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SOLAR RADIATION AND WEATHER STUDIES

(With Three Plates)

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Secretary, Smithsonian Institution

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INTRODUCTION

Many years ago the late Secretary Langley expressed the hope that the studies of the Astrophysical Observatory on the intensity of the sun's radiation would lead to long-range weather forecasting. His hopes were encouraged when in 1903 our studies seemed to indicate a considerable change in the sun's output of radiation [1] associated with a marked drop of temperature over the Northern Hemisphere. This, which now seems to have been a chance coincidence, led to a campaign of "solar constant" determination which is still in progress. It has involved the establishment of observing stations at high altitudes in 10 different localities, 5 in the United States, 2 in Chile, 1 each in South-West Africa, Algeria, and Egypt. Three of these are now in occupation. Part of the expense of these observing stations was borne by the Government, but a considerable fraction was defrayed by grants from Mr. John A. Roebling and from the Hodgkins Fund of the Smithsonian Institution. The National Geographic Society also made a large grant which supported the establishment and continuation of 5 years of the station in South-West Africa.

After an excellent series of nearly daily solar-constant observations of 12 years length became available, analysis showed that what at first sight seemed chance variations of the sun's output really comprised a summation of at least seven [2] regular periodicities. Although these were of the order of only 1 percent or less, it seemed advisable to see if they appeared to be associated with weather changes of significance. A study of this question was made by the aid of the long-term records of temperature and precipitation contained in "World Weather Records," published recently by the Smithsonian Institution with the assistance of Mr. John A. Roebling.

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[1] See Ann. Rep. Smithsonian Inst. 1903, pp. 81-84, 1904; and Astrophys. Journ., vol. 19, pp. 305-321, 1904.

[2] In the latest analysis, given below, covering the years 1920-1934, 12 periodicities are found in solar variation.

Analysis of weather records appears to show that each of the various solar periodicities above referred to influences both temperature and precipitation to a significant degree. At least five (perhaps six) other periodicities in weather elements, closely associated in length with the original seven, are also significant. Inasmuch as all of these 12 or 13 periodicities are very nearly aliquot parts of 23 years, it follows that their combined effect produces in the weather a large number of features more or less pronounced during a period of 23 years. Succeeding intervals of 23 years tend to bring repetitions of these features. For some of these periodicities, however, 46 years appears to be the critical interval. Hence there is a somewhat closer correspondence at some times and some stations between weather features 46 years apart. Owing to certain modifying influences in the sun itself, to which reference will be made below, and to the complexity of the terrestrial agencies through which the solar influences act, these repetitions of weather features are subject to moderate displacements in time, and to modifications in amplitude. Actual reversals of phase, as will be shown, sometimes occur after 23-year intervals. Nevertheless, special weather features remain recognizable in many instances by comparison of successive 23-year curves.

Based on these grounds it becomes possible to make forecasts of weather conditions for years in advance which appear to be significantly more representative than normal values. The modifying factors referred to above detract as yet greatly from the accuracy of such forecasts, but further study may lead to greater perfection. The following paper gives the evidences for these statements.

The evidence to be presented being extensive and complex, and certain parts of it—as, for instance, the studies of periodicities in the temperature of Berlin—being apt to prove tiresome to some and controversial to others, it is suggested that high spots of the demonstration may be picked out as follows:

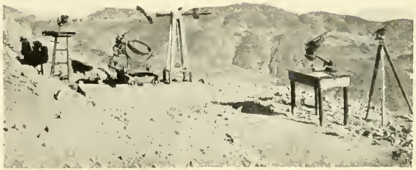
1. Turn to captions 3 and 4, pages 6 and 10, and note the results expressed by figures 4, 6, 7, and 8.

2. Turn to captions 14-Ba, 14-Bb, and 15, pages 35, 38 and 53, and note the results expressed by figures 15, 16, 17, 19, and 23.

3. Turn to captions 17 to 25, pages 56 to 75, and note at least a part of the results expressed in figures 24 to 37, inclusive.

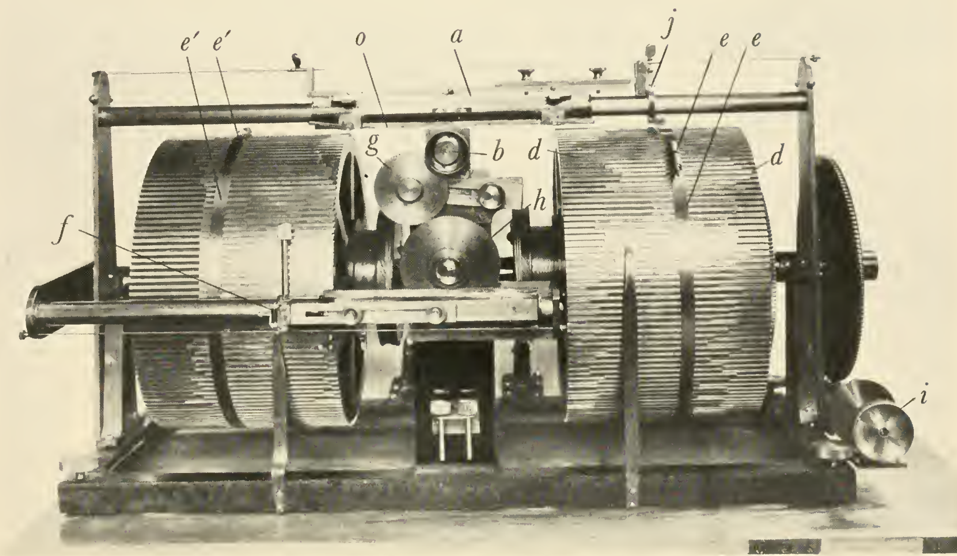
4. Finally, with these results in mind, read the Summary, pages 88 and 89.

In this way it is hoped that the reader will obtain briefly such a view of the more remarkable parts of the investigation as will arouse his curiosity to pursue the entire course of the demonstration.



SMITHSONIAN SOLAR RADIATION STATION, MONTEZUMA, CHILE

Upper left, coelostat and pyrheliometric apparatus; upper right, hauling materials from Calama for the solar observing station; lower left, dwelling house at the solar observing station; lower right, peak on which the observing station is located.



THE PERIODOMETER, AN INSTRUMENT FOR DETECTING AND EVALUATING PERIODICITIES IN LONG SERIES OF DATA

I. SOLAR RADIATION MEASUREMENTS[3]

1. Objects and Stations

We measure at the earth's surface the total intensity of solar radiation, its spectral distribution, the losses its various rays meet in traversing the atmosphere; and we compute its intensity and spectral distribution outside the atmosphere, and the variations of its intensity from day to day as they occur in the sun itself before the rays enter the atmosphere. At present, the Smithsonian Institution carries on these measurements at three high-altitude desert stations chosen for their cloudlessness and other favorable conditions. They are Table Mountain, Calif.; Montezuma, Chile; and Mount St. Katherine, Egypt. Their respective altitudes are 7,500, 9,000, and 8,500 feet, approximately. Other Smithsonian stations formerly occupied have included Washington, D. C.; Hump Mountain, N. C.; Mount Wilson and Mount Whitney, Calif.; Mount Harqua Hala, Ariz.; Bassour, Algeria; and Mount Brukkaros, South-West Africa. Plate 1 shows the station at Mount Montezuma. Besides these terrestrial stations, a self-recording instrument for measuring total solar radiation was raised by sounding balloons from Omaha, Nebr., July 1914, to a level of over 15 miles. It made good records of the intensity of solar radiation at that high level where only 1/25 of the atmospheric pressure remained above. The mean value of the solar constant of radiation as computed from mountain stations is 1.94 calories per square centimeter per minute. Balloon pyrheliometry indicated 1.84 calories at 15 miles elevation. Correction of balloon pyrheliometry for loss in the highest atmosphere gives 1.88 calories, which agrees with mountain solar-constant results within the experimental error of the balloon observations.

2. Instruments and Methods

For measuring total solar radiation at the earth's mountain surface we have hitherto depended[4] on the silver-disk pyrheliometer and the water-flow pyrheliometer. The former is a secondary instrument whose readings are converted into absolute units (calories per square centimeter per minute) by comparisons with the water-flow pyrheliometer.[5] These instruments are shown diagrammatically in figures

[3] This section is for the most part abbreviated from vols. 1-5, Annals of the Astrophysical Observatory of the Smithsonian Institution.

[4] We are now (1935) introducing the Ångström electrical compensation pyrheliometer as a cooperating instrument.

[5] See improved water-flow pyrheliometer as described in Smithsonian Misc. Coll., vol. 87, no. 15, 1932, and vol. 92, no. 13, 1934.

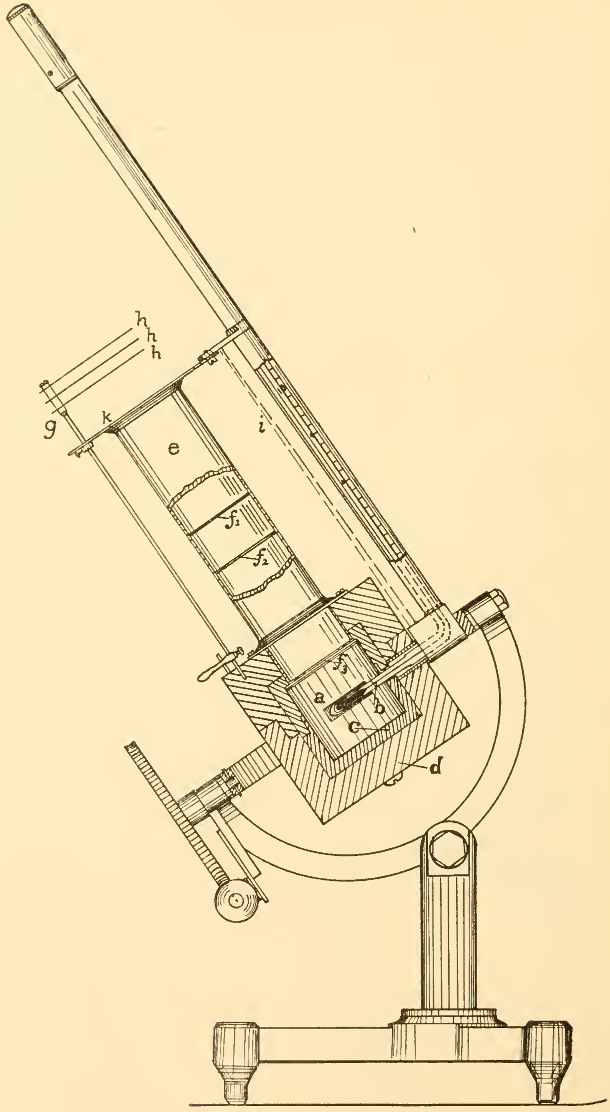


FIG. 1.—Diagram of silver-disk pyrheliometer.

1 and 2. Their sources of error, corrections to their direct readings, and other details regarding them are published in volumes 2, 4, and 5 of the Annals of the Smithsonian Astrophysical Observatory and in papers nos. 3182 and 3288 of the Smithsonian Miscellaneous Collections. Intercomparisons of silver-disk pyrheliometers made at intervals over a period of about 20 years indicate that the scale of observing has not changed appreciably. These intercomparisons are

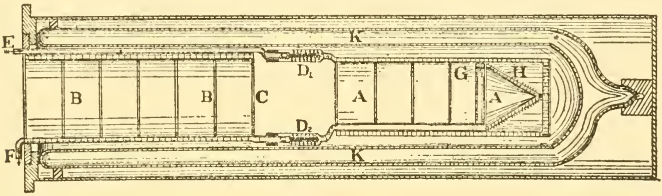


FIG. 2.—Diagram of water-flow pyrheliometer.

Solar rays are mainly absorbed on the cone in A, but some are scattered about the walls of AA. Their heat is given up to water which flows in a spiral channel about the cone and tube AA. The rise of temperature of the water due to solar heating is measured by the electrical thermometer D_1D_2. Test quantities of electrical heat introduced at G or H may be measured as a check.

published extensively in the Annals, volume 4, pages 94-97, and volume 5, pages 139-145. Table 1 gives one typical example.

TABLE 1.—*Long-continued Series of Intercomparison of Pyrheliometers S.I. 1 with A.P.O. 8_{bis}*

Year	1911	1911	1912	1913	1915	1916 [a]	1917	1917	1920
Ratio	1.0357	1.0246	1.0268	1.0324	1.0343	1.0119	1.0360	1.0330	1.0352

[a] It is believed that owing to maladjustment S.I. 1 was not properly exposed on this occasion.

The distribution of energy in the solar spectrum before it enters the atmosphere approximates roughly that of the perfect radiator at 6,000° K. Hence, nearly all of its energy is contained between wave lengths 0.3 and 3.0 microns. Rays beyond 0.3 micron in the ultraviolet are almost wholly cut off by ozone in the higher atmosphere, and those beyond 3.0 microns in the infrared by water vapor in the lower atmosphere. Between these limits not only these and other atmospheric vapors, but also dust and even the permanent gaseous molecules of the air, absorb or scatter the sun's rays both selectively and generally, so that the solar beam is both changed in spectral distribution and generally weakened during its passage through the atmosphere. In order to evaluate these losses, energy spectral measurements are required. These are made at our stations several times on each observing day by means of the spectrobolometer. This instrument, shown diagrammatically in figure 3, is explained in the Annals. Plate 3 shows a group of successive solar spectrobolometric observations made at Montezuma, Chile, July 7, 1924. The relative losses of radiation suffered at different wave lengths in transmission through the spectrobolometer are measured and allowed for as described in the Annals, volume 2, pages 50-52, and volume 3, pages 27-29.

Knowing the sun's altitude, and thereby the length of path of the sun rays in the atmosphere compared to the length of a vertical path therein, taken as unity, these several curves may fix the atmospheric transmission coefficients at all wave lengths. Thereby the spectral energy curves can be reduced in form and height to what they would have been if observed outside the atmosphere. This reduction is explained in the Annals, volume 2, page 56, and volume 3, page 28. The total area included under such a spectral energy curve is proportional to the total energy of the solar beam as it would be observed with the pyrheliometer. Hence, the ratio of areas included under two spectral energy curves, one computed as of outside the atmosphere, and the other observed as at the earth's surface, is the factor by which the pyrheliometer measurement is to be multiplied to yield the intensity of the sun's radiant energy outside the atmosphere. Including also, as a factor, the square of the ratio of the earth's actual solar distance to its mean value, we arrive at the " solar constant of radiation."

In the year 1919 it was discovered that a mere measurement of the brightness of the sky surrounding the sun could be made to yield closely enough the coefficients of atmospheric transmission at all wave lengths. This measurement is made with the instrument called the pyranometer. It thus becomes possible to make five solar-constant determinations in one morning and reduce them within the time formerly occupied with one determination. The method as now developed is explained in the Annals, volume 5, pages 110-120.

3. The Variation of the Sun's Radiation

Figure 4 shows superposed in the form of 10-day means the solar-constant results obtained at Montezuma, Table Mountain, and Mount Brukkaros from 1925 to 1930. The order of excellence of the stations is the order just given. This is indeed plain from the relative smoothness of the three curves of figure 4. But though differing in details, the three stations agree in showing in common certain principal trends, and thereby indicate a real variation of the sun.

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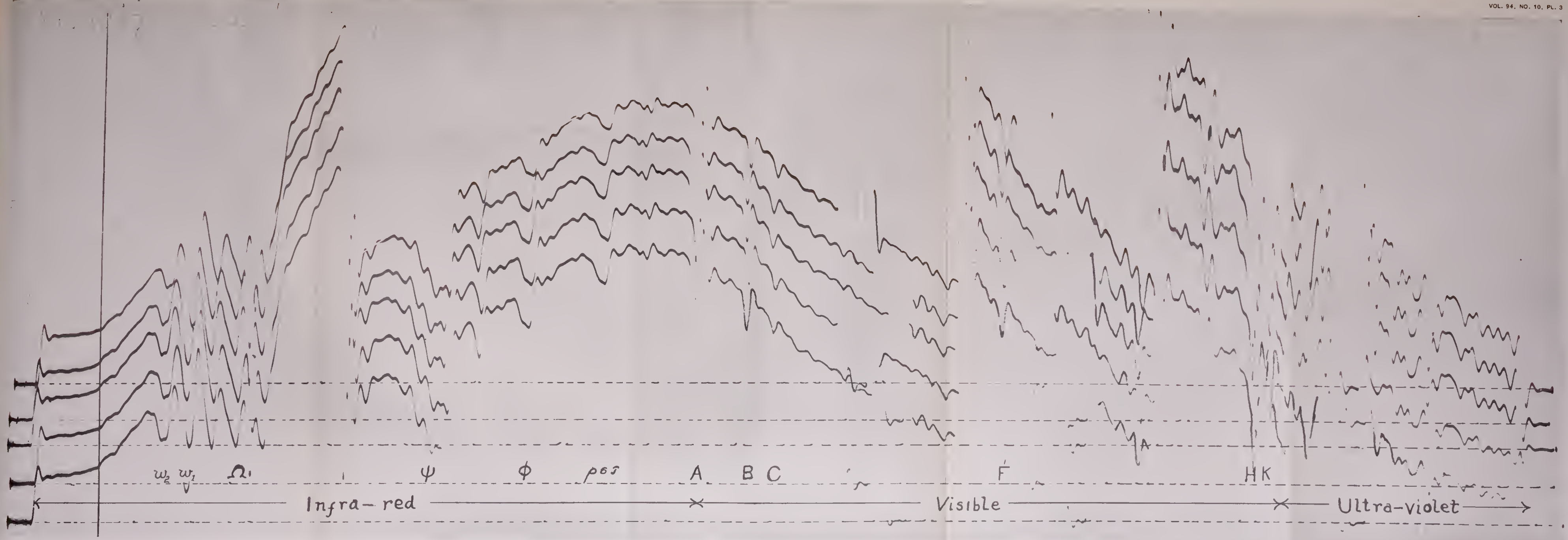

Bolographs of the Solar Energy Spectrum

Observed at Montezuma, Chile, July 7, 1924. Precipitable water, 0.03 cm

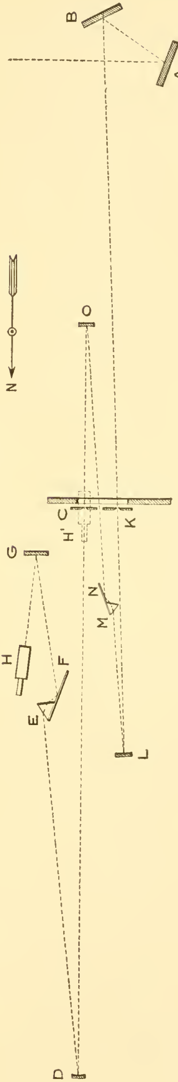

FIG. 3.—Diagram of spectrobolometer.

In daily observing, the sun ray is reflected directly from B through the slit C, is analyzed into the spectrum at EF, and a selected ray falls on the bolometer H. The double spectroscope, as shown, is used occasionally to measure the transmission of the optical train CDEFG.

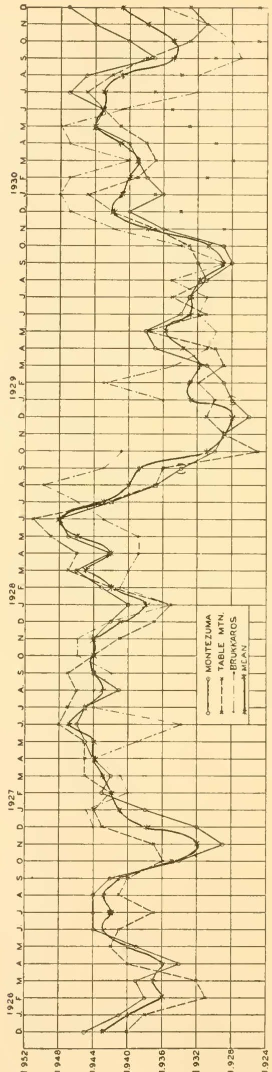

FIG. 4.—Solar-constant values, three stations, 1925-1930.

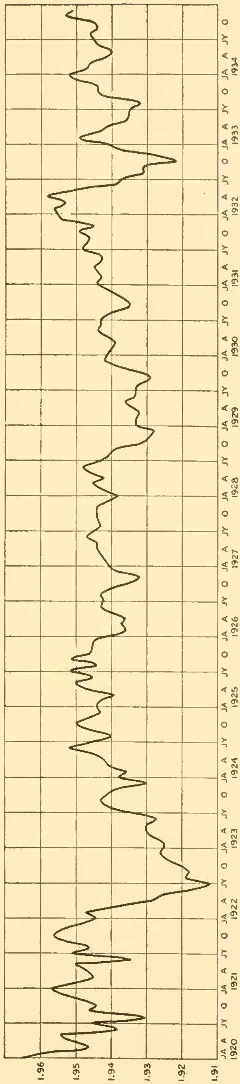

FIG. 5.—March of solar variation, 1920-1934.

Figure 5 illustrates the mean result of all the evidence from 1920 to 1934, inclusive. It depends on observations at Calama and Montezuma, Chile; Mount Harqua Hala and Table Mountain in the United States, and Mount Brukkaros, Africa.[6]

The range of variation of solar radiation as indicated by the 10-day mean values of the solar constant is given by table 2.

4. Periodicities in Solar Variation

To casual inspection the solar variation is irregular. More careful inspection discloses an 8-month periodicity. Eleven other periodicities have also been found and evaluated. As successively discovered, they have been removed by subtraction, one by one, from the numerical record so as to simplify the search for other solar periodicities. The process of evaluating and removing periodicities is illustrated for an 11-month period by table 3 and figure 6. Plate 2 shows a machine capable of doing the same thing.[7, 8]

The reader will note that this computation of the 11-month solar period is separated into several parts nearly similar to each other whose mean result is to be repeated consecutively and added to consecutive repetitions of other periodicities to produce the second curve in figure 7. The partial mean curves computed in table 3 are seen to differ somewhat in form and amplitude, but to agree fairly closely as to the phases of maximum and minimum values of solar radiation. These independent determinations at different epochs, all yielding 11-month periodicities in nearly the same phase, seem to strongly support the veridity of the 11-month solar period. The third group, indeed (1930-1934) shows about 3 months lag in phase. As will be shown in sections 14B and 25 below, there is some reason to anticipate a change of phase of some of the periodicities about January 1934. Possibly this is the cause of the observed phase-shift. Later observations will settle it.

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[6] The values given in fig. 5 and table 2 are provisional for the years 1931-1934 and may be altered in revision.

[7] See The Periodometer, Smithsonian Misc. Coll., vol. 87, no. 4, 1932.

[8] In the analysis of curves, most investigators employ developments of Fourier's methods. That is, they represent the observed curve as a summation of a number of arbitrary harmonic curves of integral periodic relationships. These constituent harmonic curves have the periods $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, . . . $1/n$ of the entire unit length of the curve analyzed. In such a case as that of a harmonic analysis of the sun-spot numbers, none of the constituent harmonics have any independent physical significance whatever. Nor is it to be supposed that the harmonic form itself represents at all closely the march of any physical quantity connected with the phenomenon. It has seemed to me preferable to discard this tedious and arbitrary procedure, and to compute the actual mean forms of the solar periodicities as illustrated by table 3 and fig. 6.

5. Analysis of the Solar Variation

The curve of solar variation contains, however, not only a number of regular periodic constituents, but also accidental errors of nearly as great amplitudes as the periodic terms themselves. These various constituents, accidental and periodic, are confused together, and mutually influence the graphic expressions of each other in the observed curve. For the purpose of simplification it has seemed best to remove the several periodic terms one by one, beginning with those of shortest period. In the presence of confusing variations from other causes, these short-period curves may be the most accurately investigated of any because they present the largest numbers of cases which may

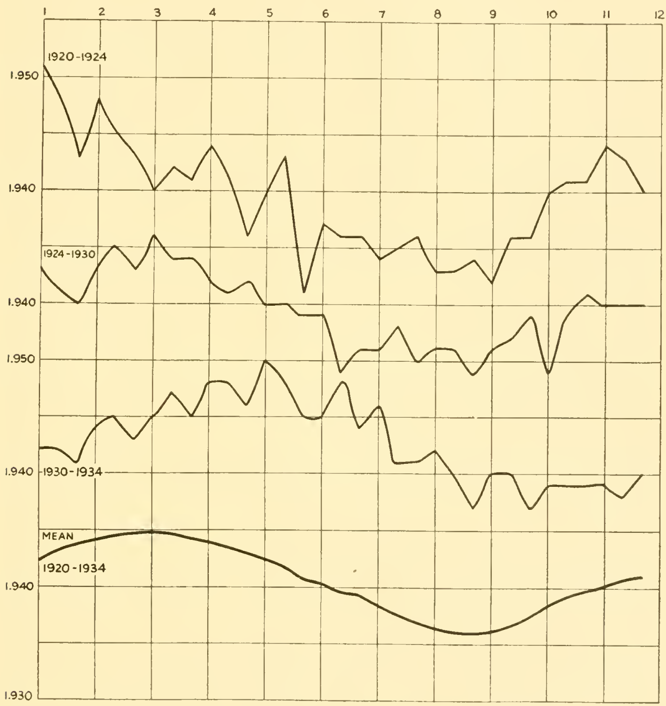


Fig. 6.—The 11-month periodicity in solar variation.

