ is 42.5 km/sec, this number is 2.5. For impacts between the fore and rear points along the orbit the ratio will be less. Until the ideas in this paper have been subjected to critical consideration by those learned in this field, it does not seem worthwhile to go more elaborately into this question. But it seems reasonable to take 1.5 as an order-of-magnitude estimate of the over-all factor by which the total coronal energy flux into the atmosphere on the moving planet exceeds what it would be on a stationary planet for which v_P and T have the supposed values. The motion of the planet will cause the energy flux to be greater over the forward than over the rear hemisphere; this must produce a daily variation of temperature in the high layer of the atmosphere that absorbs the coronal particles.

When we insert the orbital-motion factor 1.5 in the expression for the energy flux into the atmosphere, this becomes

$$2.4\times10^{-3} n a_P^2. \quad (25)$$

This energy is kinetic energy; each electron-proton pair also brings its energy of ionization, 13.6 ev. This increases the energy supply by nearly 20 percent, if this additional energy is "absorbed" by the atmosphere. The kinetic energy will be absorbed, and will be increased by the gravitational pull of the planet, but the increase per proton-electron pair, for a planet like the earth, is less than 0.6 ev, and may be left out of account.

Applying this calculation to the moon, supposed to be devoid of an atmosphere and any hindering magnetic field, a_P must be identified with the actual radius of the moon. Hence the flux of coronal kinetic energy to the surface is $2.4\times10^{-3}n/4\pi$ or $1.9\times10^{-4}n$ ergs/cm² sec. If $n=10^3$, this is 0.19 erg/cm² sec. At the subsolar point of the moon, the flux of incident solar radiation is 2 cal/cm² minute or 1.4×10^6 erg/cm² sec. Thus the heat flux from the corona is about 10^{-7} of the maximum solar radiation flux on the sunlit half of the moon. Hence the coronal gas seems unlikely to have any appreciable influence on the temperature of the dark side of the moon. This temperature is now inferred, from the study of microwave thermal radiation (Piddington and Minnett, 1949; Jaeger, 1953), to be of order 150° K, superseding the much lower values still sometimes quoted (Duncan, 1946).

If the planet has an atmosphere, the coronal particles will penetrate only its outermost layer, even on the forward hemisphere. The kinetic energy of the coronal particles will be shared with the particles of the atmosphere; if these likewise are mainly protons and electrons, the energy can be shared only as kinetic or heat energy; if they include other kinds of ions and neutral particles, a fraction of the coronal energy may go into excitation and ionization, but most of it will be converted into local heat energy. In a steady state this must be removed at the same rate as it is supplied; the removal will be by downward conduction. The energy will descend as heat to lower layers where it can escape, probably by long-wave radiation from polyatomic molecules. This downward heat flux will be superposed on the other processes of supply and dispersal of energy, by solar wave or corpuscular radiation. It implies an upward increase of atmospheric temperature towards a high value, perhaps of the same order as that of the coronal gas. This rise of temperature will facilitate the escape of lighter gases of the atmosphere, beyond the rate that would obtain in the absence of the coronal gas.

Zodiacal dust

The zodiacal light (Allen, 1946; van de Hulst, 1947) indicates the presence round the earth of multimolecular particles ranging in radius up to 0.03 cm (van de Hulst, 1947), or on another view, 0.001 cm (Allen, 1946); such particles are supposed to be similar in size to telescopic meteors. Van de Hulst considers that only a small proportion of the zodiacal light particles have speed sufficient to render them visible when they enter our atmosphere. Watson (1937) estimated that about 5 tons of meteoric matter falls daily upon the entire earth (Whipple, 1955); van de Hulst estimated the total accretion to be 10^4 times as great, mainly by zodiacal particles. If the bulk of this matter enters the earth's atmosphere with the speed simply of free fall (about 10 km/sec), the energy flux over the earth will be of order 3×10^{17} erg/sec. Some of this energy may be absorbed below 100 km, but the main contribution, by the smaller particles, would be made at higher levels.

