7741

A FINITE ELEMENT ANALYSIS OF MULTI-LAYERED ORTHOTROPIC MEMBRANES WITH APPLICATION TO OIL PAINTINGS ON FABRIC*

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1 INTRODUCTION

In 1975, a research program was initiated at the University of Maryland to develop an analytical procedure capable of predicting the effects of various environmental actions on fabric-supported oil paintings. The basic concept applied was that the behavior and response of a typical fabric-supported painting to external agents depends on the interaction of all components of the composite system, that is, stretcher, fabric, size, ground and paint film.

It was recognized that the success of the project depended on the completion of the two following major tasks:

1. the development of a mathematical model to evaluate accurately deformations and stresses in the composite system;
2. the accurate quantification of pertinent mechanical properties of the constituent materials.

The overall purpose of the work undertaken was to contribute to a better understanding of the mechanisms of deterioration of paint-film surfaces, thereby providing a means of evaluating current and proposed conservation treatments.

Obviously, the magnitude of the task compared with the limited resources available prevented a comprehensive consideration of all aspects of the problem. However, it is believed that the work completed to date provides for the first time a sufficient basis for the rational evaluation of the principal agents and mechanisms of deterioration.

This presentation concentrates on the numerical procedures developed. The equally important complementary work related to the material properties of the constituent materials has been presented by Mecklenburg [1].

2 MATHEMATICAL MODEL

The stress analysis of complex continua has developed rapidly in recent years due to the advent of the high-speed electronic digital computer. Thus it is now possible to employ numerical discretization techniques to obtain approximate solutions to previously insoluble problems. The most powerful of these numerical techniques is the finite element method (FEM). Originally developed intuitively, the technique has been generalized using variational principles of mathematical physics and has been applied successfully to such fields as seepage, heatflow, hydrodynamics, soil and rock mechanics, and bio-engineering.

In general, problems of mathematical physics may be specified in one of two ways:

1. differential equations governing the behavior of a typical infinitesimal region are given;
2. a variational or extremum principle valid over the entire region of interest is postulated.

These two approaches are mathematically equivalent, and the finite element method is based on the second approach [2]. The appropriate variational theorem relating to static problems of structural analysis is the well known theorem of minimum potential energy which may be derived from the principle of virtual work.

* 'Orthotropic' describes materials whose elastic properties vary in different directions, e.g. canvas.

Exact mathematical solutions of the governing differential equations are only possible for relatively simple problems. As a result, approximate solutions must suffice. Two general methodologies are as follows: a) an approximate solution of the mathematical equations governing the actual problem; or b) an exact solution of the mathematical equations governing a simplified problem.

This latter approach forms the basis of the finite element method. Thus, the actual problem is replaced by a simplified problem for which an exact solution is obtained. Provided the simplified problem does not differ significantly from the actual problem, the solution may be considered valid. The concept is illustrated in Figure 1, in which a value for the circumference of a circle is sought. Numerical solutions may be obtained by using straight line segments to model the curved geometry. Note that the accuracy of the approximation increases as the number of segments used increases. The approximation is, therefore, of a physical nature. There is,