TABLE 2.—*Preferred Solar Constants. Ten-day Mean Values. Collected and Adjusted from Observations at Several Stations*

Date (decade)		1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934
Jan.	1	1.968	1.964	1.938	1.934	1.936	1.945	1.939	1.940	1.937	1.929	1.935	1.944	1.959	1.946	1.955
	2	1.967	1.956	1.945	1.924	1.940	1.939	1.940	1.939	1.932	1.933	1.938	1.944	1.952	1.942	1.950
	3	1.959	1.950	1.948	1.918	1.941	1.947	1.934	1.937	1.937	1.932	1.936	1.943	1.954	1.935	1.950
Feb.	1	1.958	1.942	1.944	1.925	1.936	1.941	1.935	1.939	1.944	1.933	1.934	1.942	1.953	1.952	1.960
	2	1.954	1.954	1.949	1.934	1.935	1.949	1.941	1.944	1.940	1.927	1.940	1.945	1.962	1.946	1.943
	3	1.956	1.954	1.948	(1.915)	1.937	1.940	1.930	1.942	1.941	1.930	1.943	1.941	1.952	1.946	1.943
Mar.	1	1.959	1.954	1.947	1.925	1.944	1.941	1.938	1.936	1.947	1.929	1.940	1.932	1.944	1.947	1.944
	2	1.948	1.939	1.941	1.930	1.941	1.936	1.939	1.943	1.946	1.930	1.937	1.945	1.954	1.940	1.943
	3	1.932	1.942	1.932	1.930	1.939	1.941	1.934	1.943	1.945	1.931	1.939	1.948	1.953	1.938	1.951
Apr.	1	1.948	1.949	1.930	1.933	1.939	1.945	1.930	1.944	1.943	1.932	1.941	1.948	1.959	1.939	1.959
	2	1.956	1.945	1.930	1.925	1.943	1.950	1.935	1.947	1.939	1.940	1.938	1.948	1.960	1.949	1.943
	3	1.952	1.946	1.925	1.931	1.944	1.946	1.940	1.945	1.942	1.937	1.939	1.938	1.954	1.934	1.938
May	1	1.950	1.950	1.923	1.927	1.943	1.946	1.938	1.946	1.942	1.938	1.942	1.936	1.952	1.937	1.942
	2	1.961	1.949	1.932	1.930	1.946	1.950	1.938	1.942	1.949	1.934	1.942	1.948	1.948	1.934	1.942
	3	1.950	1.950	1.924	1.934	1.947	1.954	1.943	1.945	1.946	1.935	1.942	1.947	1.942	1.938	1.938
June	1	1.943	1.927	1.920	1.918	1.951	1.943	1.939	1.950	1.946	1.935	1.945	1.943	1.935	1.937	1.948
	2	1.934	1.939	1.915	1.932	1.953	1.943	1.945	1.944	1.951	1.932	1.944	1.948	1.936	1.936	1.945
	3	1.938	1.936	1.912	1.932	1.953	1.948	1.941	1.945	1.945	1.932	1.940	1.946	1.942	1.932	1.938
July	1	1.945	1.952	1.900	1.934	1.946	1.952	1.941	1.948	1.943	1.933	1.944	1.954	1.943	1.933	1.948
	2	1.940	1.953	1.913	1.928	1.950	1.954	1.944	1.945	1.939	1.932	1.950	1.949	1.932	1.937	1.942
	3	1.951	1.948	1.923	1.944	1.943	1.947	1.941	1.946	1.940	1.934	1.946	1.943	1.937	1.935	1.942
Aug.	1	1.930	1.944	1.917	1.942	1.950	1.949	1.943	1.943	1.941	1.931	1.944	1.948	1.933	1.927	1.946
	2	1.927	1.957	1.919	1.940	1.940	1.941	1.941	1.940	1.934	1.932	1.944	1.949	1.932	1.932	1.946
	3	1.932	1.937	1.921	1.941	1.930	1.942	1.942	1.942	1.938	1.930	1.941	1.943	1.928	1.936	1.947
Sept.	1	1.951	1.950	1.921	1.945	1.941	1.956	1.941	1.941	1.941	1.928	1.939	1.946	1.930	1.938	1.945
	2	1.944	1.957	(1.915)	1.943	1.950	1.946	1.938	1.941	1.935	1.928	1.934	1.948	1.935	1.941	1.946
	3	1.944	1.950	1.919	1.940	1.946	1.950	1.943	1.948	1.923	1.932	1.941	1.945	1.929	1.943	1.941
Oct.	1	1.942	1.955	1.926	1.942	1.950	1.942	1.936	1.945	1.929	1.931	1.937	1.951	1.921	1.944	1.941
	2	1.951	1.961	1.921	1.942	1.950	1.949	1.937	1.943	1.932	1.933	1.937	1.945	1.915	1.945	1.944
	3	1.938	1.953	1.914	1.939	1.949	1.946	1.931	1.941	1.927	1.930	1.938	1.951	1.929	1.943	1.951
Nov.	1	1.952	1.958	1.928	1.934	1.947	1.944	1.932	1.943	1.925	1.932	1.938	1.947	1.930	1.941	1.954
	2	1.948	1.952	1.925	1.943	1.949	1.948	1.930	1.943	1.929	1.935	1.939	1.946	1.926	1.948	1.954
	3	1.943	1.955	1.920	1.941	1.944	1.944	1.932	1.943	1.930	1.940	1.935	1.941	1.929	1.940	1.950
Dec.	1	1.957	1.953	1.925	1.942	1.942	1.944	1.935	1.944	1.929	1.941	1.940	1.956	1.936	1.951	1.947
	2	1.957	1.950	1.922	1.940	1.947	1.945	1.934	1.938	1.926	1.939	1.943	1.953	1.939	1.949	1.954
	3	1.949	1.948	1.930	1.922	1.939	1.946	1.935	1.938	1.932	1.939	1.939	1.946	1.939	1.950	1.952

TABLE 2.—*Preferred Solar Constants. Ten-day Mean Values. Collected and Adjusted from Observations at Several Stations*

Date (decade)		1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934
Jan.	1	1.968	1.964	1.938	1.934	1.936	1.945	1.939	1.940	1.937	1.929	1.935	1.944	1.959	1.946	1.955
	2	1.967	1.956	1.945	1.924	1.940	1.939	1.940	1.939	1.932	1.933	1.938	1.944	1.952	1.942	1.950
	3	1.959	1.950	1.948	1.918	1.941	1.947	1.934	1.937	1.937	1.932	1.936	1.943	1.954	1.935	1.950
Feb.	1	1.958	1.942	1.944	1.925	1.936	1.941	1.935	1.939	1.944	1.933	1.934	1.942	1.953	1.952	1.960
	2	1.954	1.954	1.949	1.934	1.935	1.949	1.941	1.944	1.940	1.927	1.940	1.945	1.962	1.946	1.943
	3	1.956	1.954	1.948	(1.915)	1.937	1.940	1.930	1.942	1.941	1.930	1.943	1.941	1.952	1.946	1.943
Mar.	1	1.959	1.954	1.947	1.925	1.944	1.941	1.938	1.936	1.947	1.929	1.940	1.932	1.944	1.947	1.944
	2	1.948	1.939	1.941	1.930	1.941	1.936	1.939	1.943	1.946	1.930	1.937	1.945	1.954	1.940	1.943
	3	1.932	1.942	1.932	1.930	1.939	1.941	1.934	1.943	1.945	1.931	1.939	1.948	1.953	1.938	1.951
Apr.	1	1.948	1.949	1.930	1.933	1.939	1.945	1.930	1.944	1.943	1.932	1.941	1.948	1.959	1.939	1.959
	2	1.956	1.945	1.930	1.925	1.943	1.950	1.935	1.947	1.939	1.940	1.938	1.948	1.960	1.949	1.943
	3	1.952	1.946	1.925	1.931	1.944	1.946	1.940	1.945	1.942	1.937	1.939	1.938	1.954	1.934	1.938
May	1	1.950	1.950	1.923	1.927	1.943	1.946	1.938	1.946	1.942	1.938	1.942	1.936	1.952	1.937	1.942
	2	1.961	1.949	1.932	1.930	1.946	1.950	1.938	1.942	1.949	1.934	1.942	1.948	1.948	1.934	1.942
	3	1.950	1.950	1.924	1.934	1.947	1.954	1.943	1.945	1.946	1.935	1.942	1.947	1.942	1.938	1.938
June	1	1.943	1.927	1.920	1.918	1.951	1.943	1.939	1.950	1.946	1.935	1.945	1.943	1.935	1.937	1.948
	2	1.934	1.939	1.915	1.932	1.953	1.943	1.945	1.944	1.951	1.932	1.944	1.948	1.936	1.936	1.945
	3	1.938	1.936	1.912	1.932	1.953	1.948	1.941	1.945	1.945	1.932	1.940	1.946	1.942	1.932	1.938
July	1	1.945	1.952	1.900	1.934	1.946	1.952	1.941	1.948	1.943	1.933	1.944	1.954	1.943	1.933	1.948
	2	1.940	1.953	1.913	1.928	1.950	1.954	1.944	1.945	1.939	1.932	1.950	1.949	1.932	1.937	1.942
	3	1.951	1.948	1.923	1.944	1.943	1.947	1.941	1.946	1.940	1.934	1.946	1.943	1.937	1.935	1.942
Aug.	1	1.930	1.944	1.917	1.942	1.950	1.949	1.943	1.943	1.941	1.931	1.944	1.948	1.933	1.927	1.946
	2	1.927	1.957	1.919	1.940	1.940	1.941	1.941	1.940	1.934	1.932	1.944	1.949	1.932	1.932	1.946
	3	1.932	1.937	1.921	1.941	1.930	1.942	1.942	1.942	1.938	1.930	1.941	1.943	1.928	1.936	1.947
Sept.	1	1.951	1.950	1.921	1.945	1.941	1.956	1.941	1.941	1.941	1.928	1.939	1.946	1.930	1.938	1.945
	2	1.944	1.957	(1.915)	1.943	1.950	1.946	1.938	1.941	1.935	1.928	1.934	1.948	1.935	1.941	1.946
	3	1.944	1.950	1.919	1.940	1.946	1.950	1.943	1.948	1.923	1.932	1.941	1.945	1.929	1.943	1.941
Oct.	1	1.942	1.955	1.926	1.942	1.950	1.942	1.936	1.945	1.929	1.931	1.937	1.951	1.921	1.944	1.941
	2	1.951	1.961	1.921	1.942	1.950	1.949	1.937	1.943	1.932	1.933	1.937	1.945	1.915	1.945	1.944
	3	1.938	1.953	1.914	1.939	1.949	1.946	1.931	1.941	1.927	1.930	1.938	1.951	1.929	1.943	1.951
Nov.	1	1.952	1.958	1.928	1.934	1.947	1.944	1.932	1.943	1.925	1.932	1.938	1.947	1.930	1.941	1.954
	2	1.948	1.952	1.925	1.943	1.949	1.948	1.930	1.943	1.929	1.935	1.939	1.946	1.926	1.948	1.954
	3	1.943	1.955	1.920	1.941	1.944	1.944	1.932	1.943	1.930	1.940	1.935	1.941	1.929	1.940	1.950
Dec.	1	1.957	1.953	1.925	1.942	1.942	1.944	1.935	1.944	1.929	1.941	1.940	1.956	1.936	1.951	1.947
	2	1.957	1.950	1.922	1.940	1.947	1.945	1.934	1.938	1.926	1.939	1.943	1.953	1.939	1.949	1.954
	3	1.949	1.948	1.930	1.922	1.939	1.946	1.935	1.938	1.932	1.939	1.939	1.946	1.939	1.950	1.952

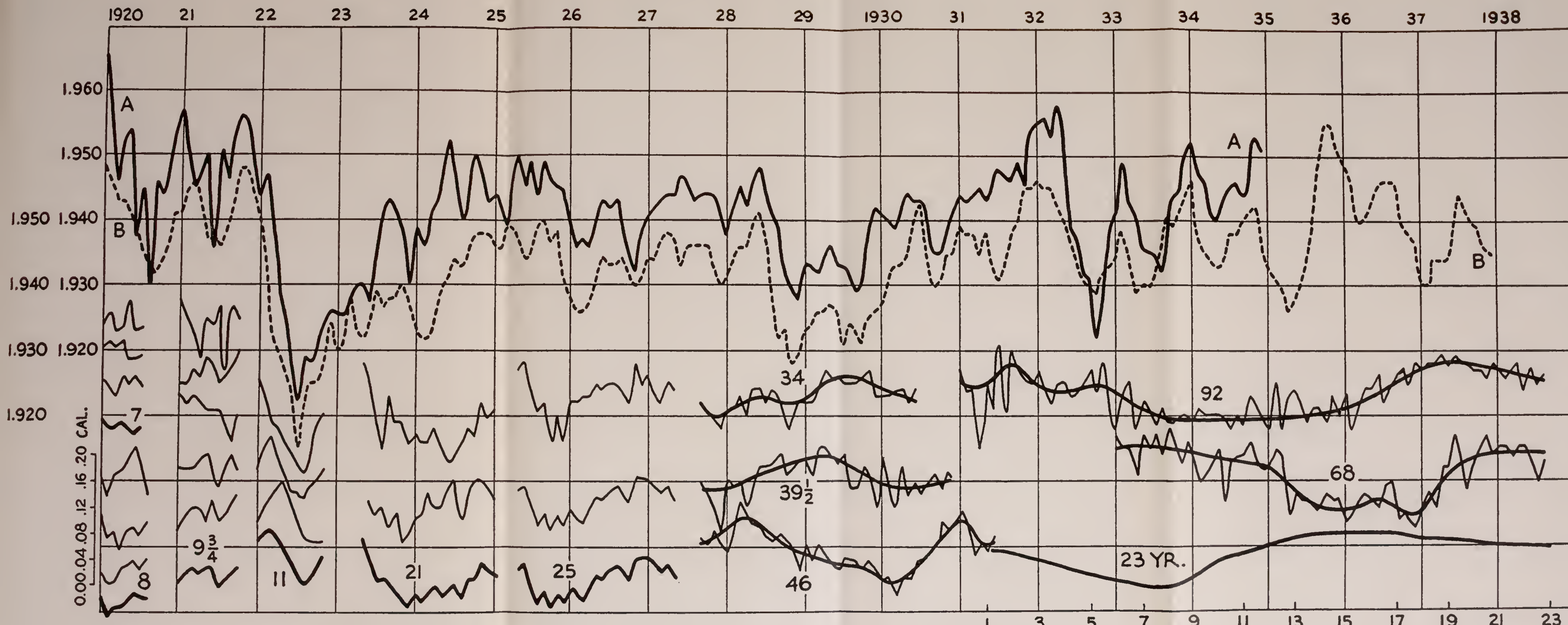


FIG. 7.—Analysis and synthesis of solar variation 1920-1934. The synthetic curve B is drawn below the observed curve A to avoid confusion. Successive derivations of the shorter periodicities precede their general mean. The 23-year periodicity presents as yet only 15 years of its course and is partly estimated.

be combined to determine their mean forms. Interferences from other periodicities and from accidental errors are largely eliminated when occurring in different phases in so many independent cases.

As stated above, an 8-month periodicity in solar variation was seen by the first inspection, and this was first determined. Then the original curve was modified by subtracting from its ordinates a sufficient num-

TABLE 3.—*The 11-month Periodicity in Solar Variation*[a]

Jan. 1920–July 1924					Mean	Aug. 1924–Jan. 1930						Mean	Feb. 1930–Aug. 1934					Mean	General Mean[b]
68	59	58	28	43	51	47	52	37	44	45	30	43	36	44	54	33	42	42	1.9421
67	57	51	24	40	48	38	54	43	41	42	30	41	43	43	50	28	44	42	1.9431
60	48	53	16	40	43	29	48	41	44	44	31	40	46	42	44	29	43	41	1.9437
60	62	50	29	41	48	39	51	42	50	43	30	43	43	40	59	37	41	44	1.9441
57	53	48	26	41	45	49	44	44	45	49	38	45	39	44	53	41	48	45	1.9445
58	48	47	22	39	43	45	44	41	46	45	35	43	39	41	55	41	39	43	1.9446
60	41	38	27	34	40	50	57	41	48	44	36	46	42	32	55	46	49	45	1.9447
49	53	45	22	42	42	51	46	39	45	49	33	44	40	45	65	41	46	47	1.9447
34	54	49	29	39	41	50	49	40	47	45	34	44	41	47	55	34	48	45	1.0443
50	54	46	32	39	44	47	40	39	45	44	35	42	42	46	47	50	55	48	1.9441
56	38	52	21	38	41	49	47	37	43	39	33	41	41	45	56	45	51	48	1.9435
51	40	50	16	21	36	45	46	42	44	40	33	42	41	37	53	46	51	46	1.9431
48	46	46	24	36	40	44	45	36	42	39	33	40	43	36	60	47	62	50	1.9424
58	43	42	33	40	43	50	48	38	41	32	32	40	43	49	62	40	46	48	1.9418
48	45	34	15	42	31	41	44	32	47	36	35	39	40	48	56	37	46	45	1.9407
42	50	32	25	38	37	46	42	32	43	39	33	39	44	45	52	37	47	45	1.9404
33	49	30	29	38	36	39	43	30	41	34	35	34	50	51	47	46	45	48	1.9396
38	51	24	28	39	36	46	44	33	41	22	32	36	45	49	41	32	51	44	1.9389
45	29	21	30	45	34	39	37	37	44	29	29	36	42	57	33	37	60	46	1.9384
39	42	29	23	42	35	47	39	37	43	33	28	38	41	51	35	35	45	41	1.9376
49	38	22	30	41	36	40	33	37	43	28	31	35	39	43	42	39	40	41	1.9371
27	53	19	27	41	33	42	35	41	42	25	29	36	39	49	43	39	42	42	1.9365
25	54	14	30	43	33	36	42	39	36	29	31	36	35	51	32	39	41	40	1.9362
31	50	12	35	43	34	41	31	36	36	31	30	34	42	45	36	35	37	37	1.9359
51	46	00	20	41	32	43	38	37	35	31	31	36	39	46	31	36	46	40	1.9361
44	57	12	35	43	36	48	39	42	31	29	35	37	40	47	29	39	44	40	1.9366
45	36	21	34	45	36	44	35	42	36	34	40	39	41	44	26	35	38	37	1.9374
44	58	14	35	50	40	44	32	37	44	30	39	34	41	49	30	28	48	39	1.9385
54	54	17	29	52	41	49	38	43	41	33	37	39	41	44	36	34	42	39	1.9392
40	48	20	46	53	41	53	42	43	42	31	37	41	35	51	30	38	41	39	1.9398
53	54	21	44	46	44	43	39	42	47	31	35	40	41	47	23	38	44	39	1.9401
49	60	15	40	49	43	44	38	45	46	25	39	40	45	46	18	40	43	38	1.9406
45	53	20	40	41	40	49	42	43	46	30	37	40	41	40	32	42	45	40	1.9410

[a] The figures in the table are to be understood as subjoined to 1.900. Thus, for 68 read 1.968 calories, etc.
[b] Computed from smooth curves representing the three groups.

ber of successive repetitions of the mean form of the 8-month periodicity. Thereupon inspection seemed to indicate an 11-month periodicity. With this also removed, a 7-month periodicity showed itself. Proceeding in this way, periodicities of 7, 8, 11, 21, 25, and 45 months were successively removed.[9] The residual curve remaining after their removal showed very plainly as its major feature a periodicity of 68 months. It has the largest amplitude of any of the solar periodicities.

[9] In our latest analysis, extending from 1920 to 1934, additional solar periodicities of $9\frac{3}{4}$, 34, $39\frac{1}{2}$, 92 months, and one of 23 years were added to the above list.

Unfortunately, accurate solar-constant determinations have not been available long enough to fix the lengths of these periods very accurately. In the discussion of weather periodicities below, evidence is presented indicating corrections of plus 1 day, minus 3 days, and plus 1 month, respectively, to the periodicities stated above as 8, 11, and 45 months.

6. Synthesis of Solar Variation

Having resolved the curve of observation of solar variation into 12 periodicities of approximately determined lengths and amplitudes, the next step was to synthesize these constituents and see how well their summation represents the original curve of observation. This operation is shown graphically in its details and completion in figure 7. The average of residuals between the original curve A and the synthetic curve B is only 0.0036 calories, or 0.19 percent. It appears that the whole solar variation displayed by the observed monthly means is comprised in these 12 periodicities. The small average deviation may reasonably be ascribed to experimental error.

7. Long-Range Predictions of Solar Variation

The curves in figure 7 represent the third analysis and synthesis of solar variation. A 3-year forecast of solar variation is given there. This analysis is based on so much longer a period of observation than the first and second analyses that several new long periodicities are disclosed which add decidedly to the accuracy of the representation. The first and second analyses were published each with a 2-year forecast attached. (See Smithsonian Misc. Coll., vol. 85, no. 1, 1931; vol. 89, no. 5, 1933.) These predictions and the events are shown in figure 8. The average of residuals for the first prediction is 0.0079 calorie, or 0.41 percent. The reader will observe that the first prediction indicated an expectation of values all above normal, although at the time the prediction was made the solar radiation had been almost continuously below normal for many months. The event generally confirmed this expectation.

Unfortunately, a volcanic eruption in Chile interrupted the continuity of the solar-constant observations at Montezuma, so that this series of 2-years' observations is at a disadvantage. It is probable that part of the discrepancy, May to November, 1932, is caused by the volcano. Only Montezuma values are used in preparing the figure. The second prediction was made from data closing in September 1932, and again a prediction of solar variation for 2 years in advance was ventured. The average of residuals between predicted and observed values is 0.0071 calorie, or 0.37 percent. Although maxima and minima are well placed, there is a decided separation of the curves near the end in figure 8. This is cured in figure 7, and in that figure the average deviation for the curves thus far observed is reduced to

FIG. 8.—Predicted and observed solar variation. The maxima and minima occur in the two curves at nearly identical phases. The observed curve may be faulty in 1932 owing to the Chilean volcanic eruption. The separation of the curves toward the end is due to a 23-year periodicity not taken account of.

0.0036 calories, or 0.19 percent. As explained in caption 26, on page 86, there may possibly be a change of phase in solar variation about 1934, tending to modify the 3-year forecast given in figure 7.

II. WEATHER RESPONSIVE TO PERIODIC SOLAR CHANGES

8. Sun-Spot Influence

Having strong indications of 12 long-continued periodic fluctuations in solar radiation, statistical studies were made to seek for their effects on temperature and precipitation. First taking the departures

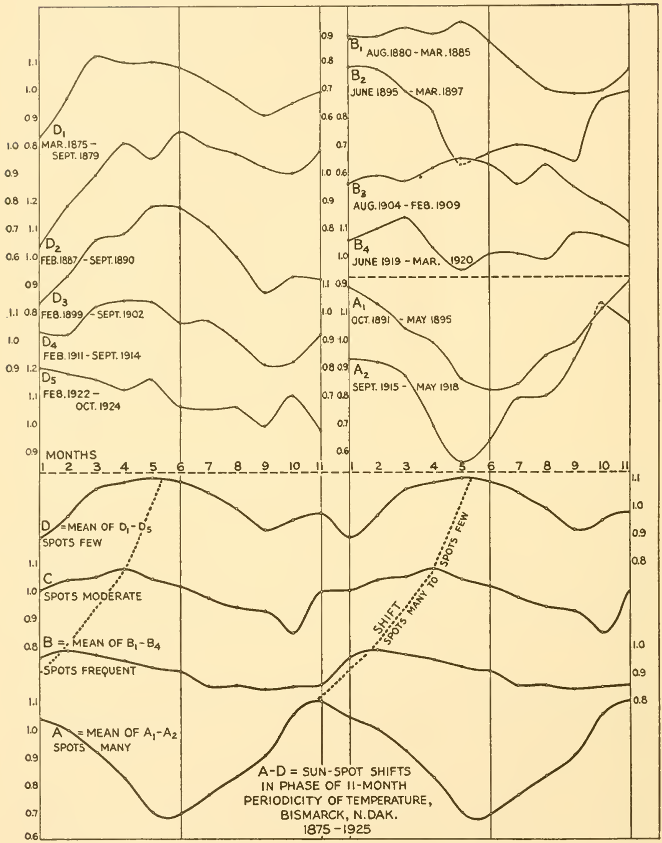


FIG. 9.—Sun-spot numbers and phase changes. The 11-month periodicity in temperature departures at Bismarck, N. Dak.

from normal temperature at Bismarck, N. Dak., from 1875 to 1925 as computed from " World Weather Records," computations similar to those illustrated in connection with table 3 were made. It was soon found that evidences of terrestrial counterparts of each of the seven solar periodicities then known were apparent for short intervals, but changes of phase occurred, showing that continuity is lacking. Further studies seemed to show that these puzzling changes of phase were absent if the computations were restricted to intervals when the sun-spot activity as measured by Wolf's numbers is nearly constant. Later, when longer series of weather records were studied, another phase relationship of much more importance was disclosed. But of this we shall write later.

Figure 9 shows the results of analyses of Bismarck temperatures aimed to disclose and evaluate the 11-month periodicity during the interval 1875-1925. The data are segregated into four groups in which low, medium low, medium high, and high sun-spot numbers prevailed. The dates included in this classification are indicated on the curves. It will be seen that a gradual shift of the maxima of the 11-month periodicity amounting in total to fully half a period is disclosed by the mean values.