Influence of the coronal gas upon the outermost atmosphere of the earth

The geomagnetic field complicates the consideration of the terrestrial influence of the proton-electron gas. This question has already been discussed by Dungey (1954), in the light of the theoretical discussions of magnetic storm phenomena by Ferraro and myself (Chapman and Ferraro, 1931; 1932; 1933; 1940). As Dungey says, the problem is difficult, and no attempt is made to solve it here. Dungey concludes that, as in the theory of magnetic storms, the geomagnetic field will exclude the coronal gas from a space around the earth. He estimates the radius of the cavity in the plane of the equator to be $9a$, where a denotes the earth's radius.

When a solar cloud or stream of fast-moving gas is first projected towards the earth, a cavity will be formed in it; this is a passing event. But if the coronal gas surrounds the earth for long periods, it seems to me probable that the cavity will slowly fill up. Suppose the coronal temperature were to fall to a low value, causing the equilibrium density n near the earth to become inappreciable. If, then, T_0 rose again to a steady value of 10^6 degrees or more, the corona would expand, and on reaching the earth a cavity would be formed in it. The particles, protons and electrons, would be magnetically deflected back into the gas at the surface of the cavity. Collisions there would be rare, but when they occurred they would scatter the colliding particles, which could remain in the vicinity or spiral along the lines of force towards the earth. They could also slowly diffuse laterally into the cavity, by the action of further collisions, with spiralling motion. If the density of the coronal gas remains fairly uniform over long periods, the cavity might thus fill up with gas, forming an extension of the terrestrial atmosphere. At first the surface of the cavity would act like an almost perfect reflector, and no energy would be lost to the coronal gas. If the cavity filled up, the extended atmosphere would trap some of the energy of the impinging coronal particles, and this energy could be conducted downwards to lower levels in the atmosphere. The heat conductivity along the lines of force is unimpeded by the magnetic field, but the conduction transverse to them is reduced by the field. The expression (25) for the kinetic energy brought to the radius of penetration gives an upper limit to the downward flux of energy.

In the terrestrial case, suppose that the radius a_P of the penetration level is $5a$, and n is 10^3. If the air temperature rises to be of order 10^5 at this level, the mean temperature gradient between this level and the F_2 peak would be about 4°/km. The maximum coronal heat available to the whole of the earth's atmosphere at this level would be 2.4×10^{19} erg/sec.

Whether any significant fraction of this energy is actually trapped by the earth must remain uncertain until the trapping process above suggested, or some alternative process, is established by theory or until air temperature measurements above the F layer show that there, also, heat energy is flowing downwards.

The temperature of the outer ionosphere

The temperature at the F_2 peak itself is still uncertain, but several types of evidence indicate a value of about 1000° K; this is supported by Gallet's studies (unpublished) of the absorption of radio waves in the F layer.

The thermal economy of the upper atmosphere has been discussed by Spitzer (1952), Bates (1951), Johnson (1956), and others. Some of these discussions (Spitzer, 1952; Bates, 1951) mention the possibility that a significant energy contribution may come from particles in the space around the earth, but mainly they proceed on the supposition that there is no such contribution. Spitzer drew attention to the importance of thermal conduction. He estimated K (using a rigid-elastic-sphere formula) to be 4.1×10^3 erg/°cm sec. He determined the temperature gradient required to enable conduction to dispose of the energy absorbed in the F layer from solar radiation. This energy has not yet been measured, and it was assumed to be X times the energy in black body solar radiation at 6000° K for wavelengths shorter than 910 A. The conclusions drawn were: If $X<5$, T will be constant above the O_2 dissociation level, probably about 150 km; if $5<X<10$, T will be determined jointly by the absorbed radiation and by conduction, and an average value of 1500° K, reached some distance above 300 km, would be consistent with all known facts; if $X>100$, there will be radiative equilibrium, with $T>2000°$ from about 300 km upwards (according to Spitzer (1952, p. 239), "this value is much too great for an average temperature, but might reasonably be expected during a burst of intense ultraviolet radiation from the sun").