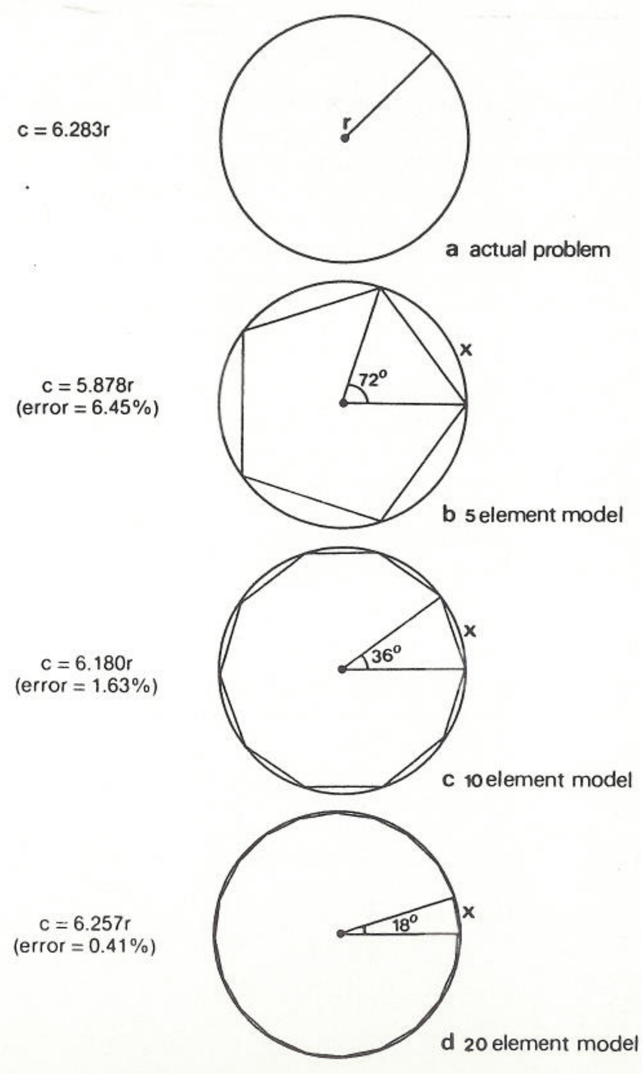

Fig. 1 Simplified finite element model.

however, no approximation in the mathematical analysis of the modified structure. Provided that the solution of the modified problem is approximately equal to that of the actual problem, this approach has tremendous potential. Substantial experience in solving a wide variety of complex problems has identified three general criteria which, if satisfied, will guarantee that the method yields a known bound of the correct answer and that if the number of elements is increased, the accuracy of the solution is increased.

The steps involved in a finite element analysis may be summarized briefly as follows:

2.1 *Discretization of the Continuum (Structure)*

In this step, the body is subdivided into a system of component elements. In many cases, good results may be obtained with relatively crude divisions and this is therefore not a difficult or crucial step in the procedure.

2.2 *Selection of Element Displacement Model*

This is the crucial step in the analysis procedure, and the selection of an appropriate function to represent the displacement within each element determines to a considerable extent the validity and the accuracy of the final solution.

For simplicity, polynomial functions are usually selected and rules have been established for selecting the proper number of terms in order to guarantee accuracy of the solution.

2.3 *Determination of Element Stiffness**

Using the appropriate variational theorem, the element stiffness can be determined. The basic parameters affecting element stiffness are the displacement model previously selected, the element geometry, and the element material properties and strain-displacement relations. Appropriate strain-displacement may be obtained from mechanics. With respect to the analysis of oil paintings the stress-strain relationships of the constitutive materials are of paramount importance.

2.4 *Assembly of Governing Equations*

The equations governing the response of the discretized structure may be assembled by appropriate merging of the element stiffnesses and forces applied to the structure using a standard procedure known as the direct stiffness method. This step in the analysis procedure is well standardized, and application to oil paintings does not introduce any complexities to the process.

2.5 *Solution for Unknown Displacements*

Once the governing equations have been generated, standard procedures for solving large systems of equations may be used to evaluate the displacements of the structural system. Depending on whether or not the material stress-strain relations are linear or non-linear, the solution process may require an iterative solution procedure which must be continued until a convergent solution is obtained.

2.6 *Computation of Element Stresses*

From the element displacements, element strains and subsequently element stresses may be obtained without difficulty.

The basic advantage of the FEM outlined above is that the method can consider without major difficulty complex variations in the following four significant factors: 1) structure geometry; 2) support conditions; 3) loadings or stimuli; 4) material properties.

Thus, irregular-shaped structures with complex supports subjected to variable external actions can be readily analyzed in a typical FEM computer program. In addition, structures composed of elements with highly variable material properties can also be included without excessive computational difficulties.

* Stiffness may here be defined as resistance to deformation.