Here, as in what follows, the reader is reminded that owing to the presence of other periodicities, and of accidental fluctuations besides, it is not fair to expect perfect correspondence between periodic curves of a given length of period, when these are determined from rather brief intervals containing but a few repetitions of the periodicity in question. Specifically, for instance, the curve D_5 of low sun-spot number statistics in figure 9 differs at months 1, 2, 3 in its trend from the other four. Also the four curves B_1 to B_4 corresponding to medium high sun-spot numbers, show considerable disagreement, although each of them has its maximum in the first half of the period. But when it is recalled that curves D_5, B_2, and B_4 in figure 9, which are the most unsatisfactory of those shown, represent, respectively, only two, two, and one recurrences of the 11-month periodicity, it does not seem surprising that they deviate as much as they do from the better determined mean forms with which they are associated. Naturally, the effects produced by the influences which determine all other periodic and accidental changes of temperature departures cannot be eliminated by taking the mean of only one or two recurrences of the 11-month periodicity.

9. Preparation of Weather Data

When a large program of computation of periodicities in weather departures was undertaken, it was soon found that the monthly fluctuations from normal values of temperature and rainfall, as derived from the tables of " World Weather Records " were so large that they obscured the principal trends which might reveal periodicities corresponding to those found in solar radiation already mentioned. As the computations proposed were very laborious and the available computers inexperienced, it seemed necessary to restrict the smoothing process to be employed to one of great simplicity. Hence the traveling mean of 5 months was chosen. For instance, the values employed for March and April of any year would be represented as follows:

$$\text{March} = \frac{\text{Jan.} + \text{Feb.} + \text{Mar.} + \text{Apr.} + \text{May}}{5}$$

$$\text{April} = \frac{\text{Feb.} + \text{Mar.} + \text{Apr.} + \text{May} + \text{June}}{5}$$

In computing the monthly departures themselves, the mean values used throughout were those found in the first volume of " World Weather Records," neglecting those found in the second volume. It was desirable to use the same normals at all times because we wished the departures used to be homogeneous throughout the entire interval of years examined. Departures from these normal values were computed for the monthly mean temperatures of a great many stations in all parts of the world, and the 5-month traveling means were computed from these departures as described above.

With regard to precipitation a modified course was pursued. It is well known that the precipitation at most stations is seasonal, and at many stations the seasons present extreme variations in normal values. Hence a departure from the normal value, expressed in inches or centimeters, which would be moderate if it occurred in the rainy season, would be immense and perhaps unheard of if it occurred in the dry season. But it was indispensable for our purpose that the departures from normal should be comparable whether occurring in the wet or the dry season. Hence the monthly mean precipitations were first expressed in percentages of the normal values, and then smoothed by taking 5-month traveling means. It would perhaps have been preferable to smooth the percentage values by taking the fifth root of the product of five values, but for simplicity the monthly mean percentage values of the normal were smoothed in exactly the same way as the departures from normal temperatures.

10. Amplitudes of Periodicities Diminished by Smoothing

It was appreciated that the 5-month traveling means of weather data could not yield the full amplitudes of periodicities as short as

7 or 8 months. The fractional diminution produced thereby on the amplitude of the 7-month periodicity is estimated to exceed one-half. For the 8-month and $9\frac{3}{4}$-month periodicities the effect is still considerable, though smaller. For periodicities of 11 months or more it is believed to be inconsiderable. No corresponding effects of diminution occur in the analysis of the solar-constant variation itself because the solar data are not smoothed by 5-month traveling means as are the weather data.

11. Sun-spot Data

As indicated under caption 8, there was evidence to indicate that changes of phase in weather periodicities occur when the activity of the sun alters as measured by the Wolf sun-spot numbers. Therefore, before entering upon statistical computations from weather data extending over the past century, the first step was to assign the beginnings and ends of intervals throughout which sun-spot numbers were approximately equal. To this end the monthly mean sun-spot numbers given in "World Weather Records" were plotted as in figure 10. In preparing figure 10, a 23-year arrangement of the sun-spot data has been adopted. It will be noticed that, excepting the first of the 23-year cycles shown, there is a very fair constancy of positions of maxima and minima in the successive 23-year intervals. From this plot the following intervals were selected as of comparable sun-spot activity:

(*a*) *Sun-spot numbers generally below 40.*

Jan. 1811–Aug. 1815; Aug. 1818–Feb. 1826; Jan. 1832–Nov. 1834; May 1841–Oct. 1844; Aug. 1853–Aug. 1857; Apr. 1865–Sept. 1868; Aug. 1874–May 1880; Nov. 1889–June 1891; Dec. 1897–Sept. 1903; Mar. 1910–Oct. 1914; June 1922–Apr. 1925.

(*b*) *Sun-spot numbers generally above 40 but below 80.*

(b_1) Ascending values (or ascending and descending values contiguous). Sept. 1815–July 1818; Mar. 1826–Dec. 1831; Dec. 1834–July 1835; Nov. 1844–July 1846; Sept. 1857–Mar. 1865; Oct. 1868–Apr. 1869; June 1880–Oct. 1886; July 1891–Jan. 1892; Oct. 1903–Feb. 1910; Nov. 1914–Feb. 1917; May 1925–Dec. 1929.

(b_2) Descending values. Aug. 1839–Apr. 1841; July 1849–July 1853; Jan. 1873–July 1874; June 1894–Nov. 1897; July 1919–May 1922.

(*c*) *Sun-spot numbers generally above 80.*

Aug. 1835–Aug. 1839; Aug. 1846–June 1849; May 1869–May 1873; Feb. 1892–May 1894; Mar. 1917–June 1919.

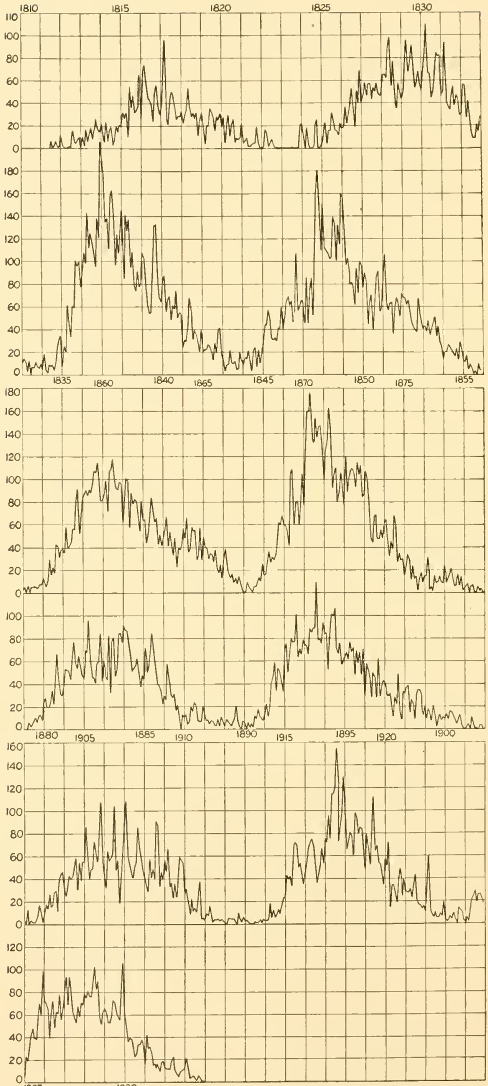


FIG. 10.—Wolf sun-spot numbers, 1810-1933.

It was recognized that this arrangement was very imperfect because of the irregular wide fluctuations of sun-spot numbers. Hence, if, as seemed indicated, the phases of weather periodicities actually alter with sun-spot activity, it could not be hoped that any such arrangement would eliminate altogether these phase changes. Therefore, some dissimilarity between the periodic curves computed for the different intervals of time given above must certainly be expected. All that could be hoped for would be that periodicities in weather of the lengths found in the solar variation would seem to persist without more than a few months of shifting backward or forward, as between the individual intervals stated above, while during the century there would be so persistent and obvious a tendency for maxima and minima to recur in a certain unchanged phase as to justify a belief in the veridical existence of the periodicity in question.

12. Corrections of Solar Periods

It was apparent that since the interval during which daily solar-constant work has been carried on continually is only a little over a decade of years, it is unlikely that the supposed solar periods are determined in length to within several percent of probable error. It was hoped that if these periodicities were really reflected in the weather, the records of such stations as Berlin, Helsingfors, Copenhagen, and others which are published for over a century, might enable the lengths of the solar periodicities to be determined to very high percentage accuracy. A change of periodic length shows itself if the successively determined forms of any assumed period, as for example 11 months, are plotted successively vertically over one another. The maxima and minima will be found to shift steadily to the left or the right according as the true period is less or greater than 11 months. The first station records worked upon were those of Berlin.

13. Full Lines Required in the Statistical Tabulations

It is well known that the temperatures and precipitations frequently tend to depart from normal values continually in a given sense during considerable intervals of time. This must be so if the assumption of a plurality of regular periodicities in weather is a true one, for the combination of several periodicities must lead to prevailingly high values at some times and prevailingly low values at other times. Hence, if a table for computing a periodicity is arranged as indicated above in caption 4, it is improper and leads to error if the first and last lines of the table are left incompletely filled.[9a] Owing to influences aside from the periodicity in progress of being computed, the variable under investigation may be particularly high or particularly low throughout the intervals of the time represented by the first line or the last of the table. To use a part but not the whole of such a line in a short table must produce distortion of the averages. Yet the total intervals given under caption 11 are so short that one can ill spare any part of them. The best course seems to be to fill the first and last lines of the tables by extending the table a little past the limits set by equality of sun-spot activity as represented in the caption 11. Yet this may also lead to a distortion of the curve of averages owing to changes of phase produced by changes of solar activity. Perfection under these circumstances is unattainable, and some indulgence to irregularities is to be given on these accounts in criticizing the results.

14. Berlin, Germany. Departures from Normal Temperatures

It will be difficult, within the allowable limits of tabular and graphical illustration, to demonstrate the findings of this research so thoroughly as to lead the minds of readers to conclusions such as impressed themselves on those of us who followed all the computations from day to day.

A. SEGREGATED WITH REFERENCE TO SUN-SPOT NUMBERS

a. The 11-month Periodicity

Recalling that, owing to the smoothing by 5-month traveling means, the 7-month and 8-month results must necessarily be unrepresentative, let us take up first of all the 11-month analyses. In figure 11 are given all of the 11-month mean curves for Berlin temperature departures obtained by the process outlined above in captions 4, 5, 9, 10, and 11. In order that the reader may more vividly grasp the nature of this work the periodicity computation for low sun-spots for the interval January 1811 to July 1815 is given in table 4. To avoid printing numerous decimal points, the values as given are the 5-month smoothed departures from normal monthly temperatures expressed in tenths of degrees Centigrade.

Table 4.—*Sample Computation of 11-month Periodicity*

	−6	−5	1	16	21	16	15	12	7	7	3
	3	−6	−17	−21	−18	−22	−20	−14	−10	−13	−23
	−27	−18	−20	−14	1	3	−5	−8	−13	−16	−14
	−10	−13	−25	−29	−24	−32	−30	−13	−8	−17	−14
	−8	−9	−16	−8	−2	−2	−3	6	−3	−9	−13
Mean	−9.6	−10.2	−15.4	−11.2	−4.4	−7.4	−8.6	−3.4	−5.4	−9.6	−12.2

[9a] In table 3, what is here called a line is there a column.

FIG. 11.—Eleven-month periodicity in Berlin temperatures. Low, medium, and high sun-spot numbers. A broken line connecting curves indicates a slight defect from full 11 months in the periodicity. Alternate full and dotted pairs of curves cover 23-year cycles measured from 1819.

In this sample the table contains five lines. Of 11 such tables illustrated in figure 11 by plots depicting the 11-month periodicity from low sun-spot temperature departures, there are four tables of four lines, four tables of five lines, one of six lines, and two of seven lines. It is clear enough that the mean values from columns as short as these are subject to a large fractional error. As remarked above, the presence in the data of other periodicities than that sought, and of accidental departures, cannot but distort mean curves depending on so few values per point.

If, now, the general mean value is taken at all times of low sun-spot numbers through more than a century, it results as follows. The unit is still the tenth of 1 degree Centigrade. An arbitrary zero is chosen to give positive and negative values about equally.

0.4	1.2	0.8	1.1	1.3	—0.4	—0.6	—0.5	—0.3	—0.4	0.7

But if it is assumed that the true period is 11 months minus 3 days, then the corresponding general mean is as follows:

—1.5	—0.5	—0.1	0.7	1.7	2.0	0.9	—0.3	0.0	—0.6	—0.3

The latter periodicity has an amplitude of 0.°35 C., about twice the amplitude of the former. It results from 56 lines of smoothed values of temperature departures covering all periods of low sun-spot numbers from 1811 to 1925. The method of allowing for the 3-day decrease of period is partially indicated by the broken inclined line of figure 11. In detail the method is as follows: In the computation of the general mean, the 11 means which represent individual periods of few sun-spots were arranged in a table in such a manner that the values connected by the broken inclined line in figure 11 composed together one vertical column. The mean form, with phase chosen to agree with that expected of the top curve, a_1 of the figure, is given in curve l_1 at the bottom of figure 11.[9b] It is obvious that curve a_1, just singled out for numerical illustration is not in the expected phase, but is 3 months out of phase with the best periodicity. This selection for illustration was, indeed, made to draw attention to occasional irregularities of phase, to which we shall recur. Had I permitted myself to alter arbitrarily the phases of two or three of the mean curves by 2 months each, on the plea of accidental displacement by terrestrial influences, then the general mean would have had an amplitude of a full half degree Centigrade.

It seems difficult to avoid the conclusion that a periodicity lacking 3 days of 11 months in length, and with an average amplitude of 0.°35 C., persists in the temperature of Berlin during times of low sun-spot activity for the interval of 114 years covered.

[9b] The mean for 11 m. 0 d. is given by curve l_1^1.

But the amplitudes alter widely from time to time among the 11 curves shown. Not only do they thus vary, but the forms of the curves differ widely also. When these features are carefully scanned, there seems to be disclosed an interesting regularity. *Beginning with the year 1819,* the forms and amplitudes may be arranged *in pairs* with very good effect. The only deviation from noticeable similarity among these pairs occurs for the pair covering the interval March 1886 to July 1903. Of this pair of curves the first covers that period when the sky was still filled with dust from the tremendous volcanic eruption of Krakatoa. Dr. W. J. Humphreys has called attention to the disturbance of weather which volcanic dust produces.[10] We shall recur frequently to the similarity of such pairs when considering other data.

Curves a_2 to k_2, figure 11, similarly deduced, cover the intervals of time in caption 11 when the Wolf monthly mean sun-spot numbers lay generally between 40 and 80. In part of the data the sun-spot activity was increasing, and in the other part it was decreasing. But no appreciable difference in the data seems to arise thereby. It appears that neither 11 months 0 days nor 11 months minus 3 days gives the maximum amplitude of the periodicity in this case. The best period is 11 months minus 1½ days. The following mean values show this:

Assumed period	Mean values										
11 m. 0 d.	—0.9	0.0	—1.1	0.0	0.8	0.7	0.3	0.1	1.8	0.4	—0.5
11 m. minus 3 d.	0.2	1.4	1.0	1.4	0.9	0.3	—0.3	—1.2	—0.6	—0.4	0.4
11 m. minus 1½ d.	1.5	1.3	0.3	—0.9	—1.9	—2.2	—2.8	—1.2	—0.2	1.5	2.0

The relative amplitudes as just given are 2.9, 2.6, and 4.8, respectively, which show a decided preference for 11 months minus 1½ days. In each case the phase given is the same as that expected for the interval 1815-1819. Here, as before, it is noted that the curves show decided similarity when grouped in pairs beginning with the second curve. The only exception is the last pair which presents dissimilarity. Curve l_2 gives the mean result, assuming a period of 11 months minus 1½ days.[10a] It depends on 58 lines of temperature departures, and shows a range of 0.°48 C., and therefore, like the case already discussed, may fairly be regarded as demonstrative.

Turning now to the temperature data corresponding to Wolf sun-spot numbers exceeding 80, these are graphically expressed in curves a_3 to e_3 of figure 11. These curves rest on few data, only 4, 3, 4, 3, and 2 lines, respectively. Excepting a_3, they are closely similar. The curve a_3 is in fact displaced 5 months in phase from all the others.

[10] Journ. Franklin Inst., vol. 176, pp. 131-172, 1913.
[10a] The mean for 11 m. 0 d. is given by curve l_2^1.

No explanation for this displacement is offered. The mean curve, l_3 is computed by transposing the phase of a_3 by 5 months and taking a straight mean as of period 11 months 0 days. The result is as follows, still in units of one-tenth degree Centigrade:

0.7 —0.4 1.1 —3.6 —3.8 —0.9 —0.1 2.6 4.0 3.2 1.0

The range is nearly 0.°8 Centigrade.

The skew relationship of period between the best 11-month periodicities as determined for the low, medium, and high sun-spot activities is puzzling, but perhaps not impossible to account for. It will be recalled that the periods found were 11 months minus 3 days, 11 months minus 1½ days, and 11 months 0 days, respectively. What this implies, as far as the 11-month periodicity goes, is the advance of the temperature influence associated with high sun spots over that associated with low sun spots by 14 months in 130 periods. As the 11-month periodicity is only one of many, and produces less than a tenth of the total influence which, as we shall see, is exerted by those periodicities which are nearly aliquot parts of 23 years, the effect is not conspicuous.

B. The 8-month Periodicity

Figure 12 shows, in curves a_1 to k_1, the mean 8-month periodicity results derived from the intervals of low sun-spot activity. As shown by the inclined lines there seems to be an advance of 5 months in 110 years, corresponding to a corrected period of 8 months plus 1 day. Taking account of this modification, but preserving the same phase expected as of 1811-1815, the mean results are as follows:

—0.6 0.4 1.6 2.2 2.3 0.1 —2.4 —2.5

The range is almost 0.°5 Centigrade, which owing to the modifying influence of the 5-month smoothing, already referred to, must be less than the real average range of this periodicity. The mean curve, l_1, figure 12, is based on 75 lines covering the intervals of low sun-spot numbers from 1811 to 1925. Scanning the curves a_1 to k_1 on figure 12, the pairing tendency, already referred to in discussing the 11-month analysis, is recognizable. The only marked inconsistency of the pairs, as arranged with a beginning in 1819, occurs for curves d_1 and e_1. It will be noted that for 8-month periodicities, as with the 11-month results, the pairs palpably begin with the *second* curve, about 1819.

Turning to the intervals when the Wolf sun-spot numbers lay between 40 and 80, we again find the greatest amplitude by assuming a period of 8 months plus 1 day. Choosing the phase to agree with

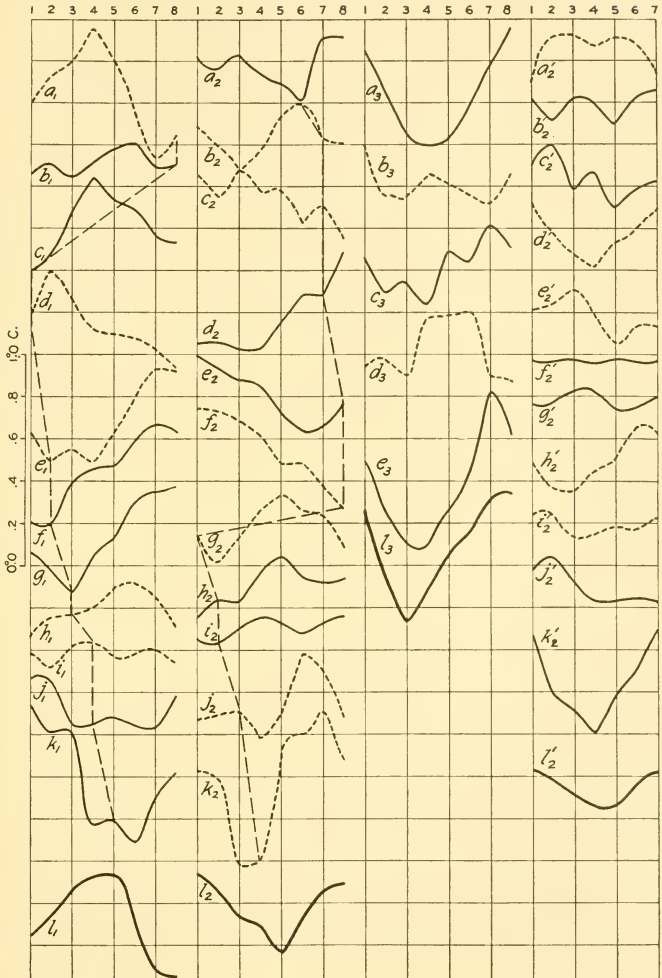


FIG. 12.—Eight-month periodicity in Berlin temperatures. Low, medium, and high sun-spot numbers. A broken line connecting curves indicates a slight excess over 8 months in the periodicity. Alternate full and dotted pairs of curves cover 23-year cycles measured from 1819.

that expected for the first interval, the mean values as thus reduced run as follows:

Assumed period	Mean form								Range
8 m. o d.	0.1	—0.6	—1.1	—1.0	0.3	0.4	0.6	—0.3	1.7
8 m. plus 1 d.	1.9	1.0	—0.1	—0.6	—1.9	—0.3	1.1	1.4	3.8

The average range of the 8 months minus 1 day periodicity is almost 0.°4 Centigrade (see fig. 12, l_2). This range is more than twice as great as for the periodicity 8 months 0 days.

Pairing is not so well marked in these curves as appears in figure 12, a_1 to k_1. It, indeed, shows strongly as between curves h_2, i_2, and j_2, k_2, but the curves a_2 to g_2 seem inclined to change form at *every* sun-spot period of 11+ years instead of every *second* period, as in former cases. We shall note this tendency frequently in other connections.

Curves a_3 to e_3 of figure 12 relate to intervals when the Wolf numbers generally exceeded 80. As in the corresponding case of the 11-month periodicity, they show no definite deviation from the originally assumed period, 8 months 0 days. There is on the whole a good agreement between the curves. Only the curve d_3 runs counter to all of the others, but it is at the same time one of the weakest, representing the mean of but four lines. The general mean is represented in the curve, l_3 and runs as follows:

2.2 —1.0 —2.6 —1.7 0.0 1.1 2.6 2.9

Its range is over 0.°5 Centigrade.

C. The 7-month Periodicity

The 7-month periodicity, as already stated is much modified by the 5-month smoothing. However, in the curves representing intervals with sun-spot numbers between 40 and 80 there is such an excellent case of the pairing which starts with the year 1819 that these curves are given, a_2 to k_2 annexed to figure 12. There is no exception to the similarity of the pairs from curve b_2 to curve k_2. Two excellent pairs are found corresponding to low sun spots, but generally these forms change with each new sun-spot period.

There is no indication in any of the analyses of the 7-month periodicity of a departure in length from the period assumed. The following are the mean forms and ranges. The ranges may be assumed to be only about half as great as would be found without 5-month smoothing.

Sun-spot numbers	Seven m. o d. mean periodic forms							Range
Below 40	0.7	1.8	0.0	—0.1	0.4	—0.6	—1.4	0.°3 C.
40 to 80	0.8	0.3	—0.4	—0.9	—0.9	0.1	0.7	0.2 C.
Above 80	1.2	0.8	—1.2	—1.9	—0.6	0.6	1.7	0.4 C.

D. Dependence of Phase on Sun-spot Activity

In caption 8, evidence was presented showing that the phases of the 11-month and other periodicities observed in temperature departures of Bismarck, N. Dak., altered as a function of the sun-spot activity. Referring now to figure 13, the data for Berlin are not wholly consistent with that conclusion. As not all of the 11-year sun-spot periods show high Wolf numbers, let us restrict our inquiry to the periods culminating about 1837, 1847, 1871, 1893, and 1918. Consider first the 11-month periodicity. In table 5 are given the months within the 11-month period when maxima prevail. The results cover times of low, medium, and high Wolf numbers. The shift of maxima for medium and high Wolf numbers is indicated in the fourth column. In the last column are given without details the corresponding shifts found for the 8-month periodicity data.

Table 5.—*Shift of Phase, Berlin Temperatures, Attending Sun-spot Activity*

Years covered	Wolf numbers	Months of maxima	11-month shifts	8-month shifts
1831-1834	Below 40	11 to 4	0.0	0.0
1834-35 and 1839-41	40 to 80	1 to 5	+1.0	—1.0
1835-1839	Above 80	2 to 4	+1.0	—4.0
1841-1844	Below 40	9 to 2	0.0	0.0
1844-46 and 1849-53	40 to 80	6 to 11	—2.5	—2.0
1846-1849	Above 80	8 to 9	—3.0	—1.5
1865-1868	Below 40	10 to 11	0.0	0.0
1868-69 and 1873-75	40 to 80	5 to 8	—4.0	+2.0
1869-1873	Above 80	8 to 10	—1.5	0.0
1886-1890	Below 40	11 to 2	0.0	0.0
1890-92 and 1895-97	40 to 80	10 to 2	—0.5	—0.1
1892-1895	Above 80	9 to 11	—2.0	—1.0
1910-1914	Below 40	3 to 5	0.0	0.0
1914-17 and 1919-21	40 to 80	1 to 5	—1.0	—3.0
1917-1919	Above 40	8 to 11	{ —6.5 or +5.5 }	—2.0

There appears a prevailing tendency for the phase to be earlier with higher sun-spot activity, but it is not as conspicuous or regularly progressive a tendency as appeared in the Bismarck data. In fact the evidence seems to show that though there is a small change of phase toward earlier dates within the cycles, when Wolf numbers increase, yet this effect is small compared with changes of phase which, as we are about to point out, occur at integral multiples of $11\frac{1}{2}$ or of 23 years, counted from January 1819. Such changes of phase will next be demonstrated.