Bates (1951) gave estimates of K using a formula showing a greater dependence on T than the formula used by Spitzer. For $T=1000$ he found $K=5.9\times10^3$ erg/°cm sec. He estimated the supply of solar radiation energy needed to ionize the F layer as observed. He found the chief loss process to be conduction, but with the estimated energy supply this would imply a temperature gradient of only 0.4°/km (at sunspot minimum); this would not accord with the supposed high temperature at the F_2 peak. Hence he sought other possibilities of energy supply, and suggested tentatively that this is mainly associated with unobserved F layer ionization.

The total heat flux descending below the F_2 peak may be estimated as 9.5×10^{17} ergs/sec, taking $K=5\times10^3$ and dT/dh as 3.5°/km. This is about 4 percent of the maximum supply here estimated as possibly coming from the coronal gas; it is thrice as much as the estimated amount that zodiacal dust would supply. All three estimates, of course, are uncertain, particularly those of the heat possibly coming from particles outside. The zodiacal dust, though it may be photoionized, is not so subject to the influence of the geomagnetic field as the coronal gas, and is likely to make some contribution to the heat balance of the atmosphere. The estimation of the supply from the coronal gas involves more difficult considerations. In view of the other unsolved difficulties in the problem of the heat balance of the upper atmosphere, it seems desirable to bear in mind, and to examine, the possibility that the observed heat flux below the F layer has come *through* the F layer from above, and is not merely a loss process of the F layer itself. If this be the case, our picture of the outermost atmosphere must be much modified. I hope elsewhere to deal with these ionospheric questions more in detail.

In conclusion, I wish to thank Dr. Eugene N. Parker and Dr. E. W. Dennison for helpful discussions on the subject of this paper.

This research, in part, was supported by the Air Force Cambridge Research Center, Geophysics Research Directorate, and by the Boulder Laboratories of the National Bureau of Standards.

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Abstract

For a model solar corona, static and spherically symmetrical, the radial distribution of temperature T and number density n is considered. For coronal gas composed solely of protons and electrons, the thermal conductivity varies as $T^{5/2}$, and depends little on n. A steady conductive heat flow to infinity ($T=0$) implies that T varies as $1/r^{2/7}$, and that if the coronal temperature T_0 is of order 10^6, the interplanetary gas is hot (about 200,000° near the earth). If in the hottest level of the corona the number density (n_0) is about 2 x 10^8 cc, near the earth n is of order 10^3, depending rather sensitively on T_0. This accords with zodiacal light indications, but differs from them as regards their suggestion of a rapid decrease of n near the earth. It accords also with inferences drawn from radio studies of "whistlers."

It is shown that the coronal gas will convey little heat to the dark side of the moon.

It is suggested that the hot coronal gas surrounding the earth may be the cause of the downward flow of heat below the F_2 peak in the terrestrial ionosphere; but difficult questions related to the influence of the geomagnetic field upon the fully ionized coronal gas render this suggestion speculative. It might be settled by observing the density or temperature above the F_2 peak.

The satellites to be launched during the International Geophysical Year will move in regions where such observations will be of special value in this connection.

Supplementary Note[1]

By Harold Zirin[2]

The preceding remarks on the corona by S. Chapman raise the following important question: Is the flux, assumed constant in equation (3), indeed independent of R? To answer this it is necessary to consider the nonconductive heat loss of the outer corona.

Since the mean molecular weight of the metals in the corona is around two or three compared to $\mu=\frac{1}{2}$ for hydrogen, the concentration of metals will fall off sharply with height. Furthermore, whatever processes of loss of coronal energy take place at lower levels (say less than 0.2 solar radii above the photosphere) are inextricably bound up with the unknown processes of energy input. We are therefore limited in our consideration of the losses of heat by the outer corona to emission by hydrogen and possibly helium ($\mu=\frac{4}{3}$).