3 DESCRIPTION OF THE IDEALIZED SYSTEM

A fabric-supported oil painting will be considered herein as a thin multi-layered orthotropic membrane possessing no flexural rigidity. Lateral loadings and displacements are not considered herein and a two-dimensional plane stress condition is, therefore, assumed. The basic concept of the structural behavior and response is that all components of the painting contribute to the overall resistance to deformation. It is assumed that there is no relative movement along the layer interfaces. Each material layer may have different material property values, with variable humidity conditions possible over the continuum surface but constant through the thickness of the structure. At any given point in time, the materials are considered to possess unique linear elastic properties.

Recent clinical tests [3] along with previous information available [4, 5] have established that relative humidity, rather than the normal range of temperature variation, significantly affects the physical properties of the component layers. Thus, a major parameter considered in the problem formulation is relative humidity.

4 SOLUTION PROCEDURE

Details of the solution procedure are presented in [6]. However, a brief synopsis of the methodology is summarized below.

The solution procedure initiates with the consideration of a totally uncracked structure. The stiffness matrix is formulated for the entire structure, using material property values associated with the initial known continuum humidity condition. The applied force vector is computed, and the specified displacement boundary conditions imposed. A solution of the equilibrium equations is obtained using the Choleski decomposition procedure [2] yielding the nodal displacements. Stresses within each layer, element and subregion of the system are then computed. A comparison of these stresses with the tensile rupture stress values will indicate whether cracking has taken place.

The effect of cracking is considered by imaginary, or pseudo, loads applied at the nodal points of the cracked element. The force vector is adjusted to allow for this pseudo loading and the equilibrium equations resolved, stresses recomputed and the entire pseudo-load procedure repeated until satisfactory convergence is achieved. In this way, a convergent solution considering cracking is obtained for the initial humidity condition. In other words, the first iterative cycle is performed assuming no humidity variation.

Prior to determining the effects of changes in the initially specified humidity state throughout the continuum, it is necessary to consider that once any portion of the structure has cracked, it can no longer offer any tensile resistance across the crack and the solution procedure must therefore recognize and permit this condition. For this reason, any subregion cracked within an element is identified in the problem and further analysis of the cracked structure, for modified humidity conditions, proceeds using a structure stiffness matrix modified to account for cracking. A humidity force vector is also computed and merged with the applied load vector. To allow for material property changes, both the modified stiffness matrix and the humidity force vector, arising from the humidity variation, are formulated using material property values for the new humidity condition. Specified displacement boundary conditions are again imposed and a second series of iterations performed in which further cracking is again handled using the pseudo-load procedure.

This entire process is repeated for each desired change in humidity conditions over the structure. In this way, a stress history of the continuum is developed due to a set of applied loads, or displacements, and a desired cycling of continuum humidity variations.

5 APPLICATIONS

Three application problems are presented for a 30in (fill) by 36in (warp) painting. Due to symmetry only one quadrant of the painting is modeled, using a 30-element mesh. The painting is considered to consist of the following layers: layer 1 — linen fabric, 0.0033in thick; layer 2 — rabbitskin glue size, 0.0001in thick; layer 3 — white lead ground paint, 0.01in thick; layer 4 — titanium dioxide paint, 0.005in thick. Properties for these materials, including modulus of elasticity, E, and tensile rupture stress, f_t, obtained from Meckenburg [1] are given in Tables 1 through 4. The model is shown in Figure 2.

TABLE 1 Material Properties for Fabric Used in Application Problems: Ulster no. 8800 Linen (Orthotropic)

Relative humidity	E_{warp} *(psi)*	E_{fill} *(psi)*	f_t *(psi)*
90%	3.55×10^4	12.50×10^4	50,000
50%	0.56×10^4	9.50×10^4	50,000
20%	0.45×10^4	6.75×10^4	50,000

TABLE 2 Material Properties for Size Used in Application Problems: Rabbitskin Glue Size (Isotropic)

Relative humidity	E *(psi)*	α *(in/in/%RH)*	f_t *(psi)*
90%	1.75×10^4	2.0×10^{-4}	40,000
50%	28.5×10^4	2.0×10^{-4}	40,000
20%	47.5×10^4	3.0×10^{-4}	40,000

n.b. $\alpha = 0$ in all cases

TABLE 3 Material Properties for Ground Used in Application Problems: White Lead Ground (Isotropic)