E. Dependence of Phase on Epoch Counted from 1819

It was desired to present this phenomenon apart from changes of phase accompanying variations of sun-spot activity. Hence the data

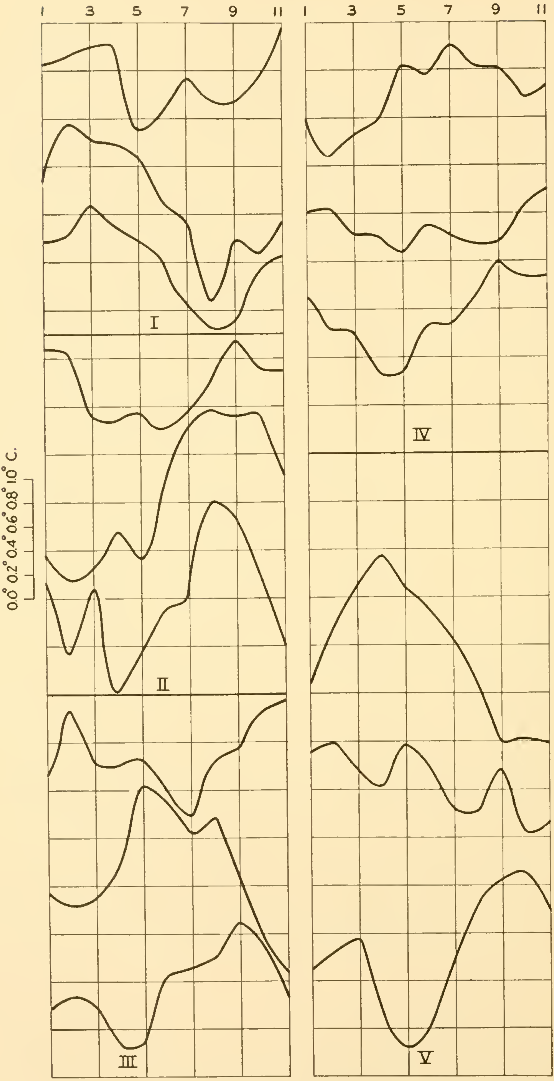


FIG. 13.—Sun-spot numbers and phase changes. Eleven-month temperature periodicity at Berlin for low, medium, and high sun-spot numbers.

were limited to times of low Wolf numbers. Smoothed departures from normal temperatures at Berlin were arranged in periodicity tables each of which fell entirely within a period of 23 years, and the beginnings and ends of such 23-year periods fell always at an integral multiple of 23 years counting from January 1819. Periodicities of 7, 8, 9¾, 11, 12, 13.6, 21, 25, 34, and 46 months were investigated in this manner. Owing to the moderating influence of the 5-month smoothing, already referred to, the 7-month periodicity was indecisive and is omitted here. Of the 12-month periodicity I shall treat separately. Figure 14 shows the results of all others. In the figure the 8-month curves are corrected in phase to the more exact period 8 months plus 1 day, and the 11-month curves are corrected in phase to the more exact period 11 months minus 3 days. The scales of abscissae and ordinates are altered in the 21-, 25-, 34-, and 46-month plots for greater convenience.

In table 6 the Roman numerals I to V refer to 23-year intervals ending respectively at one, two, three, four, and five times 23 years after January 1, 1819.

TABLE 6.—*Comparison of Phases and Amplitudes. Berlin Temperature Periodicities*

Periodicity	Phase	Amplitude
8-month	I, II, III, and V similar; IV opposed.	I and II moderate; III and V large; IV small.
9¾-month	I and V similar; II, III, and IV opposed.	I, III, and V moderate; IV large; II small.
11-month	I, II, and V similar; III and IV opposed.	II, III, and IV moderate; I and V large.
13.6-month	I, II, and III similar; IV and V opposed.	IV and V moderate; I and III large; II small.
21-month	I and V similar; II, III, and IV opposed.	Amplitudes nearly equal.
25-month	I, III, and V similar; II and IV opposed.	III, IV, and V moderate; I and II large.
34-month	I, IV, and V similar; II and III opposed.	II, III, and V large; I and IV very large.
46-month	I and V similar; II, III, and IV opposed.	All large, II, IV, and V very large.

Notes.—As all the tables were prepared from the same original smoothed departures, the influence of the unremoved shorter periodicities is very pronounced in causing irregularities in the curves representing longer periodicities. This must obviously occur because only a few repetitions (in the 46-month tables sometimes only two, sometimes three) were available for the longer periodicities. Sometimes the longer periodicities display periodic submultiples conspicuously. For instance in 46-I there is obviously a periodicity of 9.2 months superposed, while in 46-II there is obviously a periodicity of 11.5 months superposed. These two unusual periodicities correspond, respectively, to 1/30 and 1/24 of 23 years.

Referring to the table, let us now tentatively suppose that the smoothed temperature departures of Berlin were plotted in 23-year cycles for the 115 years, 1819 to 1923. Considering figure 14 and table 6, it would almost certainly be found that many features of similarity would appear in the successive plots. For so many periodic

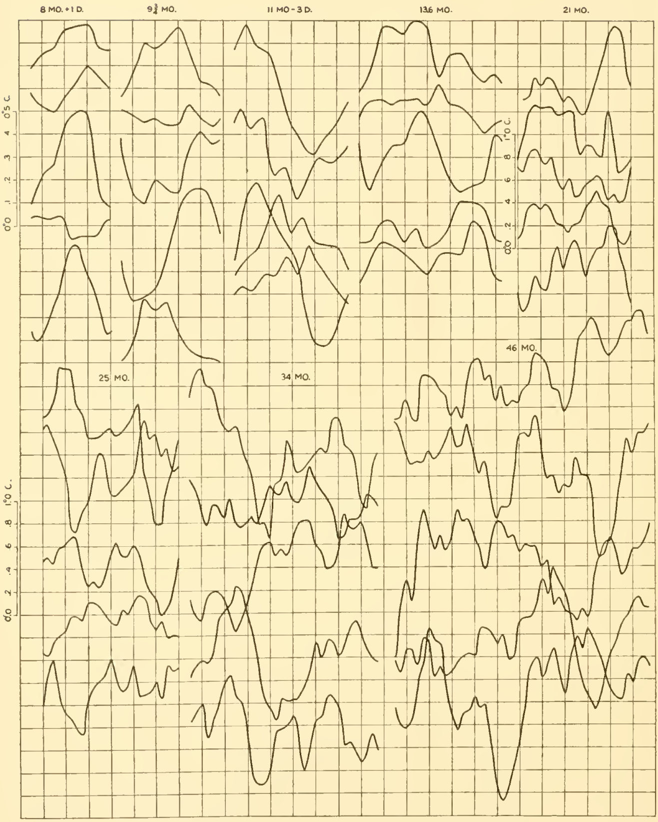


FIG. 14.—Dependence of phase in periodicities of Berlin temperatures on epoch measured from 1819. Each curve given is the mean form for 23 years. In each group the top curve starts from January 1819. Note prevailing similarity of curves I, III, V and again of curves II, IV. Exceptions noted in text.

features would recur in successive 23-year intervals in nearly the same phases that the successive complex curves formed by their summation must themselves show features of some similarity, though a little altered in phase and amplitude from one 23-year interval to another. It is clear that, of the various intervals, V would be most similar to I, because its phase is found similar to I for all periodicities given except 13.6. Interval III is next most similar to I, but IV, and next to that II, would be most dissimilar to I. On the other hand, II and IV would be found to present many features of similarity each to each. We shall recur to this when we consider the possible application of periodicities to long-range forecasting. Here I content myself with hinting that three most similar intervals, I, III, and V, and two opposed similar intervals, II and IV, have separations of 46 years.

The 12-month periodicity is particularly instructive. Meteorologists have long known that a very long interval of years does not suffice to yield monthly means of temperatures which will be closely followed in the mean during a succeeding equally long interval of years. Hence it was expected that a 12-month periodicity would be found in the departures from normal temperatures at Berlin. But it would be natural to suppose that its cause is purely terrestrial and that it would show no relation to solar periodicities. The contrary is certainly the case. Figure 15 shows clearly that the 23-year interval is of decisive influence in changing the phase and amplitude of the 12-month periodicity. This is true not only at Berlin but at all other stations which we have investigated, including Helsingfors, Copenhagen, Greenwich, Cape Town, Adelaide, and others.

In preparing figure 15, the 12-month data were not restricted to times of low sun spots as were the data for figure 14. For it was not to be presumed at first that this 12-month periodicity was due to changes originating in the sun, but rather on the earth. These more numerous data gave two tables of about a dozen lines each for each 23 years. In this way abundant evidence proves the critical importance of January 1819 and multiples of 23 years thereafter as determining points in the pairing of the curves, such as has already been referred to. Another interesting reference to these curves in figure 15 will be found below under caption 14-B.

From the studies rehearsed above under the various captions of 14-A, we conclude:

1. Certain periodicities found in solar variation are found persisting throughout more than a century in Berlin temperature departures.

2. Small corrections to the supposed lengths of two of these solar periods are indicated by these long ranges of data.

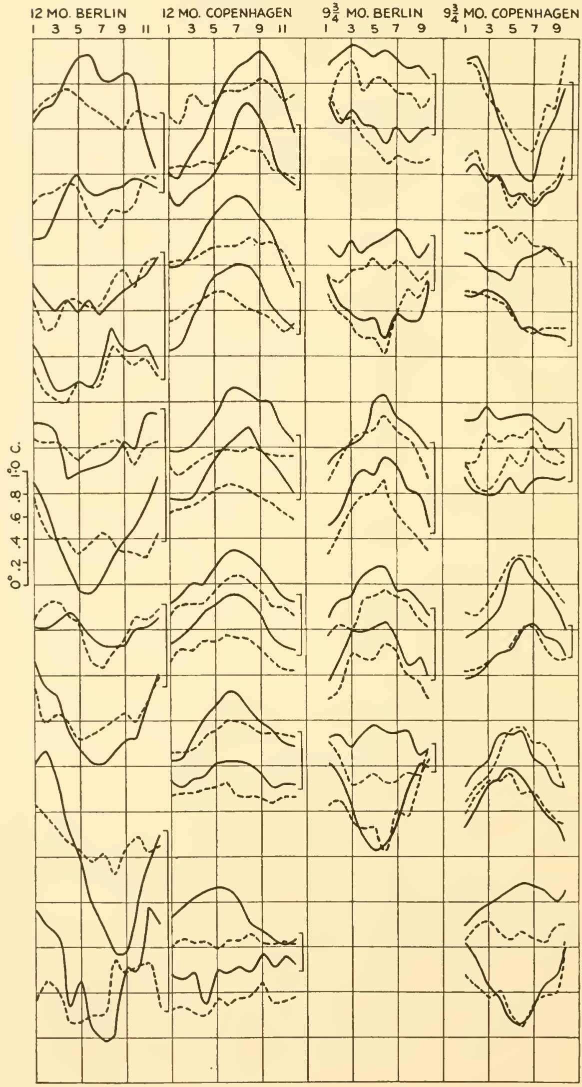


FIG. 15.—The 23-year influence on periodicities of 9¾ and 12 months. Each bracketed pair covers 23 years. Full curves are from original data, dotted curves from residuals after removing many periodicities.

3. The 23-year period, which is the approximate least common multiple of the observed solar periodicities, and is also the approximate length of Hale's solar magnetic cycle, is of dominating importance in the terrestrial weather-responses to solar influences.

4. While the intensity of sun-spot activity has some influence on the phases of the temperature periodicities, it is by no means as important as the arrival of integral multiples of the 23-year interval measured from January 1819. These define large modifications both of phases and amplitudes.

5. The 23-year period governs not only periodicities which seem to be of purely solar causation, but also the phases and amplitudes of the 12-month periodicity in departures from normal temperature. This periodicity might otherwise have been regarded as purely of accidental terrestrial origin.

6. It is not possible to arrive at definite conclusions as to the veridity of periodicities of long duration from data restricted to 23-year intervals, and further restricted to intervals of comparable sun-spot activity. Another attack on this part of the subject follows.

B. ANALYSES GROUPED IN PERIODS OF 11½ YEARS AND 23 YEARS, BASED ON JANUARY 1819, AS DATE OF DEPARTURE, AND INCLUDING ALL DATA

The preceding discussion of Berlin temperatures was restricted to intervals of comparable sun-spot activity. But though this is desirable it is not vital, and restrictions relating to 23-year intervals having been proved to be more essential, it becomes necessary to merge all data, whatever the prevailing sun-spot activity, in order to study fairly the longer periodicities. It has been proved advantageous to base our studies on the zero date January 1819.

A. THE 11-MONTH PERIODICITY

As before, we begin with the 11-month period. As there is here no intention of making a century-long comparison, no account need be made of the correction (minus 3 days), nor when we deal with the 8-month periodicity of its correction (plus 1 day). Table 7 gives, for illustration, a complete tabular determination of the mean 11-month periodicity curves from January 1819 to October 1864. The similarity of the two halves of each of the two 23-year periods covered, and the complete opposition of these two 23-year periods, each to each, are clearly shown in figure 16. It is instructive to note how abruptly the transition occurs from one type to the other just at the turn of 23 years after January 1819. The two types differ

FIG. 16.—The 11- and 21-month periodicities in Berlin temperatures. Phase dominated by the 23-year cycle from 1819. Full and dotted pairs of curves each cover a cycle of 23 years.

Under A, wording should read " Mean I, III, IV, VI, VIII, IX."

in much the same way as the right and left hands. In what follows we may sometimes speak of them as the right and left types.

TABLE 7.—*Berlin Temperature Departures 11-month Periodicity*

Jan. 1819–Dec. 1829											
	8	9	16	16	17	19	19	14	6	−7	−22
	−24	−24	−17	−5	−1	−6	0	−5	−8	−6	−7
	−10	−10	−12	2	8	0	−1	−1	−7	−4	10
	19	26	29	34	31	27	22	17	8	1	3
	5	−3	−23	−23	−26	−32	−27	−7	−7	−6	−4
	−2	3	10	14	20	18	14	8	0	−6	−9
	−5	−1	6	16	25	22	15	12	4	−6	−8
	−2	−4	−3	3	11	0	2	4	1	−6	9
	13	17	18	20	16	12	3	−12	−11	−5	−2
	3	19	16	12	10	1	3	1	−5	−7	2
	−1	3	9	6	2	1	0	0	−7	−16	−19
	−18	−20	−9	0	1	−2	−6	−13	−32	−42	−49
Mean	−22	+13	+33	+79	+82	+50	+37	+15	−48	−92	−80
Jan. 1830–Nov. 1841											
	−44	−34	−15	−2	7	3	−3	−3	2	−1	−9
	−7	−7	−6	−5	4	3	3	−7	−1	0	1
	1	5	3	6	0	0	−5	−8	−12	−7	−8
	−1	−5	4	1	−4	2	10	1	−5	−3	−12
	−15	−4	13	16	20	19	16	10	19	23	25
	20	17	10	8	11	12	9	5	2	−1	−3
	2	2	−6	−10	−10	−12	−1	5	3	4	1
	−12	−14	−5	−8	−3	4	5	−4	−5	−12	−15
	−17	−8	−7	−2	1	3	−18	−27	−28	−33	−33
	−16	−6	−11	−5	−6	−8	−6	1	−1	−4	−7
	−7	−7	−7	−4	5	5	7	3	5	2	−3
	−2	−2	−2	−2	0	−4	−5	0	−10	−9	−19
	−13	−16	0	−1	6	2	1	−1	4	13	9
Mean	−85	−61	−22	−6	+24	+22	+10	−19	−21	−22	−56
Dec. 1841–Nov. 1852											
	8	7	0	−4	0	−3	−4	−1	−5	−11	−4
	−1	−5	−4	3	−7	−14	−10	−6	−10	−5	2
	10	10	8	5	3	−4	−8	−10	−12	−15	−15
	−11	−15	−8	−19	−34	−35	−29	−30	−15	−3	−7
	−3	0	1	6	14	21	18	12	13	10	8
	8	15	12	1	−8	−13	−16	−21	−9	−6	−1
	3	4	−2	1	−6	−23	−14	−8	−7	−5	14
	7	0	−7	−4	−6	−2	0	9	7	5	5
	4	−6	−9	−9	−12	−11	−14	−21	−12	−14	−13
	−8	5	−4	−1	−5	−7	−6	−3	2	8	11
	11	2	−6	−10	−11	−15	−3	−4	2	11	14
	7	4	2	−6	−4	1	7	5	9	14	21
Mean	+29	+18	−14	−31	−63	−88	−66	−65	−31	−9	+29
Dec. 1852–Oct. 1864											
	16	6	−4	−16	−23	−17	−9	−4	0	−4	−12
	−8	−7	−6	−5	3	0	2	−1	−1	−1	−2
	−1	−1	−16	−21	−22	−29	−27	−12	−9	−7	2
	−1	−9	−6	−10	−4	−5	1	−1	−8	−6	−11
	−5	−10	−3	−2	0	−1	2	−1	1	2	7
	11	18	14	20	13	1	−8	−7	−17	−11	−3
	2	6	11	13	14	19	21	26	16	16	12
	12	9	11	11	10	1	3	1	−2	−4	2
	−4	−4	−4	−3	−6	−10	−13	−19	−12	−5	−6
	−6	8	4	−1	2	9	7	7	5	4	7
	7	10	10	8	3	1	1	1	5	13	18
	18	20	19	11	1	−1	−3	3	5	15	5
	5	2	−4	−18	−11	−13	−22	−19	−13	−15	−18
Mean	+35	+37	+20	−10	−15	−35	−35	−20	−23	−2	+1

The little double table (table 8) extracted from table 7 emphasizes this behavior. The 11-month periodicity as computed from January 1819 to December 1829, and from January 1830 to November 1841, shows high maxima at the fifth month. From December 1841 to November 1852 and from December 1852 to October 1864, on the contrary, deep minima are found at the sixth month. The transition from the first of these types to the second is abrupt. To show its abruptness the last 22 months ending with November 1841 may be contrasted with the first 22 months beginning with December 1841.

Table 8.—*Abrupt Phase-change. Berlin 11-month Temperature Periodicity*

	−2	−2	−2	−2	0	−4	−5	0	−10	−9	−19	
	−13	−16	0	−1	6	2	1	−1	4	13	9	: Nov. 1841
Mean	−7.5	−8.0	−1.0	−1.5	3.0	−1.0	−2.0	−0.5	−3.0	2.0	−5.0	
Dec. 1841:	8	7	0	−4	0	−3	−4	−1	−5	−11	−4	
	−1	−5	−4	3	−7	−14	−10	−6	−10	−5	2	
Mean	3.5	1.0	−2.0	−0.5	−3.5	−8.5	−7.0	−3.5	−2.5	−8.0	−1.0	

Obscured as they are by the influences of other periodicities and accidental effects, yet in the mean of the first two lines of table 8 the maximum occurs on the fifth month, and in the mean of the last two lines the minimum occurs on the sixth month, just as happens with the general means found in table 7. Even in details the two mean curves representing 22 months each are opposite, as shown by figure 17.

b. The 21-month Periodicity

Take as an example of another type the 21-month periodicity shown in figure 16. In this instance the transition from left to right in type usually occurs at each $11\frac{1}{2}$ years, though not invariably. One type holds for instance through the two periods of $11\frac{1}{2}$ years each from October 1841 to June 1864. But then, note the abrupt transition between the 42 months preceding and the 42 months following July 1864. The mean of the first pair of lines is almost precisely opposite to the mean of the last pair, as is shown in figure 17 and table 9.

Of the 10 curves illustrating the 21-month periodicity, numbers 1, 3, 4, 6, 8, and 9, beginning 1819, 1841, 1852, 1875, 1897, and 1910, respectively, are generally similar in phase, and not greatly different in amplitude. Opposed in phase are curves 2, 5, 7, and 10, but they are not quite so similar each to each. From this we see that during about 70 years out of 110, the 21-month periodicity, whether we regard it as true or spurious, would have produced nearly identical effects upon the temperature of Berlin. The general mean effect over 70 years, as

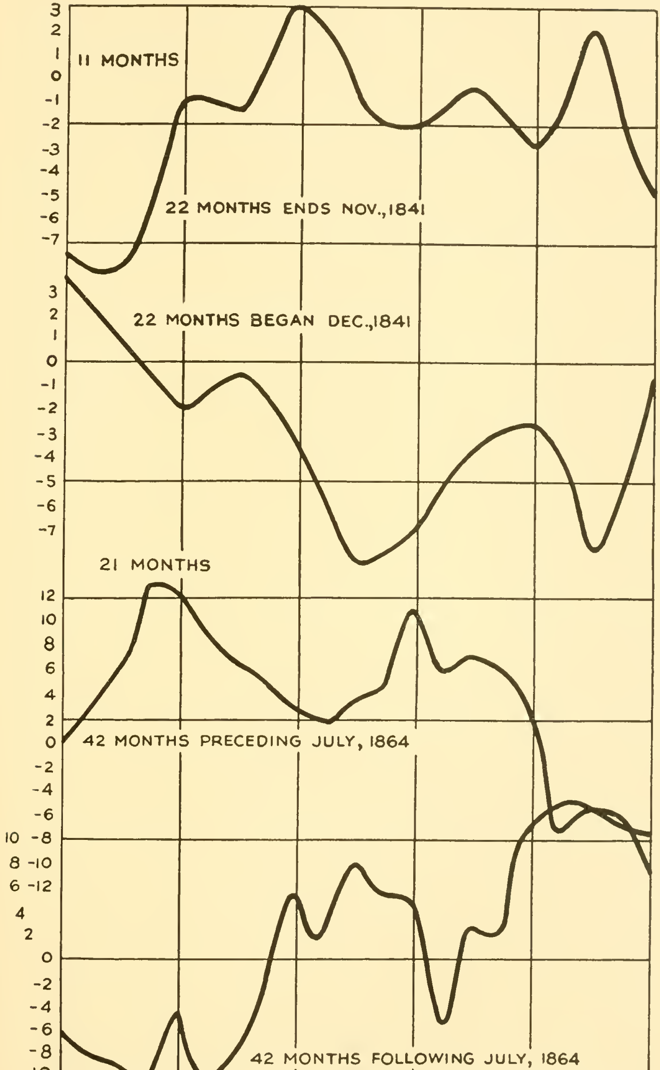


FIG. 17.—Details of the 11- and 21-month periodicities in Berlin temperatures. Showing abrupt reversal of phase.

TABLE 9.—*Abrupt Phase-change. Berlin 21-month Temperature Periodicity*

	—5	—6	—6	8	4	—1	2	9	7	7	5	4	7	7	10	10	8	3	1	1	1
	5	13	18	18	20	19	11	1	—1	—3	3	5	15	5	5	2	—4	—18	—11	—13	—22
Mean	0.0	3.5	6.0	13.0	12.0	9.0	6.5	5.0	3.0	2.0	4.0	4.5	11.0	6.0	7.5	6.0	2.0	—7.5	—5.0	—6.0	—10.5
	—19	—13	—15	—18	—10	—18	—21	—16	—1	—8	7	11	11	3	14	12	24	28	26	24	14
	8	—3	—4	—2	1	—2	5	9	12	12	9	0	—2	—14	—9	—7	—2	—2	—1	—1	7
Mean	—5.5	—8.0	—9.5	—10.0	—4.5	—10.0	—8.0	—3.5	5.5	2.0	8.0	5.5	4.5	—5.5	2.5	2.5	11.0	13.0	12.5	11.5	10.5

computed from curves 1, 3, 4, 6, 8 and 9, is as follows and is illustrated at A of figure 16. The mean values which follow are expressed as usual in units of 0.°1 Centigrade.[10b]

1.3 2.3 2.8 5.4 5.2 4.6 3.4 2.3 0.6 —0.3 —1.8
—2.8 —1.9 —0.2 1.5 3.4 5.1 3.8 2.1 1.2 0.4

The range of the general mean is 0.°8 Centigrade. This mean curve represents the tabulation of 39 lines in each of its 21 columns or 819 months in all. The contradictory results found in the remaining 24 lines, representing 504 months, themselves somewhat approach a common type. Its mean form, shown at B of figure 16 is as follows:

—4.4 —4.2 —4.4 —5.1 —5.0 —5.2 —2.8 —0.7 0.6 0.2 0.9
0.8 —0.5 1.6 2.3 1.9 2.6 0.5 —1.3 —2.9 —4.3

Being plainly associated with periodic changes in the sun, as the dates of the appearance and disappearance of contrasting phases in these curves 2, 5, 7, and 10 appear to be, the existence of these curves of a contradictory type does not, in my judgment, reasonably require us to doubt the evidence of the other 70 years or of their own 40 years that 21 months is a veridical period in terrestrial temperature, produced by a periodic solar variation.

c. Progressive Removal of Determined Periodicities

Acting on the conclusion just expressed, I have felt it justifiable to remove, one after another, the mean evaluations of the various periodicities, *and to remove them in parcels of $11\frac{1}{2}$ or 23 years at a time,* so as to eliminate them to the highest degree possible despite changes of both phase and amplitude. As I am aware that this course will be criticized and perhaps disowned by meteorologists and statisticians, I pause at this point to refer to the 12-month periodicity, as computed from the residuals of the 5-month smoothed Berlin temperatures, after removing in the way just indicated, and in the following order, the 7, 8, 11, 13.6, 21, 25, 34, 46, 68, and $9\frac{3}{4}$ month periodicities. Mean values for each $11\frac{1}{2}$ years from 1819 to 1929 are given by the dotted lines in figure 15. These results may be compared with the closely juxtaposed curves for the 12-month periodicity, as previously computed directly from the original data, and already referred to under 14-A. The very great similarity in general between the two sets of curves indicates that the removal of all of those many periodicities in $11\frac{1}{2}$-year or 23-year parcels has not ruined the residuals for the purpose of the 12-month analysis. Figure 15 also includes a similar pair of juxtaposed analyses of 12-month periodicities for Copenhagen, and

[10b] The lines of these two tables (too long for page width) are to be read consecutively like two lines of text, not staggered as might be thought.

also the $9\frac{3}{4}$-month mean curves for Berlin and Copenhagen as computed from the original data and again from the residuals after removal of many other periodicities. I hope that this may be a step toward promoting greater confidence in the procedure. But the curves about to be referred to, representing other periodicities, will furnish other grounds for confidence in these methods.