Hydrogen can emit radiation by three processes, free-free, free-bound, and bound-bound transitions. For the first two it is possible to treat the hydrogen as completely ionized. The rates of free-free and free-bound emission, respectively, are (Allen, 1955):

(1A) Free-free $\quad 1.435 \times 10^{-27}\, T^{1/2}\, n_e^2$ ergs/cm³/sec

(2A) Free-bound $\quad 5.45 \times 10^{-22}\, T^{-1/2}\, n_e^2$ ergs/cm³/sec,

where the number of protons has been set equal to the number of electrons.

The energy emitted in the line spectrum of hydrogen is equal to the number of H I atoms times the rate of collisional excitation, times the energy per transition. But the number of H I atoms is determined by the ionization-recombination equilibrium, viz.:

$$\text{(3A)} \qquad \frac{\text{N(H I)}}{\text{N(H II)}}=\frac{\text{recombinations}}{\text{ionizations}}.$$

So the hydrogen line emission is given by:

$$\text{(4A)} \quad \epsilon_{B-B}=\frac{\text{N(H II)} \times \text{recombinations}}{\text{ionizations}} \times \text{excitation} \times \chi$$

where χ is the ionization energy of hydrogen. But Bethe (1933) shows that, at the electron energies under consideration, the rates of collisional ionization and of collisional ionization of hydrogen are roughly equal. Thus the line emission is

$$\text{(5A)} \quad \epsilon_{B-B}=\text{N(H II)} \times \text{recombinations}$$
$$=3.29\times 10^{6} n_e^2 \chi Z^4 T^{3/2} \Sigma_n \frac{e^{\chi_n}}{n^3} (-Ei(-\chi_n/kT))$$

the latter formula being from Cillié (1932). At $T=10^6$ this can be very well represented by

$$\text{(6A)} \quad \epsilon_{B-B}=1.64 \times 10^{-19} T^{-1} n_e^2 \text{ ergs/cm}^3\text{/sec.}$$

We can now integrate up the loss of energy by radiation, starting at the surface of the sun

[1] This work was supported by the Office of Naval Research under Contract Nonr 393(04).
[2] High Altitude Observatory, Boulder, Colo.

and extending out to the earth's orbit. The radiation to all three processes is

(7A)

$$4\pi\int_{R_0}^{R} r^2 n_e^2\,[1.435\times10^{-27}T^{1/2}+5.45\times10^{-22}T^{-1/2}+1.64\times10^{-19}T^{-1}]dr.$$

It is necessary to integrate each contribution separately, if we assume the temperature and density distribution given by equations (8) and (14). If ϵ_R is small compared to the total outward flux, then the temperature-density model presented, which neglects this loss of energy, will stand unaltered.

Making the substitutions referred to and setting $y=R/R_0$, we get

(8A) $$4\pi R_0^3 n_0^2 e^{-2u}\int_1^y y^{18/7}e^{2uy^{-5/7}}\times[1.435\times10^{-27}T_0^{1/2}y^{-1/7}+5.45\times10^{-22}T_0^{-1/2}y^{1/7}+1.65\times10^{-19}T_0^{-1}y^{2/7}]\,dy.$$

Since $u=15.31$, the integrand falls off extremely rapidly, but increases with great distance because of the $y^{18/7}$ factor. However, evaluation of (8A), if we leave out the exponential in the integrand (as is appropriate for large y), shows that even at $y=1000$ the contribution is extremely small. Evaluation of the integral close in shows that only the free-free emission is important; this gives a value of 0.237×10^{26} ergs/sec, roughly one-eighth of the outward flux. Furthermore, most of this energy is radiated at $y<1.10$, where it is inextricably bound up with the unknown input of energy to the corona.

It should be noted that observational data on the inner corona do not show a fall-off with height of the heavy atoms relative to hydrogen. This possibility would introduce an additional large sink for heat.

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