Relative humidity	E *(psi)*	α *(in/in/%RH)*	f_t *(psi)*
90%	0.6×10^4	1.7×10^{-5}	175
50%	2.4×10^4	1.7×10^{-5}	360
20%	3.7×10^4	1.7×10^{-5}	400

TABLE 4 Material Properties for Paint Used in Application Problems: Titanium Dioxide Paint — Aged 24 Months (Isotropic)

Relative humidity	E *(psi)*	α *(in/in/%RH)*	f_t *(psi)*
90%	1.2×10^4	1.25×10^{-4}	275
50%	2.8×10^4	1.25×10^{-4}	400
20%	3.95×10^4	1.25×10^{-4}	500

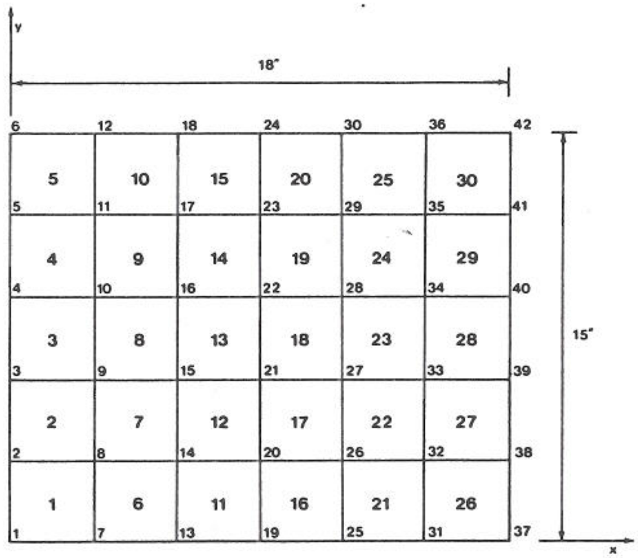

Fig. 2 Finite element model of north-east quadrant.

5.1 *Uniform Expansion with Differential Humidity*

This application assumes the following sequence of events. The stress-free 30in × 36in painting at 90% relative humidity is subjected to a uniform expansion due to swelling of the stretcher bars. Expansions of 0.09in in the fill direction and 0.18in in the warp direction are imposed. Subsequently, the relative humidity of the central portion of the painting is reduced to 50%, simulating differential drying. The resulting displacements of the north-east quadrant of the painting and the magnitudes and directions of the maximum principal stresses in each element are shown in Figure 3.

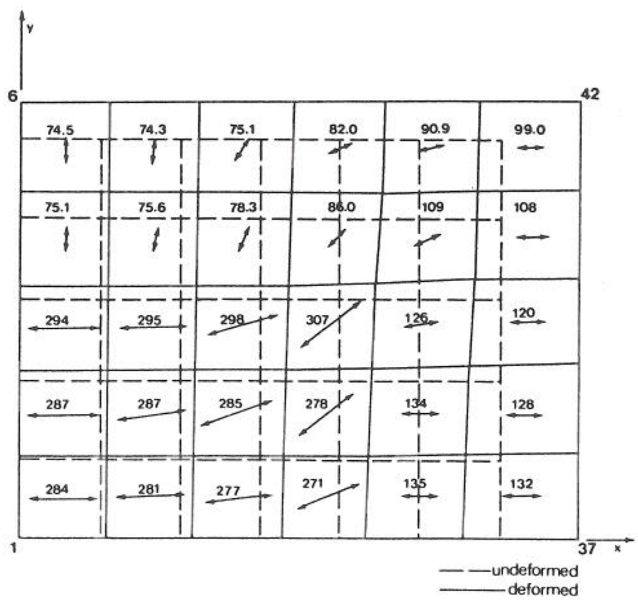

Fig. 3 Displacements and principal stresses for problem 5.1.