It has already been pointed out in the analysis of the original data that the expiration of an integral multiple of 23 years from January 1819 is often the signal which warns us of a reversal of phase impending in the temperature periodicities. As this also occurs frequently in periodicities which are computed from the residuals which remain after removal of many determined periodicities by $11\frac{1}{2}$-year and 23-year steps, it would seem to indicate that the data were not harmed by such removal. For it is to be recalled that the effect of such removal, applying as it does the actual mean values over each $11\frac{1}{2}$- or 23-year period to correct all monthly values within that very period, *must tend in the strongest way to smooth* the residual curve which remains after such removal. If then such a *smoothed* residual curve shows plainly the newly sought periodicity, and not only shows it in approximately the same phase at many intervals during a century, but also shows the reversal of its phase at the critical dates, after the manner often noted in earlier analyses—the combination of these regularities of behavior seems to strongly support the hypothesis that the computed periods are veridical, and cumulatively defends the method used in their removal.

But still another type of confirmation of veridity is available. It will be noted that in the list of 10 periodicities which were said to have been removed before seeking the 12-month periodicity, one of $9\frac{3}{4}$ months was mentioned last. This periodicity was not noticed in the original data, nor was it suspected until after the 68-month curve was determined. Then seven waves appeared so definitely in the mean curves for 68 months, as shown in figure 18, that no question of the reality of the $9\frac{3}{4}$-month periodicity could be entertained. Yet the 68-month curves themselves were not computed until after the entire previous list of eight periodicities had been removed in $11\frac{1}{2}$- or 23-year parcels. That the $9\frac{3}{4}$-month curve should have survived so much modification of the data seems to indicate that real and not spurious periodicities had been found and removed. In order further to demonstrate this argument more conclusively, I show in figure 15 the $9\frac{3}{4}$-month curves for Berlin and Copenhagen, both as computed from the original data and as computed after nine periodicities had been removed therefrom.

D. Criterion for True and False Periodicities and Limit to the Number of Periodicities

If it were the case that in long intervals of time only very small changes in phase and amplitude took place in the forms of the periodicity curves, it would be simple as well as obviously indicated to pick out, evaluate, and remove periodicity after periodicity until no more of them could be discerned in the residual temperature departures. In fact it would have been done by meteorologists long ago. But as we have now shown, this simplicity does not obtain. Although, for instance, the 11-month less 3 days periodicity may be traced at Berlin during times of low sun-spot numbers for 110 years, with an average amplitude of about 0.°4 Centigrade, there are wide fluctuations of phase and amplitude during that long interval. So the question arises, if we are to admit that obscure causes produce reversals of phase and wide fluctuations in amplitude, how shall we know if a supposed periodicity is real or arbitrary?

The quandary is much more serious for long periods than for short ones. During 11½ years there are, for instance, twelve 11-month periods and still more of 7, 8, and 9¾ months. If so many repetitions yield, as we have seen that they do, definite smooth mean curves of considerable amplitude representing the periodicity throughout these abundant repetitions, and there follows an abrupt change to another type which continues equally well verified through a second interval of 11½ years, the mere change of type, associated as Hale has shown it to be with a reversal of the magnetic status of the sun, is not a valid argument for the rejection of this otherwise excellent periodicity.

When, however, the longer periods of 21 to 68 months are in question, the number of repetitions of them in 11½ or even in 23 years is not enough to eliminate irregular fluctuations, or to inspire much confidence. For the mean curves are left very ragged. If no supporting evidences were available, they would sometimes seem probably accidental.

But let us take as a specific example the 68-month curve at Berlin, as shown in figure 18, I to V. The following observations may be made:

1. Each curve shows seven waves, indicating a periodicity of 9¾ months.

2. Removing, in imagination, the waves due to the 9¾-month periodicity, each subfigure shows a smooth curve of 68 months' period, roughly similar in form to a sine curve.

3. Each of the subfigures is the result obtained from 23 years of observation, including four repetitions of the periodicity. Although not a century, 23 years is, after all, a long time.

4. The ranges of the smoothed 68-month curves are substantial. For curves I, II, III, IV, and V, the ranges are 0.°4, 1.°1, 1.°1, 0.°5, and 1.°0 Centigrade, respectively. The extreme range of the original data before any periodicities at all were removed, but smoothed by 5-month traveling means, seldom exceeds 5.°0 Centigrade. This includes, as we have seen, several short interval periodicities of a range of 0.°5 Centigrade or more, which when combined in common phase may produce a range of at least 2.°0 Centigrade. Hence much of the original range disappears with their removal. This makes it apparent that the 68-month curves contain a very considerable part of the residual range remaining available to disclose long periods.

5. Each 68-month curve is the mean of four mutually supporting constituents. As an example, comparing the constituents of curves II and III, each of the four individual constituents in group II shows positive departures at the two ends and negative departures at the middle. Each of the four individual constituents in group III, on the contrary, shows negative departures at the two ends and positive departures in the middle. This behavior of reversal in phase, exactly at 46 years after January 1819, is precisely similar to that which we have many times referred to, relating to the short periodicities, whose validity seems unquestionable because of the great numbers of repetitions on which they depend. Thus the behavior of the 68-month curves is exactly in line with reasonable expectation.

6. Corroboratively, the curves I, III, and V, covering (with two intermissions of 23 years each) 110 years, are so nearly similar in phase as to yield the mean form VI, figure 18. It has a range of 0.°6 Centigrade.

But why, the reader may ask, have so many periodicities additional to those heretofore recognized in the variability of the sun been added in the list of terrestrial periodicities, and why are they chosen as integral submultiples of 23 years? The answer is that they are forced upon our attention by the progress of the computations. One illustration has been given. As stated above, the periodicity of 9¾ months was discovered because the curves for 68 months showed seven waves. Similarly the periodicity of 34 months was discovered because preliminary computations of the periodicity of 68 months (not here reproduced) showed the half-period curves of 34 months too plainly to

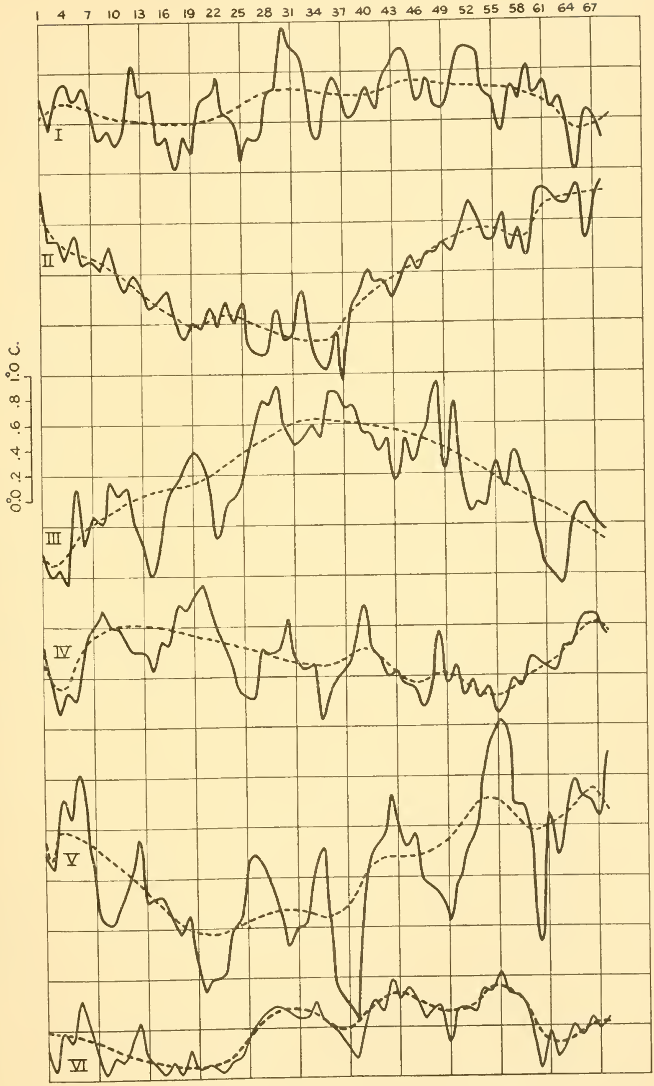


Fig. 18.—The 68-month periodicity in Berlin temperatures.

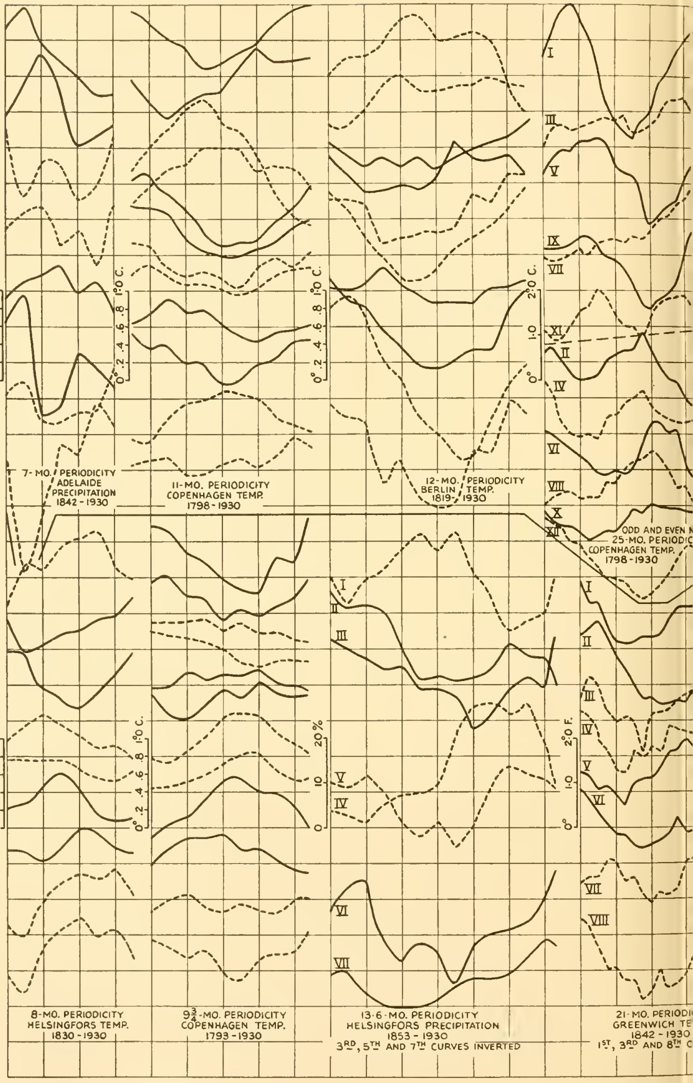
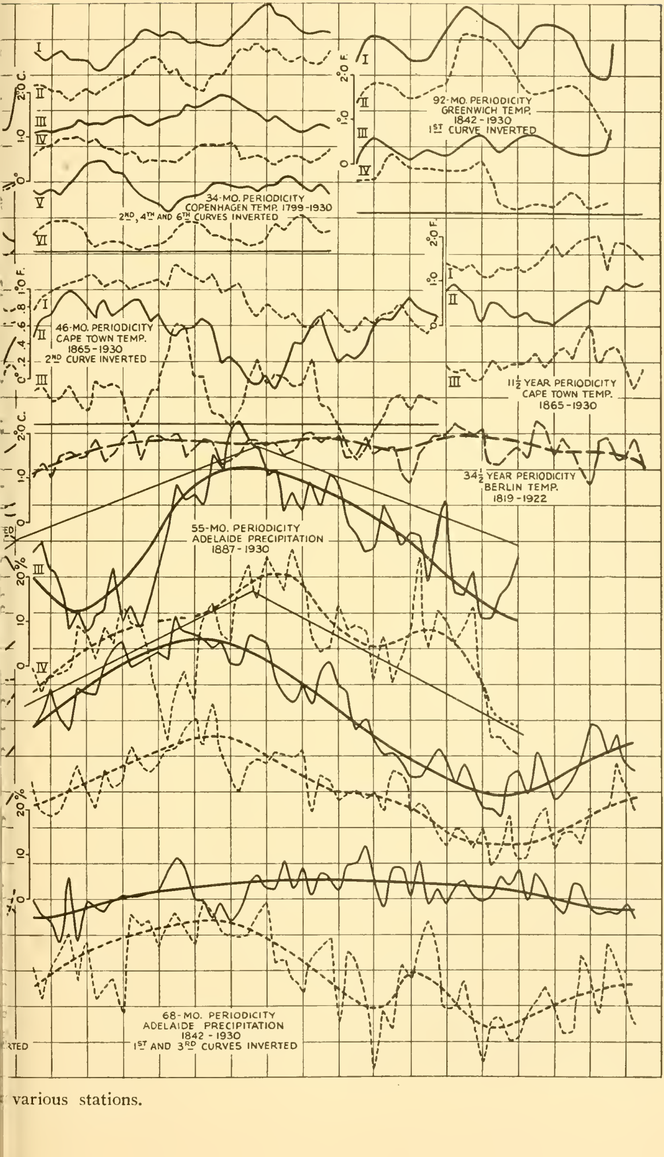


FIG. 19.—Periodic cur



various stations.

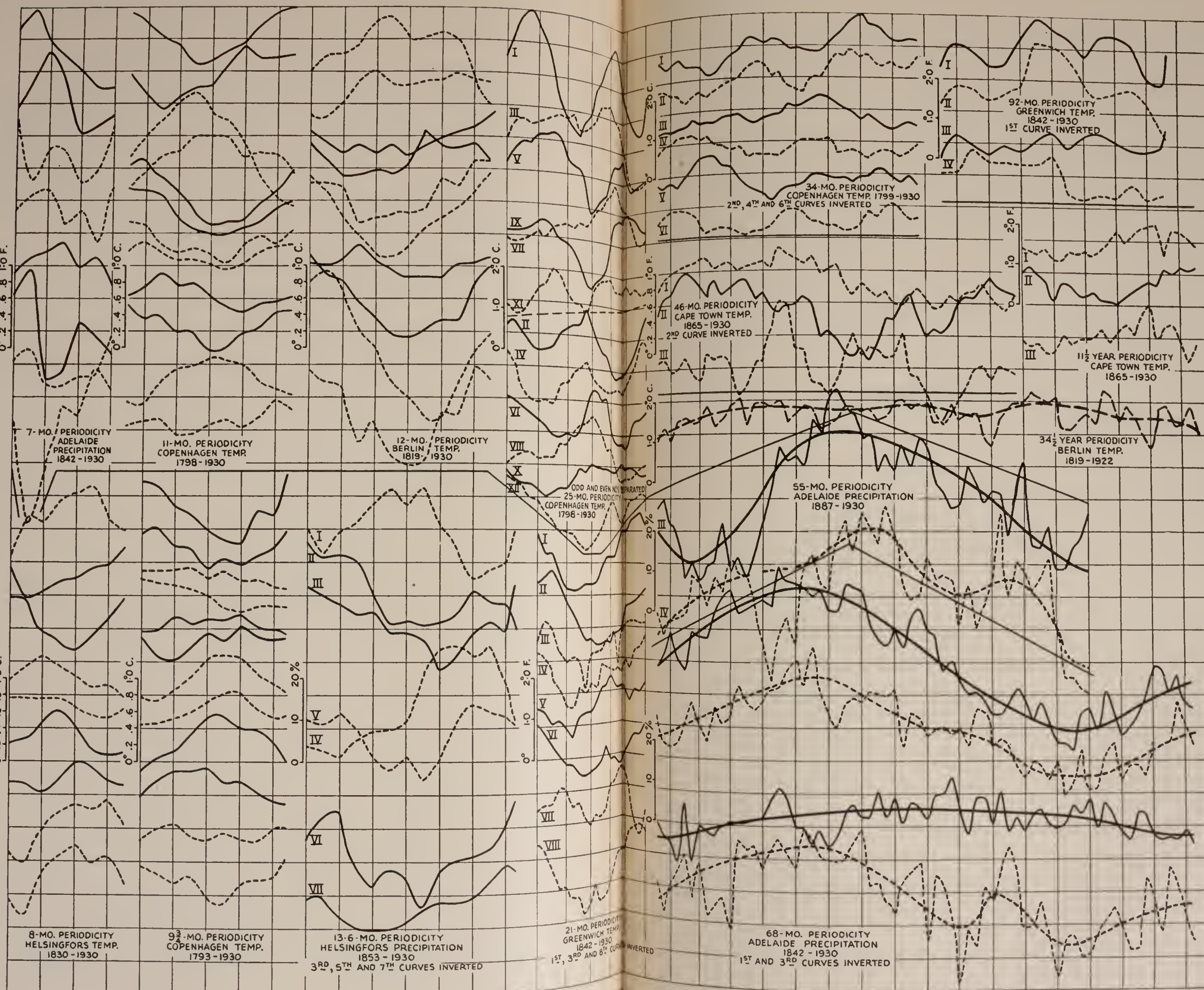


FIG. 19.—Periodic curves at various stations.

be ignored.[11] Similarly the periodicity of 13.6 months was disclosed by five waves in the preliminary curves computed for 68 months.

But it is freely admitted that if only one station had been investigated, some of the periodicities, especially those of 46 and 55 months might have been regarded as doubtful. Much support for the veridity of all of the longer periodicities is found by comparing results at several stations. In order to help the reader to appreciate the value of this support, I give in figure 19 some of the more convincing examples of each of the 14 different periodicities which are accepted as caused in terrestrial temperatures and precipitations by the solar influences integrally related to 23 years. To save space in depicting so many curves, certain special arrangements are made in figure 19, as follows: In the periodicities of 21, 34, 46, and 68 months, certain curves are inverted, as described in the legend. In the 25-month periodicity, odd-numbered curves are given separately from even-numbered curves. All of these arrangements emphasize the phase reversals already noted. As amplitudes are large in the longer periodicities, the scales of ordinates are diminished for them.

It is believed that if the reader bears in mind the abundant evidence already presented, which shows that periodicities of 8, $9\frac{3}{4}$, 11, 12, and 21 months change phases and forms radically at the expiration of integral multiples of $11\frac{1}{2}$ years after January 1819, he will be prepared to accept as veridical all of the periodicities shown in figure 19.

Accepting this evidence as proving in general the veridity of all of these periodicities because they are all so well marked at *some* stations, and almost without exception in solar radiation, as shown in figure 7, it seemed but a matter of course to compute them for each and *all* stations, and for departures of both temperature and precipitation from normal. All such computations gave more or less favorable curves. Some curves covering short time intervals, had they stood alone, might not have been regarded indeed as expressing a veridical periodicity. But reinforced by the better curves representing that same periodicity for the same station at other intervals within the century, and by such evidence as is given in figure 19, even these less satisfactory curves were acceptable.

If I am so fortunate as to have carried the conviction of the reader thus far, he will perhaps still ask, why I have stopped with 14 of the 23 periodicities which are integral submultiples of 23 years, and

[11] These two periodicities, $9\frac{3}{4}$ and 34 months, and also the 92-month periodicity, were later discovered in solar radiation. (See fig. 7.)

whether there are not other periodicities not integrally related to 23 years. The answers to these questions will be found in sections 14-C and 14-D.

C. RESIDUALS AFTER REMOVAL OF EVALUATED PERIODICITIES

Having evaluated and removed, in Berlin temperature departures, after the manner discussed in caption 14-B, periodicities of 7, 8, $9\frac{3}{4}$, 11, 12, 13.6, 21, 25, 34, 46, 55, and 68 months, mean values for each 6 months were computed from the residuals. From these 6-month mean values, periodicities of 92 and 138 months were sought. These computations were segregated into groups covering 23-year intervals. In both instances, groups I, III, and V showed considerable and nearly similar ranges of the periodicity, while groups II and IV showed slight ranges in opposite phase. These results are indicated in figure 20. The respective ranges are as follows:

Ranges of Mean Values, Berlin

	92-month period	138-month period
In mean of I, III, and V..................	1°2 C.	0°7 C.
In mean of II and IV.....................	0.4 C.	0.5 C.

After removing all of the periodicities, including the two last mentioned, the residuals remaining were compared with the original 5-month smoothed temperature departures of Berlin as shown in figure 21.

It is apparent that the range of the residuals shown in curve B of the figure is very much less than the range of the original data shown in curve A. The average amplitudes are in fact 0.°60 and 0.°90 C. *Careful scrutiny has not suggested to us any other periodicities existing in the residuals* except perhaps the Bruckner period of $34\frac{1}{2}$ years. This seems to show an average amplitude of 0.°6 Centigrade in the residuals. For reasons explained at much length above, but by no means exhaustive of all the evidence in our hands, I believe that all of the many periodicities named above have real veridity, and that the processes described in their evaluation and removal are defensible. Further evidence, however, will follow.

Nevertheless, I am sure that statisticians, if they take a snap judgment, will make the obvious remark that complex curves may be represented with much accuracy by a Fourier analysis of 14 terms, though these terms have no physical significance whatever. For an example, Dr. D. C. Miller has represented almost perfectly the profile of a girl's face by Fourier analysis in 30 terms. But I think great diffi-

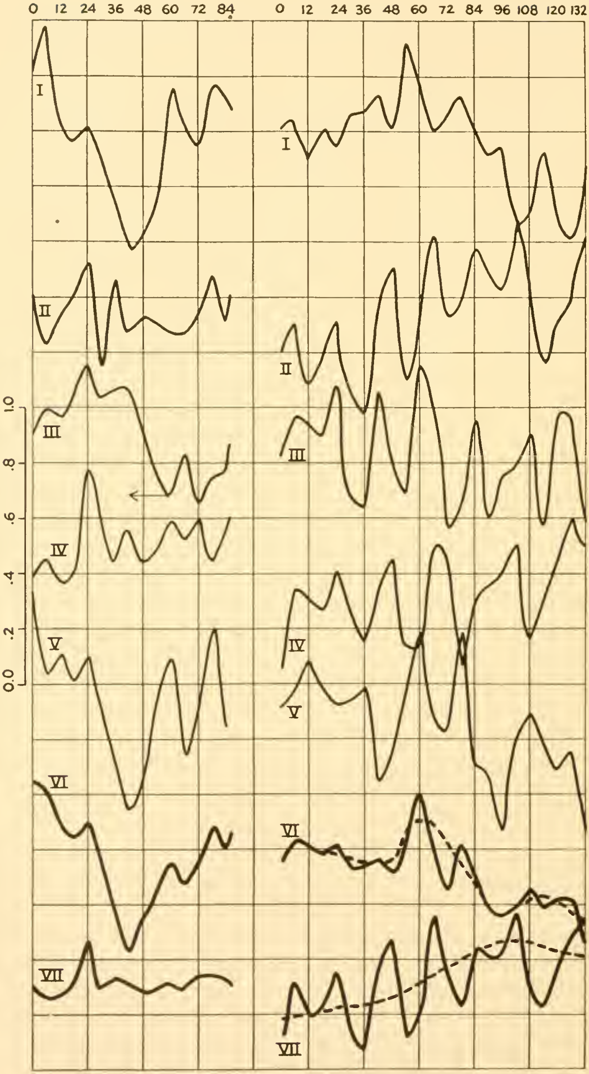


FIG. 20.—Periodicities of 92 and 138 months, Berlin temperatures.

culty would be found in making a satisfactory Fourier analysis in 14 terms of the temperature departures of Berlin from 1819 to 1929 or in discovering by that method the remarkable reversals of phase which occur at intervals which are integral multiples of $11\frac{1}{2}$ years after 1819. Furthermore, I hope statisticians will be fair enough to weigh carefully the arguments I have presented, and having done so will suspend adverse judgment until they have examined what is yet to follow.

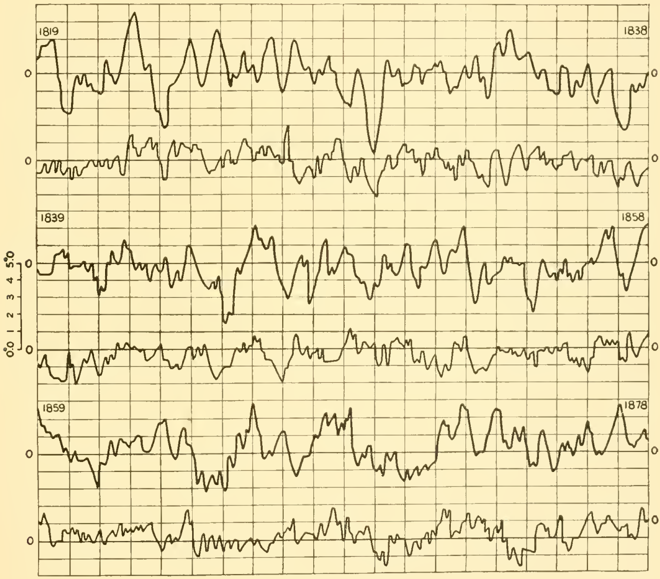


Fig. 21.—Residuals after removing periodicities, Berlin temperatures. Heavy curves, original data, light curves, residuals.