5.2 *Keying Out with Changes in Relative Humidity*

This problem investigates stresses induced by keying out 0.06in along all stretcher bars at a relative humidity of 90%. Following this, the relative humidity is reduced to 50% and finally 20%. The computed displacements and corresponding stresses due to the keying out operation are shown in Figure 4. Note that the high initial tension stress in the corner of the painting causes cracking. Stress magnitudes and extension of

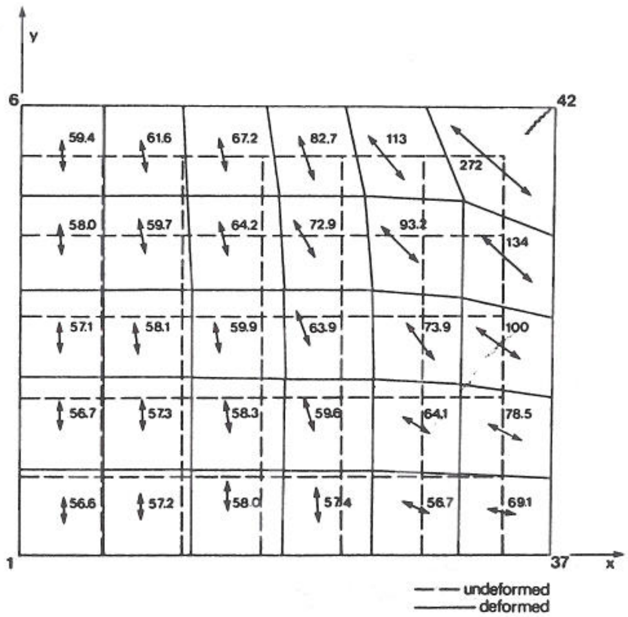

Fig. 4 Displacements and principal stresses for problem 5.2 (90% RH).

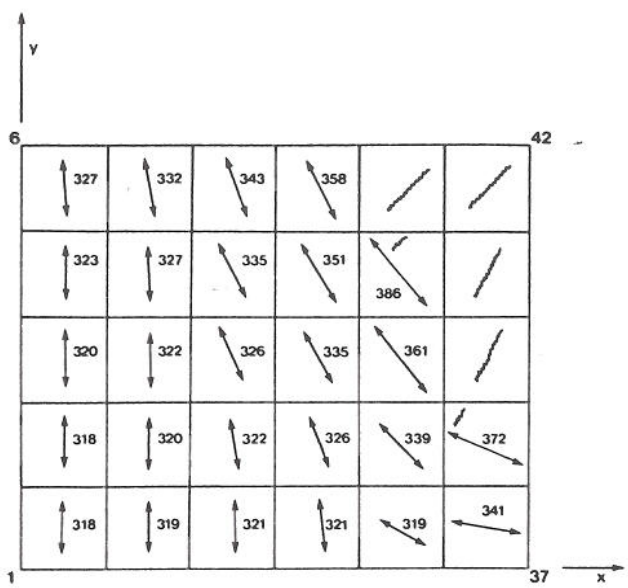

Fig. 5 Cracking and principal stresses for problem 5.2 (50% RH).

cracking following the specified reductions in relative humidity are shown in Figures 5 and 6.

5.3 *Tacking Edge Effect with Changes in Relative Humidity* In the previous applications, a continuous attachment of the canvas to the stretcher bar was assumed. In order to investigate the local effect of tacking, element no. 5 (Fig. 2) is analyzed in more detail by subdividing it into 16 elements, each 0.75in square, as shown in Figure 7. It is assumed that the nodal points along the bottom edge are fixed in the y-direction but free to slide horizontally, and the left-hand side is fixed in the x-direction but free to slide vertically. The right-hand side of the configuration is displaced 0.015in, an amount which is consistent with, and proportional to, the displacements imposed in problem 5.1. The nodal points 5 and 25 are given vertical displacements of 0.006in, an amount which is again consistent with previous displacements. Thus, the painting is

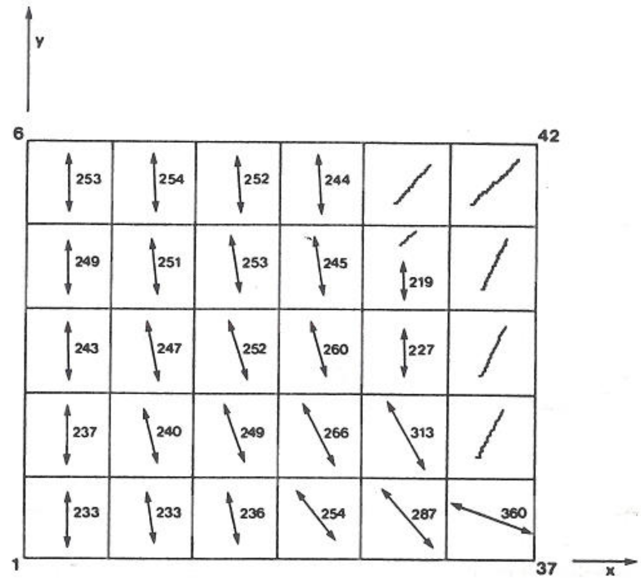

Fig. 6 Cracking and principal stresses for problem 5.2 (20% RH).

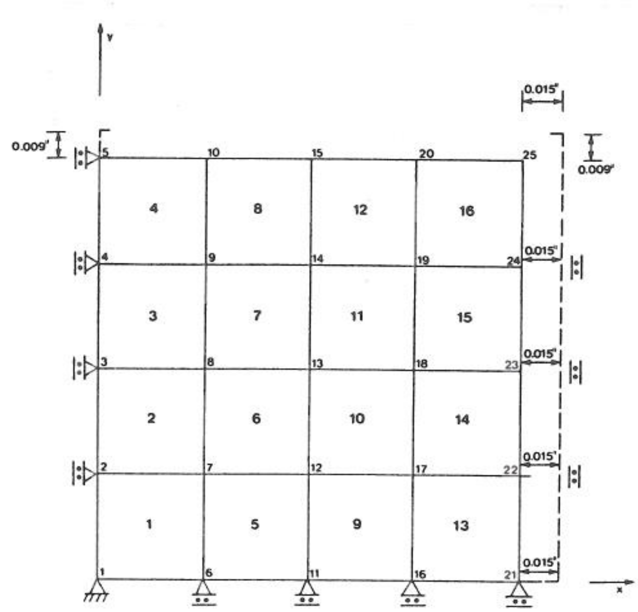

Fig. 7 Boundary conditions for problem 5.3.

modeled as if it were under an initial tensioning influence. The ensuing results may then be compared to those for problem 5.1, since one element is essentially being replaced by a finer mesh of 16 elements.

The computed displacements for the tacking edge effect at conditions of 90% RH are plotted, to an exaggerated scale, on Figure 8. Also shown are the magnitude and direction of the maximum principal stresses in each element. The displaced configuration shows that considerable distortion occurs in the elements adjacent to the tacking points (i.e., elements 4 and 16). When the humidity is reduced to 20% over the entire area, the distortion is magnified.

6 COMPARISON WITH ACTUAL PAINTINGS

The application problems studied here have been limited to a number of common influences to which a fabric-supported painting could be subjected. It is important to note that the analyses used typical dimensions, thicknesses and material property values obtained from ongoing research [3].