D. ANALYSES BY INTERVALS NOT INTEGRALLY RELATED TO 23 YEARS

The periodicities employed in the preceding discussion were selected partly because they had been found in solar variation, partly because they seemed to appear in Berlin and other temperatures and precipitations. But it will be objected by some who ignore the fact that we repeatedly scanned the curves, and sought all periodicities existing therein, that there was no reason for selecting integral submultiples of 23 years as the assumed periodicities whose validity was to be tested, or any particular time as better than another for departure, or any preferable time interval for delimiting the tables. Such hasty critics may suppose that any other periods or lengths of tables would probably have been equally successful.

To test this objection, computations were carried through to test for the existence of periodicities of $7\frac{3}{4}$, 10, 12, $12\frac{3}{4}$, $15\frac{3}{4}$, 19, and 29

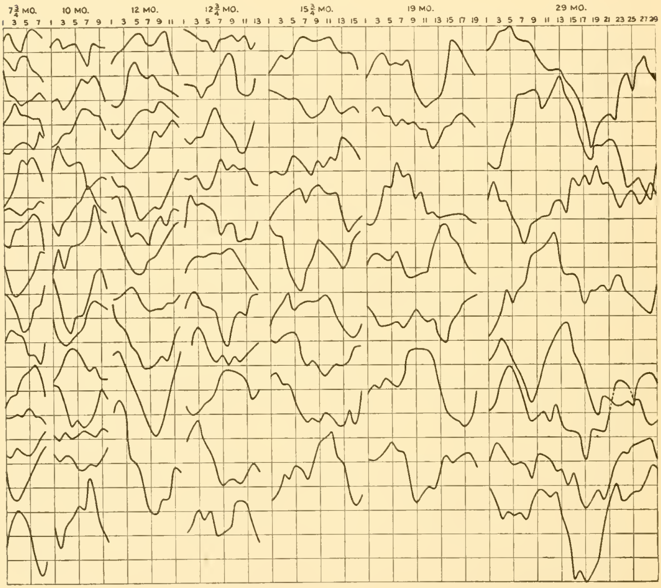


FIG. 22.—Trials of periodicities not related to 23 years. Compare with figures 19 and 23.

months in the temperature of Berlin. These tabulations, like the others, commenced with January 1819, but were arranged in tables of 10 lines. Thus they covered intervals of time having no particular relation to the 23 years which previous computations proved to be so important. The results are shown in figure 22.

With regard to the 12-month periodicity, this analysis differs but little from that shown in figure 15. The first and second 12-month curves in figure 22 cover about the same intervals of time as in figure 15. Also other pairs in figure 22, as the sixth and seventh, the eighth

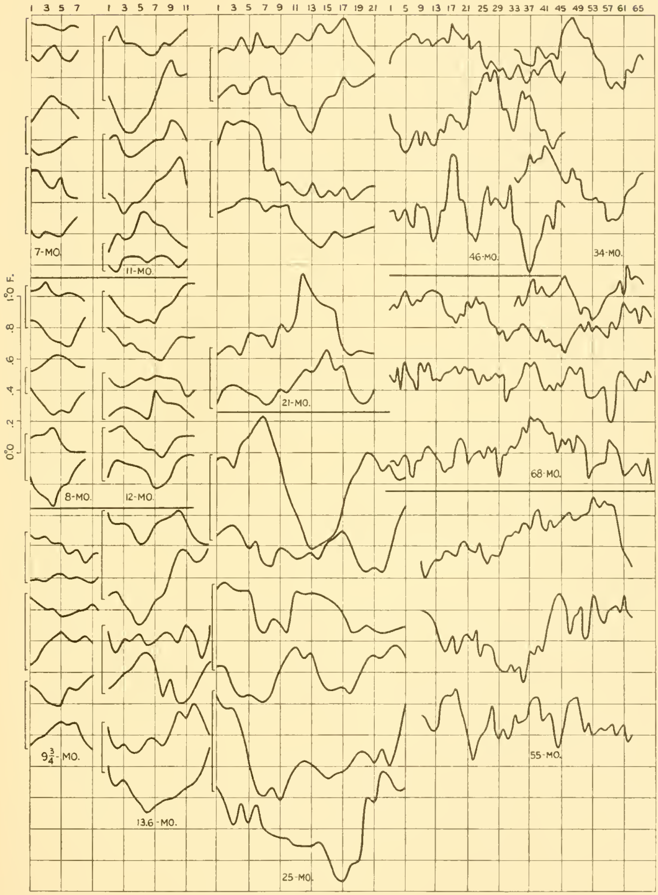


FIG. 23.—Cape Town periodicities in temperature departures. Bracketed pairs of curves each cover 23 years. For periodicities of 34 months or over only one curve is computed for each 23 years.

and ninth, and the tenth with the eleventh correspond, respectively, closely to V-VI, VII-VIII, and IX-X of figure 15 in time intervals. Hence, results were to be expected in these cases nearly parallel to those previously obtained.

But as to the other six sets of curves in figure 22, there is hardly a vestige of indication supporting the periods chosen, excepting for the last three curves among the 29-month group. The similarity of these three curves is indeed curious. In the 10-month group there is the nearest semblance to continued periodicity. Here it may be that a case could be argued for a periodicity of 10 months plus 3 days. But this would be 1/27 of 273 months, and would but add one more to the group of nearly integral submultiples of 23 years, already discussed.

15. Other Analyses

Besides the temperature of Berlin, both temperature and generally precipitation also have been analyzed with equal thoroughness at Helsingfors, Copenhagen, Greenwich, Cape Town, and Adelaide. The results were very similar to those already discussed for Berlin. The dominating importance of the 23-year period displays itself quite as conspicuously in these other analyses as in the case of Berlin temperatures. That is to say, all the periodicities which seemed to be indicated were nearly integral submultiples of 23 years. Also if the date January 1, 1819, was selected as a point of departure, changes of phase and amplitude occurred abruptly at multiples of $11\frac{1}{2}$ or of 23 years thereafter.

As it is felt that the united evidence from these widely separated stations is of great importance, excerpts from the results from various stations are given in graphic form in figure 19. In addition, the complete analysis of the temperature of Cape Town is shown in figure 23.

16. Conclusions Derived from Analyses of Berlin and Other Temperatures and Precipitations

a. It is shown that 14 apparent periodicities may be found in the smoothed temperature departures of Berlin and other stations since 1819.

b. Summing these periodicities and subtracting their sum from the original smoothed departures, the residual departures at Berlin have an average range of two-thirds of the originals. Similar results occur in the other analyses.

c. Thirteen of the supposed periods are primarily attributed to solar changes, and are approximately aliquot parts of 23 years, being, respectively, some interval between 272 and 276 months divided by the following numbers:

39, 34, 28, 25, 20, 13, 11, 8, 6, 5, 4, 3, and 2.

d. The fourteenth period is the terrestrial period, 12 months, which would certainly exist because no single expression of the march of the monthly mean temperature or precipitation fits satisfactorily over an interval of a century or more.

e. The amplitudes of the 14 periodicities vary with respect to each other and also from time to time.

f. The phases of the 14 periodicities vary from time to time.

g. In a majority of cases the periodicities retain approximately the same phases, and to a less degree approximately the same amplitudes, through either 23 or 46 years, and then abruptly alter.

h. In a minority of cases abrupt changes in phase and amplitude occur after a lapse of 11½ years.

i. The 12-month periodicity is no exception to the general rules laid down under g and h.

j. Almost without exception, when phases remain unchanged through 23 years, such a 23-year interval begins an integral number of times 23 years after January 1819.

k. The amplitudes of the periodicities disclosed in the temperature at Berlin range from 0.°2 to 1.°5 Centigrade. As stated in another form under b, these 14 periodicities combined account for about one-third of the whole range of 5-month smoothed departures from the normal in the temperature of Berlin. The amplitudes of temperature departure periodicities at other stations are of comparable magnitudes. In precipitation the amplitudes range from 20 to 300 percent. Here also the synthesis of the 14 periodicities found accounts for a substantial part of the entire departures from normal in the 5-month smoothed values. These are by no means as striking results as were found in respect to the periodic features in the solar variation reported in caption 6. But it must be remembered that the terrestrial effects are subject to various disturbing intermediate influences, besides the original solar causes.

l. Attempts to substitute some other set of periodicities, not related to the 23-year interval, are conspicuously less successful either to display continued periodic fluctuations or to bring to light any conspicuous regularities of behavior such as those stated under g and h above.

m. Other stations as widely separated from Berlin as Cape Town and Adelaide show similar results in temperature and rainfall with regard to numerous periodicities approximately integrally related to 23 years, and governed in phase and amplitude by the lapse of integral multiples of $11\frac{1}{2}$ or 23 years from January 1819. It is therefore hard to attribute these similarities of behavior to causes not extra-terrestrial.

17. Summary of Preceding Studies and Their Guidance Toward Those Which Follow

It has been shown that the sun is variable. Its variations comprise numerous periodicities. These periodicities are so definite as to justify synthetic forecasts of solar variation. Apparently, all the periodicities in solar variation are integral subdivisions of 23 years.

With this background it seemed reasonable to attempt to trace the effects of solar periodicities in weather. Analyses have been presented of temperatures and precipitation at several stations widely separated. The 23-year period is thereby found to exercise a dominating influence in weather. Numerous periodicities which are integral submultiples of 23 years seem to exist in weather. Nevertheless, changes of phase and amplitude complicate these relations. But it has been shown that these changes of phase and amplitude are apt to occur abruptly at times which are integral multiples of $11\frac{1}{2}$ or of 23 years after January 1, 1819.

These studies lead us to expect that many of the features in weather which occur apparently unordered are really produced by the summation of periodic changes integrally related to 23 years. Hence they will be apt to be found, though doubtless with considerable modification, in successive 23-year cycles. There is ground to expect that the similarity of such features will be greater after 46 than after 23 years. As these periodic changes seem to be of solar origin they should be observable throughout the world.

We may also expect that phenomena which depend intimately on the sunshine or the weather, such as the growth of vegetation, the numbers of creatures that feed on vegetation, the flow of rivers, the level of lakes, the thickness of varves, whether produced by the flow of glacial rivers or by the summer dessication of lakes, all such phenomena may display the influence of the 23-year cycle. In the remainder of this paper it will be shown in how far it has been found that these expectations are realized.

18. A Test of the 23-Year Hypothesis in the Precipitation of Southern New England

In 1934, C. M. Saville[12] published a table of annual precipitation over southern New England given as percentages of base values from 1750 to 1932. The values depend on reports from 1 to 10 stations.

[12] Quart. Journ. Roy. Meteor. Soc., vol. 60, p. 324, 1934.

In the same journal[13] I have used Saville's data to indicate evidence for a periodicity of $22\frac{2}{3}$ years. I now incline to prefer 23 years, and have some reason to trace a periodicity of 46 years as well as one of 23 years. Accordingly, I have rearranged the data, omitting decimals of percentages, as given in table 10.

These data are also shown graphically in figure 24. The first cycle of 23 years is discordant. It is, indeed, almost the exact inversion of cycle II. The latter is shown inverted by the dotted line on cycle I. A similar, though less complete inversion occurs with cycle VII. Fortunately, the cycles are in almost exact step with the important date, January 1819. This adds interest to these inversions, which, as we have seen, are apt to occur at integralmultiples of $11\frac{1}{2}$ years measured from 1819. Noting the considerable similarity of cycles III, V, VII as forming one group, and cycles II, IV, VI, VIII as forming another, I have plotted in curve IX the mean of groups III, V, VII, omitting I. In curve X, I have plotted the mean of groups II, IV, VI, VIII. Although both curves IX and X agree in many particulars, and both show a marked maximum at about the thirteenth year, they also tend to show opposition in some minor features, of the type which I have hitherto called, to give it a name, "right- and left-handedness." This tendency is apparent even in the individual 23-year cycles, for they show alternately the " left " and " right " tendency, corresponding to a 46-year period superposed on one of 23 years. The range from the first to the thirteenth year in the mean of group II, IV, VI, VIII is +18 percent, and in group III, V, VII, +9 percent. Having completed cycle VII in the year 1933, and assuming that the average march shown by group III, V, VII will now take place, we may expect nearly 10 percent more annual precipitation in Southern New England about 1945-1946 than in 1934. Should group II, IV, VI, VIII prove the more representative, then the precipitation about 1945-1946 would be nearly 20 percent above that of 1934.

19. A Lake Level Test of the 23-Year Hypothesis

By courtesy of the United States War Department, Corps of Engineers, a set of charts of the levels of the Great Lakes was obtained. These charts were cut and pasted by the present author so as to present 23-year intervals superposed. These charts all began with the year 1860. R. E. Horton, hydraulic engineer, was good enough to send me additional data covering nearly completely the 23-year period 1835 to 1859. This furnished valuable additional evidence.

Figures 25 and 26 show these data on lake levels. Figure 25 gives original data for Lake Ontario. Figure 26 gives the march of yearly

[13] Quart. Journ. Roy. Meteor. Soc., vol. 61, pp. 90-92, 1935.

TABLE 10.—*Cycles in the Precipitation of Southern New England, 1750-1934. Departures in Percentage from Normal*

I	—1	25	—11	22	9	—10	—17	—3	26	15	—9	—23	—43	—5	14	—23	—12	0	—14	—25	—3	6	15
II	—24	—12	—3	0	—17	—13	1	—4	—14	—4	17	41	47	—3	—14	—25	—5	—7	4	—26	—17	—23	10
III	—29	—8	—4	—17	6	—7	—10	2	—4	—16	—9	6	16	5	—8	12	4	25	16	7	—4	2	—5
IV	—15	3	0	—16	26	—2	—14	4	24	—5	7	11	14	6	—7	—12	—15	—6	—17	—9	—8	0	4
V	—3	9	—13	—3	—21	7	—5	—12	24	2	2	20	10	6	1	13	—4	17	—4	2	9	22	—12
VI	4	4	14	16	18	6	12	7	7	1	5	8	4	23	0	—10	4	—2	—13	9	—2	6	13
VII	12	19	20	9	—11	8	—11	—2	—4	17	25	—4	7	18	7	4	—1	—10	1	2	—11	—9	—17
VIII	—7	—7	—6	—17	6	—9	—12	—16	2	12	—12	3	—5	—17	—7	—3	10	—5	—8	—23	—5	1	(5)
General mean	—7.9	—4.1	—0.4	—0.7	20	—2.5	—7.0	—3.0	7.6	2.7	3.3	7.8	6.3	4.1	—1.7	—6.3	—2.4	1.5	—4.4	—7.9	—5.2	0.6	1.6
Mean of III, V, VII	—7.3	7.3	1.0	—3.7	—8.7	2.7	—8.7	—4.0	5.3	1.0	6.0	7.3	11.0	9.7	0.0	9.9	—0.3	10.7	4.3	3.7	—2.0	4.3	—11.3
Mean of II, IV, VI, VIII	—12.5	—3.0	1.2	—4.2	8.2	—4.5	—3.2	—2.1	4.8	1.0	4.2	15.8	15.0	2.2	—7.0	—12.5	—1.5	—5.0	—8.5	—12.2	—8.0	—4.0	8.0

FIG. 24.—Cycles in the precipitation of southern New England. Dotted curve is cycle II inverted.

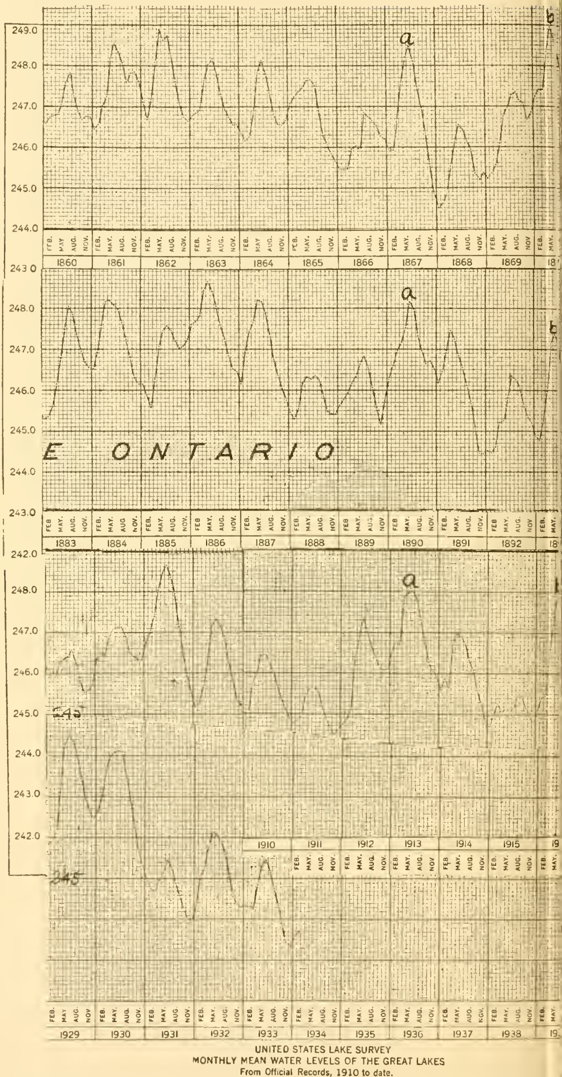
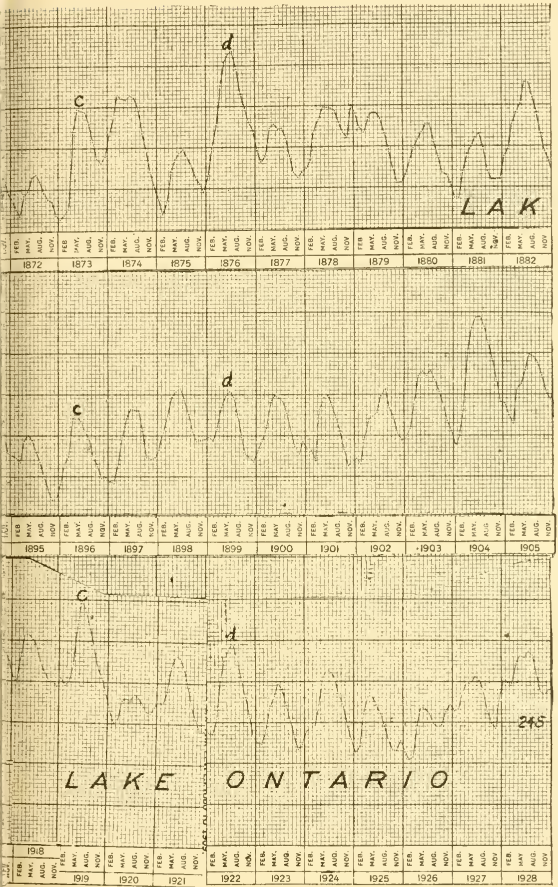


FIG. 25.—Levels of Lake Ontario, 23-year cycles. Note general s



e about the sixth year, also approximate repetition of features *a, b, c, d*.

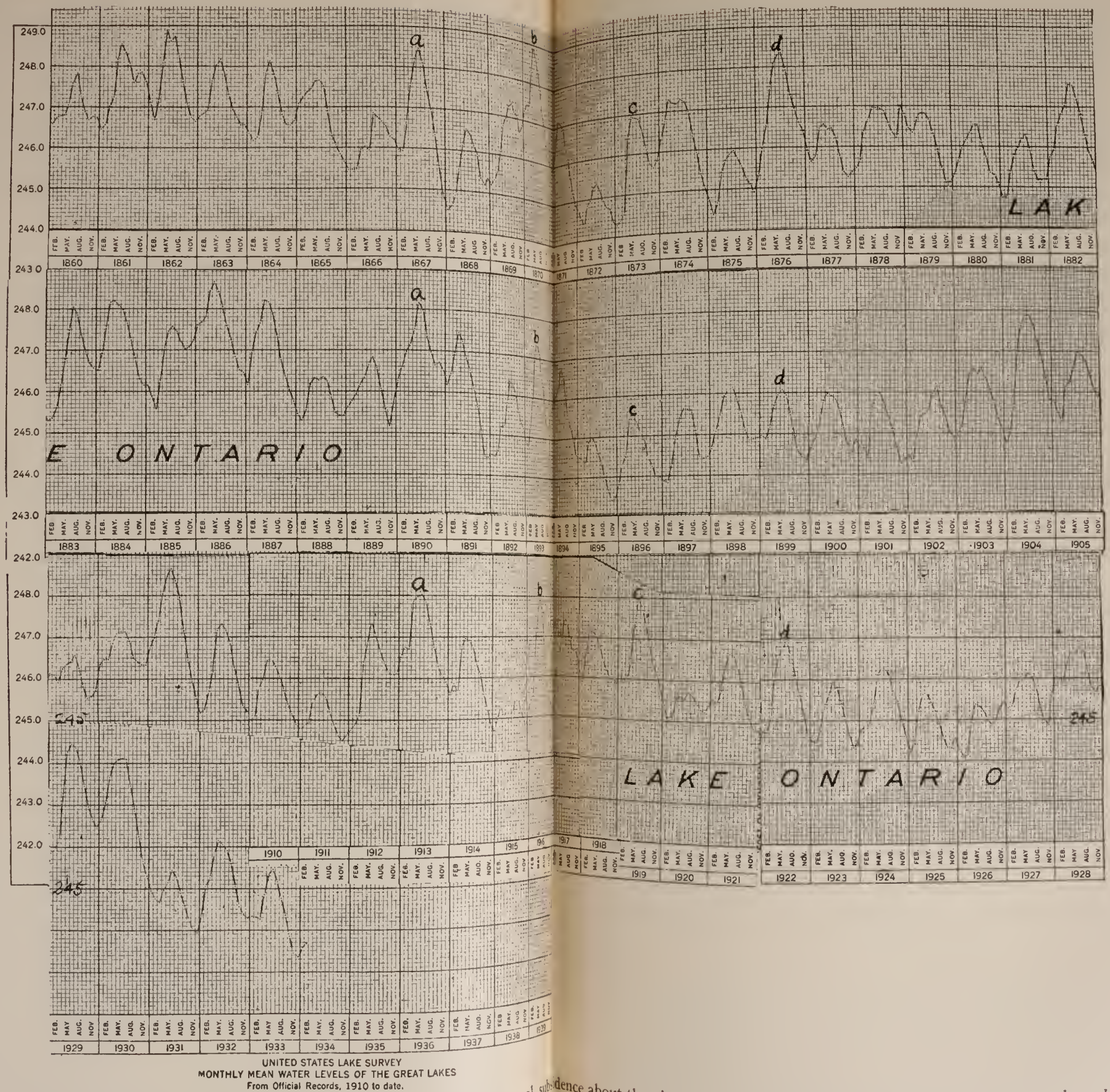


FIG. 25.—Levels of Lake Ontario, 23-year cycles. Note general subsidence about the sixth year, also approximate repetition of features *a*, *b*, *c*, *d*.

means computed from the original data for four lakes. It is unnecessary to include Lake Michigan, for its level practically duplicates that of Lake Huron. Some features, as the low levels of the intervals from

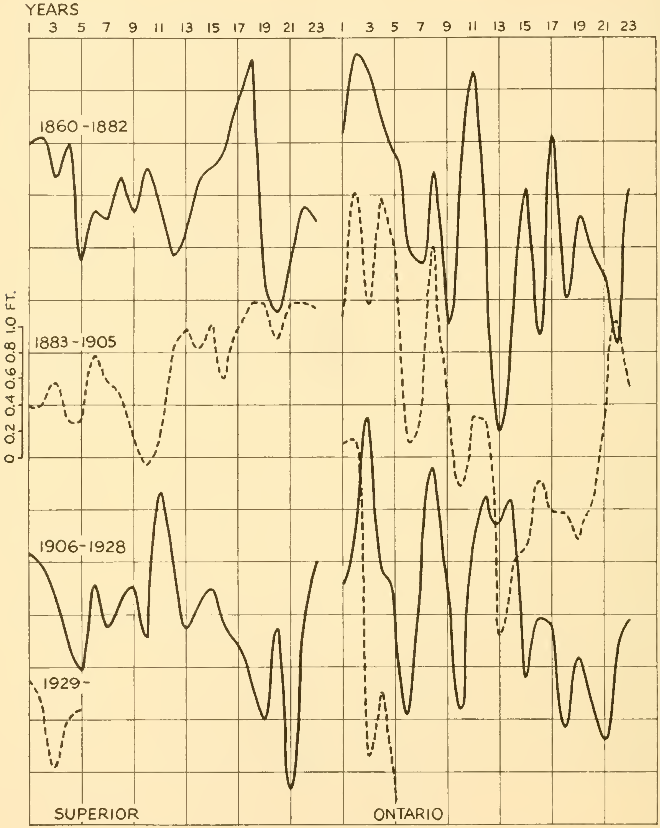


FIG. 26A.—Levels of Great Lakes, 23-year cycles.

about the fourth to the tenth year, are so conspicuous as to be striking. This shows distinctly in all of the Lakes, but least so in Lake Superior. It may be remembered in this connection that much of the drainage into Lake Superior comes from far to the north and west in Canada,

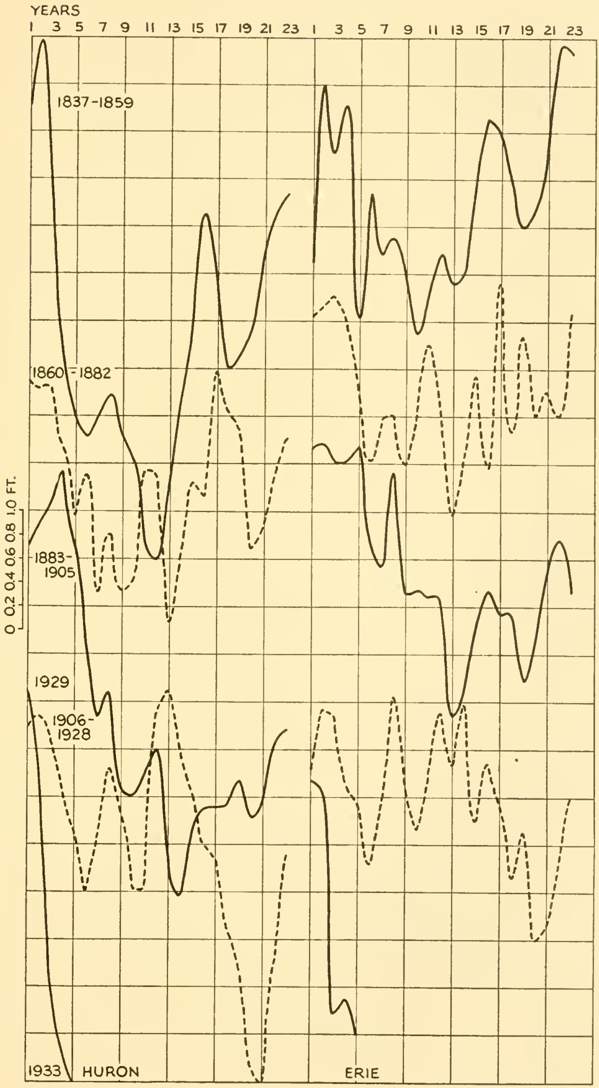


FIG. 26B.—Levels of Great Lakes, 23-year cycles. Note the marked subsidence culminating after 11 years in the full curves.

where, at least in the last few years, the severe drought which affected our Northwestern Central States was less severe, or even absent. In the levels of Lake Ontario several minor features by which the yearly ranges have been decidedly modified seem to be repeated each 23 years. These features have been marked in figure 25 with letters.