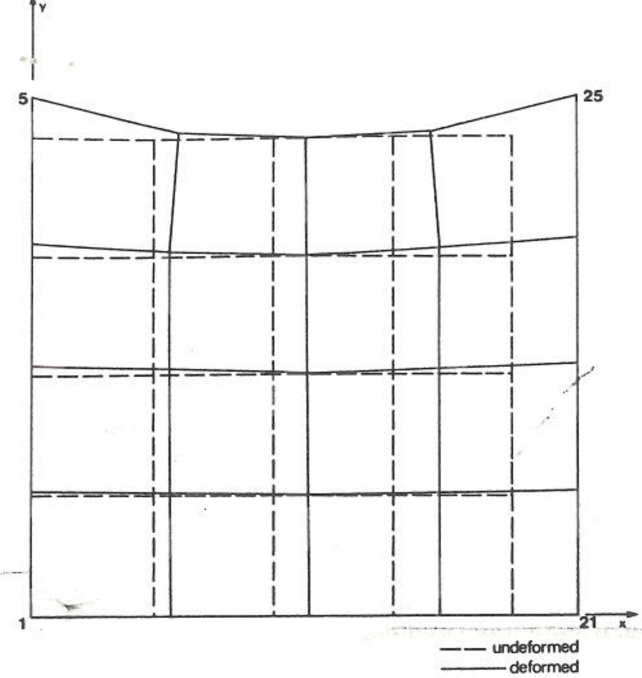

Fig. 8 Displacements for problem 5.3 (90% RH).

Interpretation of the results of the computer analyses leads to the prediction of potential cracking configurations on the entire painting as shown diagrammatically on Figure 9. Keck has reported [5] that fabric-supported oil paintings show a marked similarity in cracking configurations and that these cracks take the general patterns as depicted on Figure 10.

A comparison of the predicted and actual cracking configurations indicates that a reasonable correlation exists for the cases studied here.

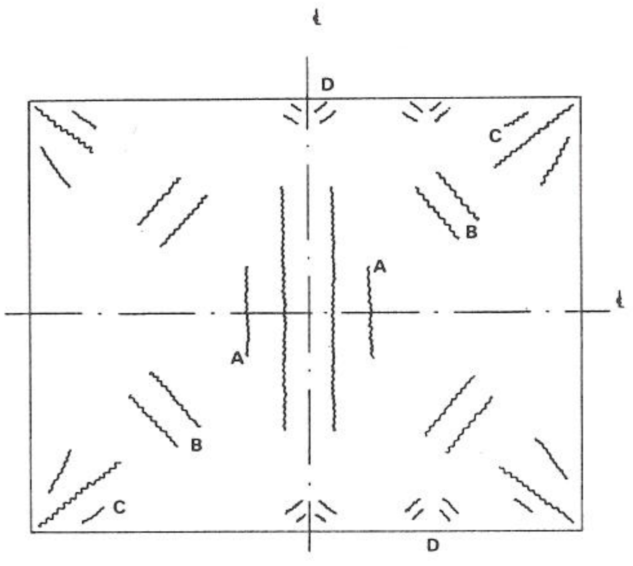

Fig. 9 Summary of predicted crack patterns.

7 CONCLUSIONS

Many current practices in conservation and restoration are performed without adequate knowledge or consideration of the mechanical stresses they induce. Thus it is possible for treatments to aggravate deterioration and damage to the paint layer.

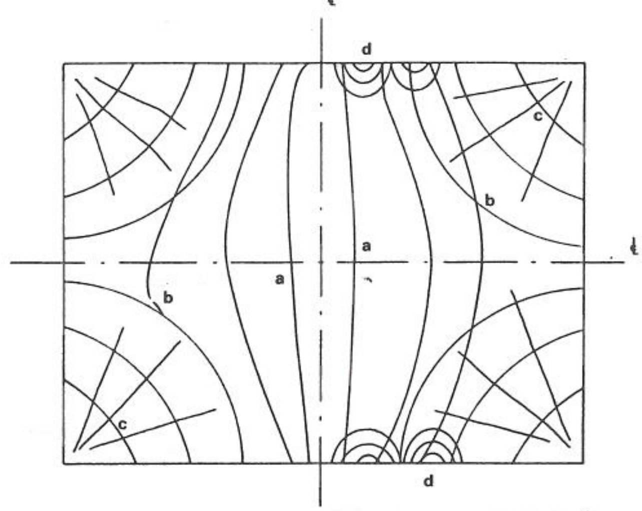

Fig. 10 Simplified diagram of observed crack patterns.

Based on the work performed and reported on briefly in this paper, it is concluded that, provided realistic data are available on material properties, the finite element model developed possesses the capability of predicting the magnitude and distribution of stresses in oil paintings caused by common stimuli.

ACKNOWLEDGEMENT
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