It is especially interesting, in view of caption 14-g, that the additional cycle for Lakes Huron and Erie furnished by Mr. Horton seems, when studied in connection with those commencing in 1883 and in 1929, to indicate a 46-year cycle. The first and third, and so much as has elapsed hitherto of the fifth 23-year cycle in the levels of these two lakes since 1837 indicate a much more conspicuous and long-continued low after about the fourth year than do the second and fourth cycles.

It is not necessary to dwell upon the association which these figures seem to bear to the drought in Northwest Central United States in recent years. The inference, if the 46-year hypothesis is sound, is obvious, and disquieting for the immediate future.

20. A Fishery Test of the 23-Year Hypothesis

Dr. Paul Bartsch, of the United States National Museum, suggested to me that since ocean fishes live upon plankton, largely a vegetable product, then if the weather is governed by 23-year cycles, the fish food would probably be subject to related changes in its abundance. Hence the fish population, as reflected by the annual catch, might vary by 23-year cycles. On my application through the Bureau of Fisheries, Dr. O. E. Sette was good enough to supply Fishery Circular 14, issued in 1933, and Bureau of Fisheries Document No. 1034, issued in 1928, which give, respectively, the catches of mackerel and cod taken since 1804. The catch of mackerel I read off from figure 1 of the first cited document. The catch of cod is taken from table 2 of the other.

Very great changes of scale in the mackerel catch occurred after 1816 and after 1885. In order to make the data fairly comparable, I omitted values of the mackerel catch 1804 to 1816, inclusive, and I multiplied the values recorded from 1886 to 1931, inclusive, by the factor 3. Five 23-year cycles remained for examination. No distortion of the 23-year cycles is produced by the alteration of scale at the date just noted, because it occurs at the beginning of a cycle.

As for the cod, the catch reported was considerably smaller during the first half of the nineteenth century than since. In order to make my data more comparable, I omitted the years 1804 to 1811, inclusive, and multiplied the values from 1812 to 1857, inclusive, by the factor 5/3 (again making the change of scale at the beginning of a cycle).

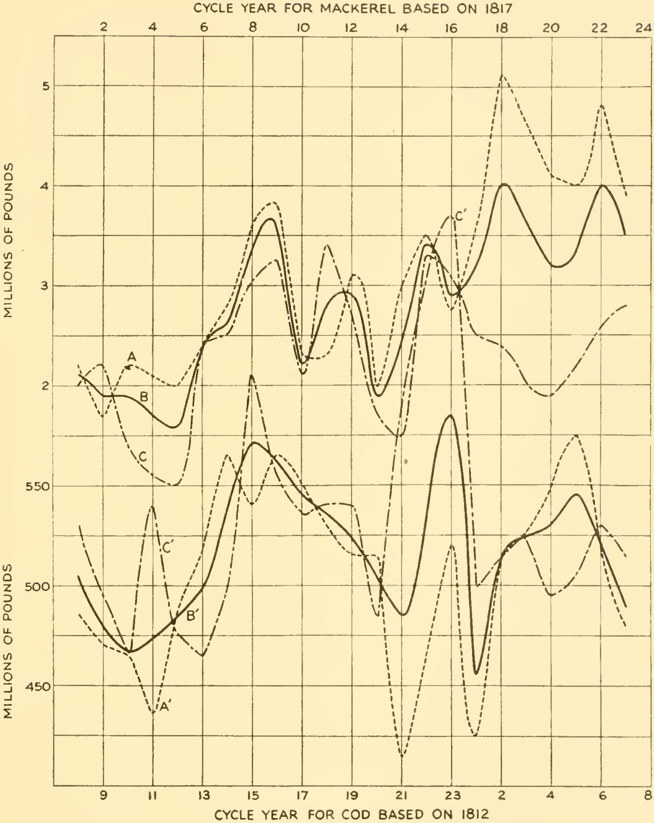


FIG. 27.—Catch of mackerel and cod in the North Atlantic, 23-year cycles from 1812 to 1931. Curve for cod shifted in phase 2 years. Dotted curves and dashed curves are means of cycles I, III, V and II, IV respectively. Full curves are general means of all five cycles.

Five 23-year cycles remained for examination, but they were based on the year 1812 for the cod instead of 1817, which latter was the basal year for the mackerel data.

The results are given in tables 11 and 12 and in figure 27. As a 46-year cycle had frequently been encountered in weather data, I took a mean of the first, third, and fifth 23-year cycles separately from the mean of the second and fourth for both mackerel and cod fisheries. As there is little definite support for a 46-year cycle in these curves, I also took the general mean in each case. Thus three curves for each fishery are given in figure 27.

The general mean range during the 23-year cycle for the mackerel fishery is astonishingly large, from 16 to 40 millions of pounds. For the cod fishery it is from 460 to 570 millions of pounds. The constituent cycles, as indicated by the curves of partial means, support the general mean very well. Also when a difference of phase of 2 years and a difference of percentage amplitude of variation are both allowed for, as shown in figure 27, the two general mean curves are surprisingly similar. As noted above, it will be observed that neither the mackerel nor the cod curves show sufficient dissimilarity as between the partial mean curves to prove definitely that a 46-year period is superposed upon the 23-year period. Yet there are some indications of it, as seen in the tendency to opposition at certain years of the cycle, contrasted with the general fair agreement between the partial means.

TABLE 11.—*23-Year Cycles in North Atlantic Mackerel Fisheries, 1817-1931. Values Given in Millions of Pounds*

Cycle					Mean of cycles	Mean of cycles	General mean
1	2	3	4[a]	5[a]	1, 3, 5	2, 4	all cycles
6	7	41	33	18	22	20	21
7	8	37	36	6	17	22	19
14	10	42	18	9	22	14	19
16	9	38	12	9	21	11	17
15	11	33	9	12	20	10	16
24	28	29	21	24	24	24	24
20	25	36	24	24	27	25	26
26	34	51	27	30	36	30	34
35	41	39	24	39	38	32	36
22	29	28	12	18	23	21	22
26	32	29	36	15	23	34	28
32	45	42	9	18	31	27	29
31	25	20	9	9	20	17	19
43	18	35	12	12	30	15	24
53	18	18	48	33	35	33	34
30	29	27	33	27	28	31	29
30	29	30	21	48	36	25	32
35	24	47	24	72	51	24	40
26	19	53	21	60	46	20	36
22	14	52	24	48	41	19	32
18	32	31	12	72	40	22	33
14	26	65	27	66	48	26	40
10	33	38	24	69	39	28	35

[a] These two columns are three times their originals, as stated in the text.

TABLE 12.—*23-Year Cycles in North Atlantic Cod Fisheries, 1812-1927. Values Given in Millions of Pounds*

Cycle					Mean of cycles	Mean of cycles	General mean
1[a]	2[a]	3	4	5	1, 3, 5	2, 4	all cycles
414	434	385	568	469	423	501	454
516	514	448	520	565	510	517	513
548	479	488	568	546	527	523	526
626	444	455	544	576	552	494	529
604	524	466	486	652	574	505	546
593	554	376	507	575	515	530	521
584	608	382	419	468	478	514	492
554	608	365	452	538	486	530	503
526	569	340	419	546	471	494	480
524	523	381	408	492	466	466	466
516	608	342	477	441	433	542	477
508	541	415	412	552	492	476	486
513	519	438	416	602	518	467	498
569	566	438	432	687	565	499	538
564	710	421	502	641	542	606	568
529	665	489	444	677	565	554	561
531	625	583	448	536	550	536	545
544	601	432	475	613	530	538	533
559	573	408	504	578	515	538	524
453	491	397	479	508	516	485	506
376	680	398	498	472	415	589	485
414	768	517	546	496	476	657	548
484	842	516	524	564	521	683	586

[a] These two columns are $1\frac{2}{3}$ times their originals, as stated in the text.

21. A Test of the 23-Year Hypothesis in the Flow of the River Nile

C. F. Talman, librarian of the United States Weather Bureau, was good enough to draw to my attention to, and lend me, Prince Omar Toussoun's " Memoire sur l'histoire du Nil," Cairo, 1925. Volume 2 of this publication gives an extended table of annual high- and low-water stages of the Nile beginning with A. D. 622. A short comparison indicated to me that the low-water stage was preferable for my purpose. It showed much smaller apparently accidental fluctuations than the high-water stage. The earliest low-water records seemed probably less accurate, for they too showed wide irregular fluctuations. After about 1430 until 1839, the low-water stage records were unfortunately fragmentary, so that these four centuries had to be omitted. After 1884 the work of British engineers so greatly modified the natural flow of the river that the records cease to be useful for my purpose. Because of these several considerations, I limited my research to a study of low stages of the Nile for 690 years from 735 to 1424, and 46 years from 1839 to 1884.

Figure 28 gives a number of individual 23-year cycles in the level of the Nile at low water. It will be seen that the early cycles of the eighth century differ from the two cycles of the nineteenth century in phase, indicating either that there is a slight deviation from exactly

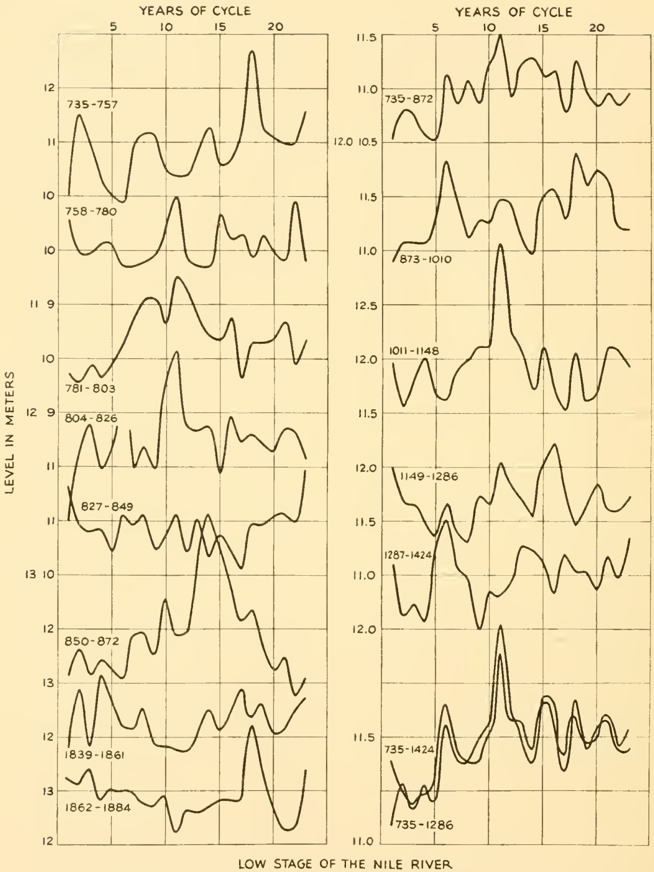


FIG. 28.—Low-level stages of the Nile River. Showing 23-year periodicity, A. D. 725-1424, and A. D. 1839-1885. See description in text.

23 years, or a mutation of phase due to unknown causes. But the individual 23-year cycles in both the eighth century and the nineteenth century show much similarity. Fortunately, they differ but little from being in step with the important date 1819. This adds interest to the apparent inversion of phase shown by cycles 1, 2; 4, 5; and 5, 6 of the early period. Figure 28 also includes five mean curves, each one the mean of 138 years or six successive 23-year cycles. Finally curves are given to represent the general mean forms of the 23-year cycle for 542 years and 690 years of observation, respectively. These latter means are taken separately, because the 138-year period ending in 1424 seems to show a change of phase tending to approach the form of cycles which prevailed from 1839 to 1884.

The general result seems to be that the Nile, before its regulation by British engineering works, showed plainly the influence of the 23-year cycle. During the 690 years preceding 1424, the average range of the low level during the 23-year cycle was about 1 meter. The extreme range of the original values during any of those centuries seldom exceeded 2½ meters, so that a very large part of it was due to the 23-year cycle. Maxima and minima repeated themselves so nearly in phase throughout the interval of 552 years from 735 to 1286 that the cycle can hardly differ by as much as 1 month from 23 years.

22. A Test of the 23-Year Cycle in the Widths of Tree Rings

In the appendices to his "Climatic Cycles and Tree Growth," Volumes 1 and 2, A. E. Douglass gives many tables of measurements of the widths of tree rings from many localities. In volume 1, pp. 117-123, we find two records of Sequoia trees, the first of 1 to 4 trees extending from 1306 B. C. to 251 B. C., the second of 11 trees extending from 274 B. C. to A. D. 1910.

I have arranged most of these data in tables of 23 columns and 5 lines, each table covering 115 consecutive years. Each group of trees just referred to gave the same general type of result, namely: At the first part of each Douglass table, where the rings are wide, there is a well-marked indication of a periodicity of 23 years, as determined from my tables of 115 years' duration. But the amplitudes of the curves diminish as time goes on. After two or three centuries, when the rings become much narrower, the 23-year periodicity practically disappears. The same thing is also observed with the long Flagstaff table, 1390-1911, found in volume 1 on page 113.

Figure 29, which contains but a few examples of my results, illustrates the preceding statements. It seems but a reasonable considera-

FIG. 29.—Cycles of 23 years in tree-ring widths. Average results of 115-year intervals. Numbers indicate percentage ranges of mean values representing 115 years. Note successive curves at Meadow Valley, Modern H, Pikes Peak, and Windsor.

tion that while a tree is young, with its roots shallow and but little extended, the water supply on which growth so largely depends would respond more directly to periodic changes in precipitation than when the tree becomes very old, with a widely extended root system, possibly tapping never failing sources of water supply at a considerable distance from its trunk.

With this view in mind, I have for the most part restricted this investigation of 23-year periodicity, and the illustrative curves to which I shall refer, so as merely to present periodic changes in the widths of tree rings from about 20 selected localities from which wide rings at the top of a Douglass table led down in a century or two to much narrower rings. In these cases it seemed most probable that his measurements had to do with young trees.

Figures 29 and 30 give the results of these investigations. It appears that in all of these cases, tabulations extending over 115 years indicated changes of tree-ring width during 23-year cycles ranging between 40 and 120 percent, and with such definiteness of gradation, from low to high and return, as seems in harmony with the idea of periodicities of 23 years in the water supply on which the tree growth depended.

In another investigation of this subject I have kept each 23-year cycle by itself, but have combined the results from five localities in southern California and Nevada. In that way I have determined individually the march of four successive 23-year cycles from 1829 to 1920 as represented by the average thickness of the rings of about 40 trees from five separate localities. Figure 30 shows these results. Not only is a 23-year cycle apparent, but many details are reproduced with such moderate alterations of phase and amplitude as to give reasonable certainty of the veridity of these minor features in all four cycles. As remarked above, the amplitudes of these features which compose the cycles tend to diminish as the trees grow older.

23. A Test of the 23-Year Cycle in Pleistocene Varves

In a paper by C. A. Reeds,[14] he gives many pages of illustrations representing the march of the thickness of glacial varves near the Connecticut and Hackensack Rivers. Independent measurements by Antevs and Reeds are shown. Continuous series represent the present thicknesses of these varves resulting, it is believed, from annual weather-reactions extending in unbroken sequence for nearly 1,000 years.

[14] Ann. Rep. Smithsonian Inst. 1930, pp. 295-326, 1931.

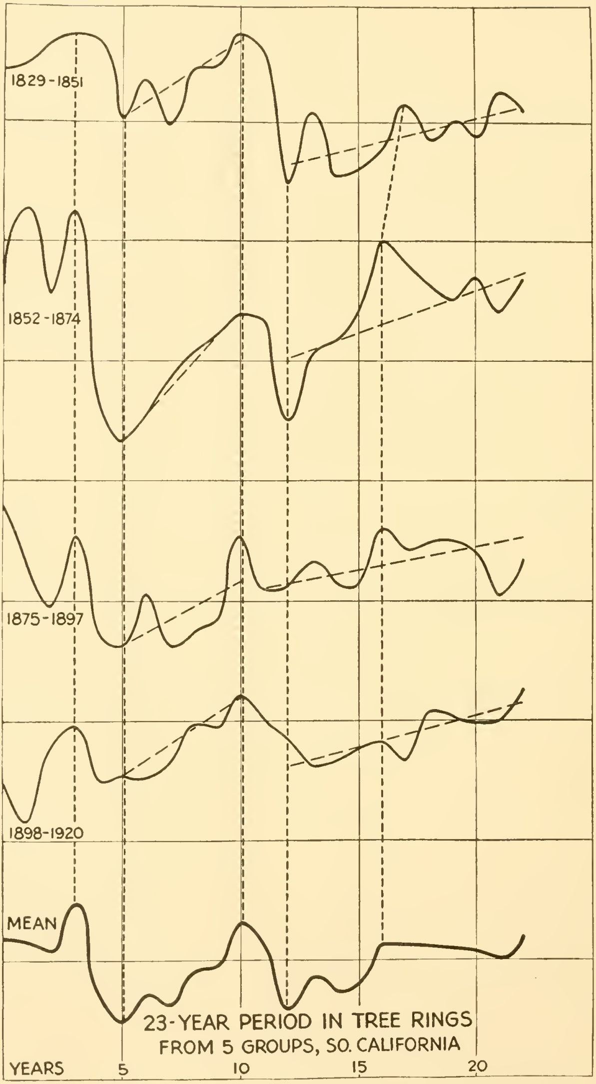


FIG. 30.—Cycles of 23 years in tree-ring widths. Individual cycles of 23 years show features which are found preserved in the mean of four cycles, or 92 years.

The varves are supposed to have been formed as follows: During Pleistocene glaciation considerable melting of the surface of the ice, as well as copious rainfall, took place during the summer of each year. This produced glacial torrents which scoured the sides of the glacial valleys and carried down sediment. Settling occurred in the quiet lakes which at the foot of the glacier intercepted the torrential flow. In such settling the coarser particles reached bottom first, and the finer particles were superposed thereon. The settling took place mainly in the colder months after the melting had greatly diminished and snow rather than rain fell, so that the turbulent streams nearly ceased. In this way each year a layer of sediment was deposited, coarser at bottom, finer above, and layer after layer formed as the years succeeded each other.

Many thousand years have since passed. Many variations of pressure, of hardness, of exposure, and of still other factors must be supposed to have affected the thickness of varves, besides the warmth and the rainfall, of which we are now to invoke them as the witnesses. Hence we can not hope to find the 23-year cycle very sharply defined in varve thicknesses. But it may be that by taking the mean values over intervals of 115 years, covering five cycles each, as was done with the tree-ring measurements, interesting results will appear.

With this anticipation I read off from Reeds' plots the thickness of varves for a continuous interval of 575 years, and arranged the values in five tables of 23 columns and 5 lines each. In figure 31, I give the results of that investigation. It seems to show that in Pleistocene time, as now, a 23-year cycle in temperature and rainfall resulted from the summations of the effects of periodic variations of the sun. Eight crests which appear in the general mean seem to be present almost without exception in very nearly the same phase in the five constituent curves. The range of values plotted in the general mean curve, F, is from 1.44 to 2.00, a range of 40 percent. The range in curve A is from 1.02 to 2.22, a range of 120 percent.

24. A Test of the 23-Year Cycle in Eocene Varves and Tree Rings

Dr. Wilmot H. Bradley, United States Geological Survey, was so good as to furnish me with several sets of measures of varves and tree rings relating to Eocene times. These data included a continuous series of varves from the Green River formation, Parachute Creek, Colo. They appear to have been formed by the annual expansion and drying up of a lake bed. These varves each presented

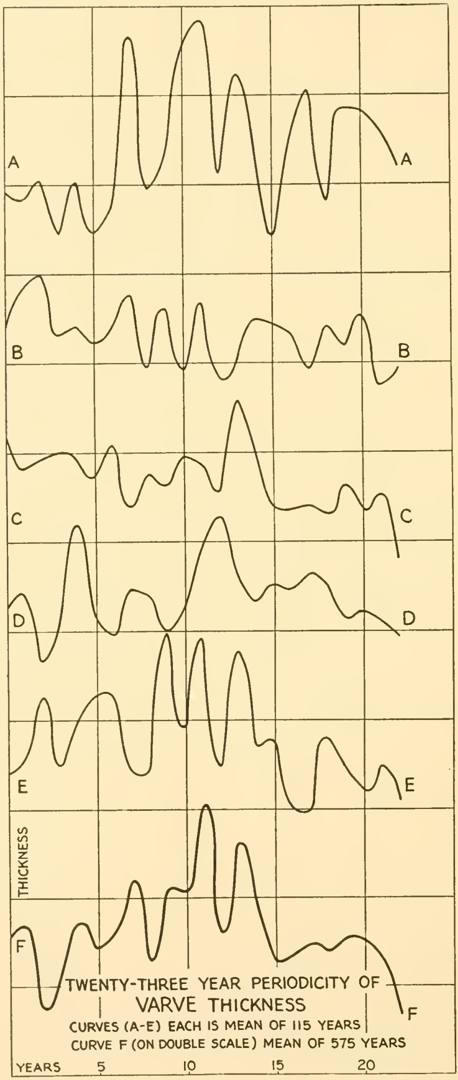


FIG. 31.—Cycles of 23 years in Pleistocene varves. Average results of 115-year intervals.

two fairly well differentiated layers, one rich in organic material, the other in mineral substance. The measurements give the thickness of each part, and I have also added them to give the total thickness.

Dr. Bradley remarks that "the organic-rich portion of each varve represents the material derived from the plankton produced in the lake each summer, and as the volume of the plankton varies directly with the amount of sunlight and the temperature (assuming an adequate food supply) it seems reasonable to expect a correlation with variations in solar energy. . . . The mineral-rich layers consist largely of carbonate, and therefore may also be expected to vary in thickness with the temperature of the lake water."

Figure 32 gives a 23-year analysis of these data. Five successive cycles of the march of the total thickness of the varves are given, and the general mean of them all, covering 115 years. In addition, I give the general mean for the 115 years of the thicknesses of the organic and inorganic parts separately. All three mean curves show a similar march, including certain details. All appear to show not only the 23-year cycle, but the approximately 11½ year cycle as well, though with alternately slightly longer and shorter intervals. The ranges of the mean curves are about 100 percent.

Dr. Bradley also furnished measurements of the widths of the annual rings extending from the center to the bark in a fossil coniferous tree of late Green River Eocene age. There were 107 successive rings measured. On arranging the data in 23-year cycles, they proved inharmonious to this arrangement. On rearranging them in five cycles of 21½ years, the result shown in figure 32 was found. In this arrangement the first two cycles are discordant, but the last three, covering over 60 years show a beautiful accord. May it not be that during some part of the Eocene, lasting millions of years, the unknown forces which govern the periodicities in solar variation acted more vigorously than in other parts of the Eocene, the Pleistocene, or the Recent?

25. A Weather Test of the 23-Year Hypothesis

As stated under caption 9 departures from normal monthly temperature and rainfall and 5-month traveling means therefrom have been computed from "World Weather Records." These relate to more than 100 stations in many parts of the world. The departures were smoothed by 5-month traveling means in order to eliminate such rapid and abrupt fluctuations as would obscure principal trends. Lack of funds prevents the publication of these valuable data.

It follows that should the working hypothesis outlined in caption 17 be a true one, then such a series of departures from normal tempera-

FIG. 32.—Cycles disclosed in varves and tree rings of Eocene age.

ture or from normal rainfall as just described must show numerous features during any 23-year cycle which would tend more or less strongly to be reproduced in each succeeding 23-year cycle. To test this probability, the data on departures from normals of temperature and precipitation for all available stations were plotted on sheets of specially prepared plotting paper. These sheets were ruled in abscissae to represent 276 months or 23 complete years, and in ordinates to present 300 millimeters, or 30 centimeters.

As an illustration, figures 33 and 34 present the percentage precipitation of Peoria, Ill., and the temperature departures of New York City. Features thought to be common in successive 23-year periods are indicated on the curves by letters. Principal trends are also to be observed. A dotted continuation of the last line of the plot covers the years 1934, 1935, and 1936. This continuation represents the mean expectation as based on former cycles. As the features in former cycles show considerable differences, such a mean can only roughly indicate their future forms. The method of drawing the mean which is the most probable expectancy may be clearly understood by observing the faint construction lines above and below the dotted continuation. Similar continuations for 1934, 1935, and 1936 were drawn before the events occurred, and may be regarded as forecasts for both precipitation and temperature for over 30 stations in the United States.

A year having elapsed, the actual departures of temperature and precipitation for all of these stations just mentioned were computed and smoothed by 5-month traveling means. These observed results for 1934 were plotted alongside of the predicted values for 1934. By inspection the agreement was then classified as " Excellent," " Fair," " Half and half," or " Bad." Under this classification the cities were grouped as follows:

A. Temperature.

Excellent, 7: Eastport, Key West, Detroit, Salt Lake, Helena, Portland, San Diego.

Fair, 17: Albany, New York, Washington, Hatteras, Mobile, Nashville, Cincinnati, Chicago, St. Paul, St. Louis, Omaha, Bismarck, Cheyenne, Denver, Santa Fe, Red Bluff, Spokane.

Half and half, 3: New Haven, Galveston, North Platte.

Bad, 4: Charleston, Little Rock, Abilene, San Francisco.

B. Precipitation.

Excellent, 11: Eastport, Burlington, New York, Detroit, Chicago, Duluth, St. Paul, St. Louis, Little Rock, North Platte, Bismarck.

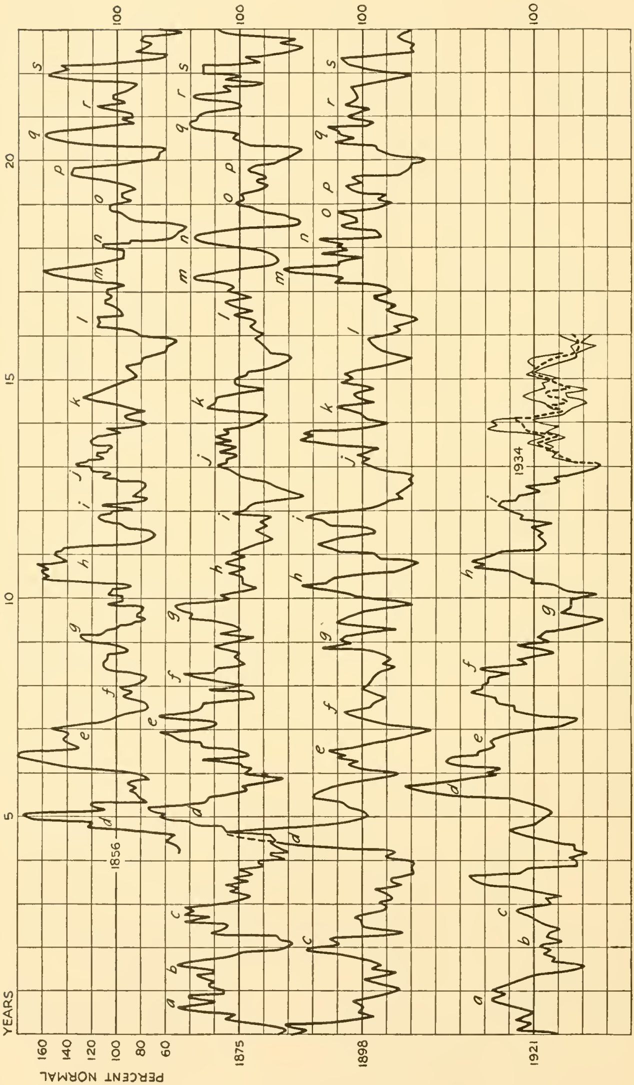


Fig. 33.—The 23-year cycle in the precipitation of Peoria, Ill. Years 1934, 1935, 1936 predicted from previous data, and expressed by the dotted curve. Corresponding features in the several curves are marked by corresponding letters.

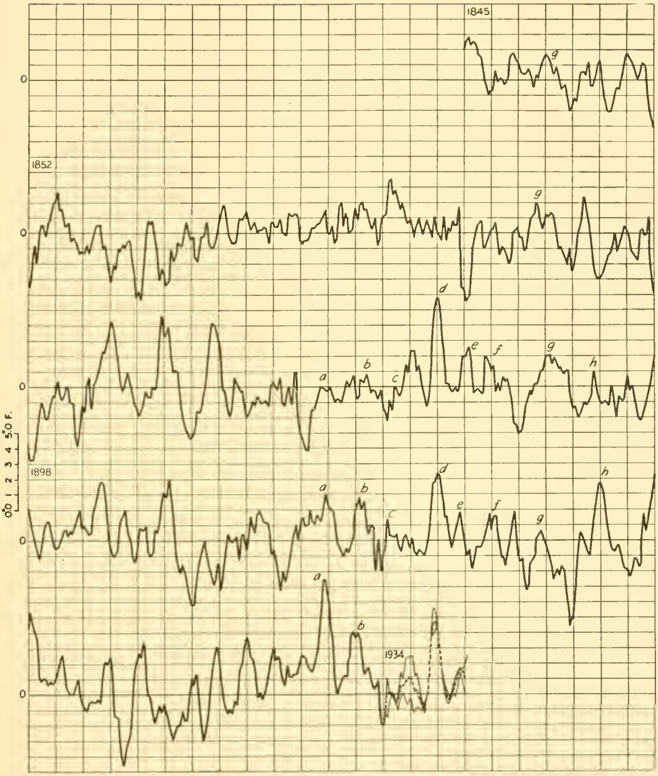


FIG. 34.—The 23-year cycle in the temperature departures of New York City. Years 1934, 1935, 1936 predicted from previous data and expressed by the dotted curve. Corresponding features in the several curves are marked by corresponding letters.

Fair, 11: New Haven, Albany, Philadelphia, Washington, Charleston, Peoria, Galveston, Santa Fe, Denver, San Francisco, Spokane.

Half and Half, 8: Key West, Cincinnati, Omaha, Helena, Salt Lake, San Diego, Red Bluff, Portland.

Bad, 5: Hatteras, Mobile, Nashville, Abilene, Cheyenne.

In order to give the reader a fair idea of this system of ranking these forecasts, I present in figure 35 a sample prediction and verification during 1934 from each group named above.

As a further comment on the basis on which these predictions rest, I refer again to figures 33 and 34 which show precipitation and temperature departures arranged in 23-year cycles. It is observed, as illustrated in figure 34 and as might be expected in view of caption 14-g, above, that frequently the resemblance is closer between cycles separated by 46 years, than by those separated by 23 years.[15] Changes of phase and of amplitude certainly exist between repetitions of the characteristic features which comprise a 23-year cycle. These must indeed have been expected in view of the discussion given above of the periodicities in the departures at Berlin and other stations. Nevertheless, in the preparation of nearly 70 three-year predictions, above mentioned, the conviction was steadily deepened that many features may nearly always be recognized in successive 23-year cycles.

Owing to the great financial importance which these predictions would assume if they could be regarded as trustworthy, it has seemed improper to publish them until the lapse of another year, or even 2 years, shall have proved to what extent they may be relied upon.

Employing only weather data previous to and including 1921, forecasts have been made, first for Bismarck, N. Dak., in one continuous interval from 1922 to 1932, and then by successive steps for Vienna, Austria, and North Platte, Nebr., in 11 intervals of 1 year each from 1922 to 1932. These forecasts and their verifications are shown in figures 36 and 37.

26. Causes

Evidence has been presented which seems to show that the radiation of the sun varies in a complex mode comprised of the summation of 12 or more periodicities, all of which are integral submultiples of 23 years. Corresponding periodicities have been traced in weather, and several other weather periodicities have been found which are also integral submultiples of 23 years. Inversions, or at least major changes in form, phase, or amplitude, have been disclosed in the periodicities

[15] Compare the general swing of curves 2 and 4 in figure 34.

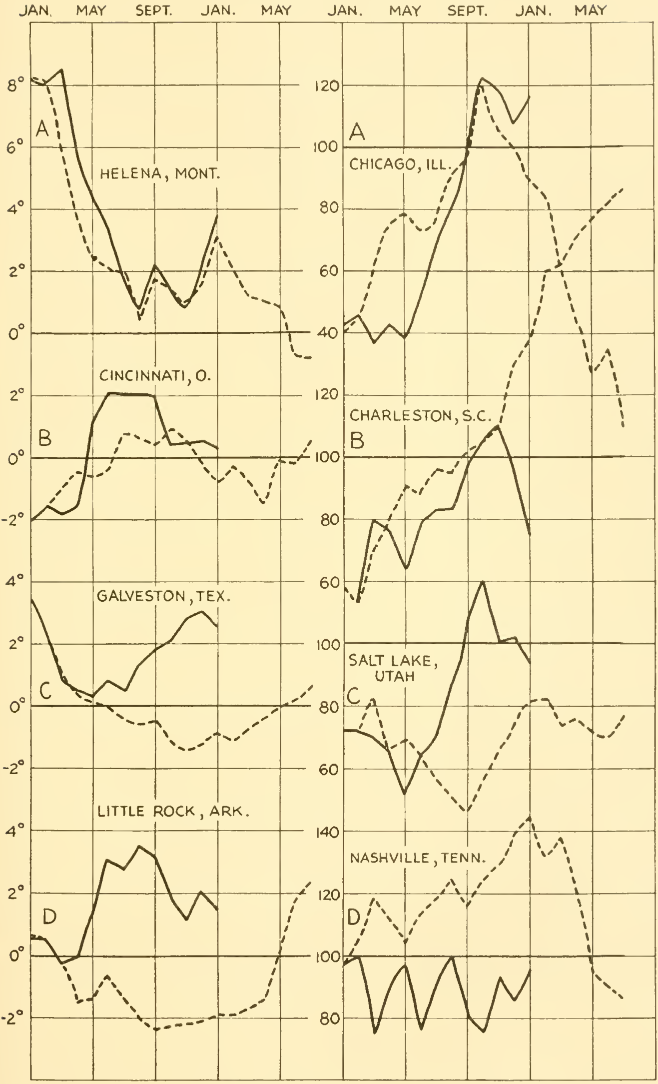


FIG. 35.—Sample forecasts and verifications. Dotted curves are forecasts. Grades of results: A, excellent; B, fair; C, half and half; D, bad. Left, temperature; right, precipitation.

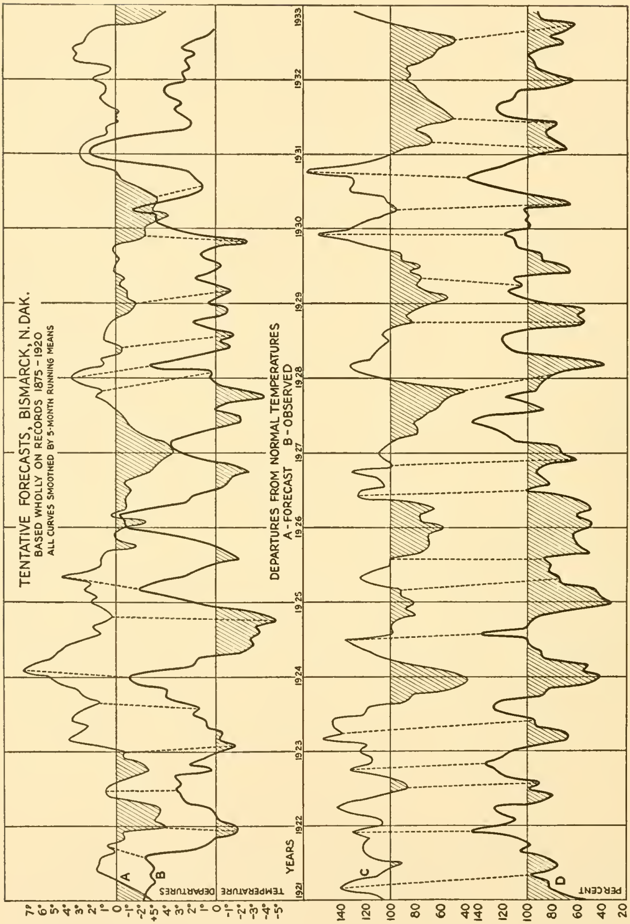


Fig. 36.—Eleven-year forecast for Bismarck, N. Dak., with verification.

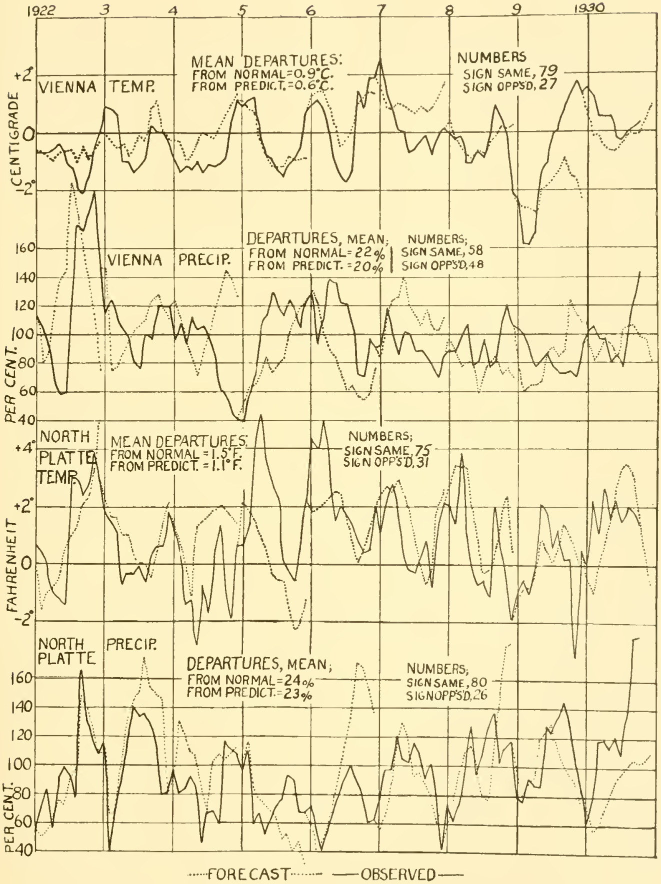


FIG. 37.—Eleven-year forecasts for Vienna, Austria, and North Platte, Nebr., with verifications. Forecasts made step by step.

of weather. These are found to occur at integral multiples of $11\frac{1}{2}$ years measured from January 1819.

These phenomena, if accepted as facts of Nature, propounded to us several problems:

A. Why should the sun, a gaseous body, emit complex pulsations of radiation which are of the nature of a fundamental and 11 or more overtones? A violin string may do this, but why should a gaseous sphere?

B. Why should the terrestrial responses to these pulsations show changes of phase, form, and amplitude at intervals intimately related to the fundamental period of 23 years?

C. Are the terrestrial responses of an order of magnitude reasonably corresponding to the solar impulses?

For question A, I confess that I have no suggestion to offer. I must leave its solution to those theorists who may be convinced by section I of this paper that there is a real body of facts which prove the existence of complex solar variation.

As for question B, the most natural hypothesis is to assume that the phases and amplitudes of the solar periodicities themselves change from time to time at intervals related to $11\frac{1}{2}$ years. Solar-constant observations are not yet of long enough standing to verify this. I have therefore sought to find some helping clue in a regularity of behavior regarding changes of phase among the different stations. In this inquiry I have compared the changes shown by the 8-, 11-, 21-, 25-, and 68-month periodicities in temperature as presented by the various stations Berlin, Copenhagen, Helsingfors, Greenwich, Cape Town, and Adelaide. It seemed superfluous to examine the precipitation which, as meteorologists are aware, is loosely dependent on temperature.

I have devised a sort of shorthand adapted to exhibit the results of this comparison. It is shown in figure 38. At the left of each subfigure will be found the approximate dates of beginning and end of each $11\frac{1}{2}$-year interval for which tabular computations of periodicities were made. Under the names of the stations appear symbols which are designed to represent the types of curves found during the various intervals of $11\frac{1}{2}$ years. These symbols are five in number, but may be combined to indicate that the first half of a curve is of one type, and the second half of another. The symbols are as follows: Numbers 1 and 2 are vertical and horizontal lines. They represent inverted phases of curves of approximately the same form. Numbers 3 and 4 are lines inclined at 45° respectively to the left and the right. They also represent inverted phases of curves of approximately the same form, but of a form essentially differing from that represented

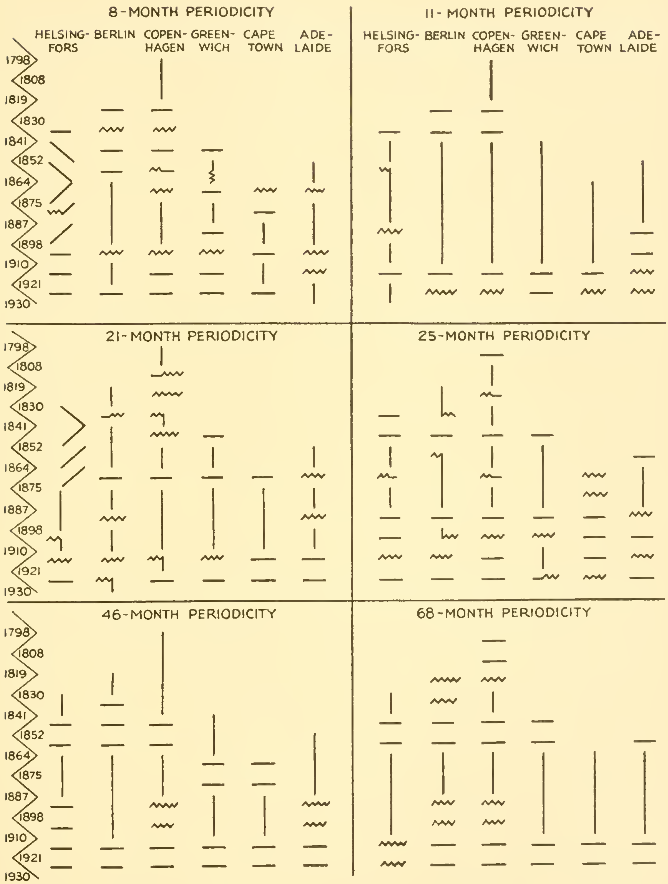


FIG. 38.—Comparison of stations with respect to phase-change of periodicities.

by symbols 1 and 2. Number 5 is a zigzag line. It represents an indeterminate form of curve not similar to those represented by 1, 2, 3, and 4. It is not intended to imply that curves 1 and 2 or 3 and 4 are always similar in form as between representations of periodicities of different stations or periodicities of different lengths. It is only implied that all curves 1 and 2 within a single vertical column of the same subfigure are approximately similar though inverted, and all those represented as 3 and 4 within a single vertical column of the same subfigure are approximately similar though inverted.

Owing to local influences, it was not to be expected that complete harmony would prevail throughout all the subfigures. But if the changes of phase and form in terrestrial periodicities to which extended references have been made, are due to radical changes in the solar radiation, it would naturally be expected that similar mutations of phase and form would tend to occur in all terrestrial periodicities and all stations at about the same time.

Figure 38 seems to show that on the whole this expectation is fairly supported by the facts. Though exceptions occur, there is a prevailing tendency for inversions to occur in all periodicities and all stations simultaneously. Thus, for illustration, at the years 1841, 1864, and 1910, reversals or at least major modifications of form occurred in nearly all cases, and this also frequently happened at the year 1887. It is believed that the exceptions are neither more numerous nor more radical than might fairly be attributed to local terrestrial influences affecting conditions differently at these widely separated stations.

If this conclusion is sound, modifications may well be expected from the prediction I have ventured of solar variation for the years 1935, 1936, and 1937 as given in figure 7. For on that basis it is very probable that a radical change in the phases or amplitudes of solar variation, or in both, will have occurred about 1934, being 115 years after 1819, and will greatly modify solar variation in subsequent years. But yet this result might not occur, for at several epochs the terrestrial periodicities appear to have continued stability for 23 years or even longer, which might call for a similarly long-lived stability in the solar variation, and no mutation of it in 1934.

As for the third query, C, let us restrict our investigation to the interval 1920-1930, for it is only then that we have actual observations of the amplitudes of the periodicities, both of the solar radiation and the terrestrial temperature. In table 13 I give the amplitudes of the periodicities expressed in percentages of the solar constant (1.94 calories per square centimeter per minute) and in percentages of the absolute temperature of the earth, which I take as 290° Centigrade.

TABLE 13.—*Comparison of Solar and Terrestrial Periodicities*

Period in months	Ranges. Observed and percentage										Ranges in percentage as ratios of solar ratios expressed in percentage			
	Solar		Berlin		Greenwich		Cape Town		Adelaide					
	Calories	%	Cent.	%	Fahr.	%	Fahr.	%	Fahr.	%	Berlin	Greenwich	Cape Town	Adelaide
8	0.004	0.21	0°.5	0.17	0°.6	0.12	0°.6	0.12	0°.3⁵	0.07	81	57	57	33
11	9	.47	1.0	.35	0.6⁵	.12	0.2	.04	0.3⁵	.07	75	26	9	15
21	10	.52	1.2	.41	1.8	.31	0.7⁵	.14	1.9	.36	79	60	27	69
25	7	.36	1.6	.55	1.4	.27	1.2	.23	1.4	.27	153	74	64	75
34	7	.36	1.2	.41	0.7	.13	0.5	.10	1.3	.25	114	36	28	70
46	10	.52	1.6	.55	1.2⁵	.24	1.0	.21	1.4	.27	106	46	39	52
68	10	.52	1.2	.41	0.4	.08	0.5	.10	0.8	.15	79	15	19	28
92	10	.52	1.2	.41	1.0	.21	0.6⁵	.12	1.4	.27	79	40	23	52
									Mean percentages		96	44	33	50

The result of this investigation indicates that the percentage change of terrestrial temperatures is from 33 to 96 percent of the percentage change of solar radiation involved in corresponding periodicities.

It might have been supposed that since the earth radiates approximately as a "black body," the relationship would be governed by Stefan's law, $R=\sigma T^4$. In that case $\frac{dR}{R} = \frac{4dT}{T}$, and we should expect the percentage temperature ranges to be only 25 percent of the percentage solar ranges. The actual figures deviate from this in the sense of showing larger temperature ranges than would be expected. Yet the discrepancy is not so great that one cannot entertain as an explanation the contributing influence of indirect causes, such as cloudiness, which might produce changes quite as great as the primary direct cause, variation in solar radiation.

SUMMARY

In the foregoing paper I have tried to present within moderate compass a general view of an investigation started by Dr. Langley more than half a century ago, carried on in recent years with the indispensable financial, intellectual, and moral assistance of Mr. John A. Roebling, the National Geographic Society, and others, and now apparently reaching definite conclusions as to the dependence of weather on the variation of the sun.

I am painfully aware that the limitations of space and funds, the extensive mass of evidence on which I base conclusions, my own ineptness in its presentation, and the preoccupation of readers with other concerns must all combine to prevent even the most interested of readers from deriving that vivid conviction of the truth and importance of these conclusions which is shared with me by those of my colleagues and friends who are most conversant with the evidence. Nevertheless, I hope I shall not have failed to convince the reader of the following propositions:

1. The output of radiation of the sun varies, as proved by simultaneous observations at three stations remote from each other.

2. The solar variation, seemingly irregular, really comprises 12 or more regular periodicities, which support successful predictions of solar changes for years in advance.

3. The periodicities in solar variation are integral submultiples of 23 years.

4. These same and other periodicities which are all integral submultiples of 23 years occur in departures from normal temperatures

and precipitations at numerous terrestrial localities. The inference is that solar changes influence weather.

5. Changes of phases and amplitudes occur in these terrestrial periodicities.

6. The changes of phases and amplitudes in these terrestrial periodicities occur at integral multiples of $11\frac{1}{2}$ years measured from January 1819.

7. On account of the integral relationships of the terrestrial periodicities to 23 years, the weather at all stations contains features which tend to repeat themselves at intervals of 23 years.

8. On account of reversals of phase of some of the periodicities at 23-year intervals rather than at $11\frac{1}{2}$-year intervals, some of these features are more accurately reproduced at intervals of 46 years than at those of 23 years.

9. Various phenomena depending on weather show the influence of the 23-year cycle. Among those examined are the level of the Nile River, the levels of the Great Lakes, the rainfall of Southern New England, the widths of tree rings, the abundance of cod and mackerel, the thickness of varves of Pleistocene and Eocene ages.

10. From tabular and graphic representations of departures from the normal in both temperature and precipitation for more than 100 stations, the weather itself has disclosed many features which repeat themselves in cycles of 23 years, and which though obscured by modifications of phase and amplitude may support predictions of future weather conditions.

11. Forecasts based on these relations having been made to cover the years 1934, 1935, and 1936 for more than 30 stations in the United States, these forecasts are fairly well verified both as to temperature and precipitation in 1934.