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PERIODICITIES IN THE SOLAR-CONSTANT MEASURES

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INTRODUCTION

This paper, based on over 40 years of observations of solar radiation, ties together the following conclusions:

- I. The sun's output of radiation varies.
- 2. It varies in at least 23 regular periodicities, all proceeding simultaneously.
- 3. The periods of solar variation are integral submultiples of 223 years.
- 4. Synthesis of curves representing the 23 periodicities reproduces the original observations of the "solar constant" to within about o.1 percent.
- 5. Synthesis of these curves for 12 years as a prediction, prior to the observations on which they depend, shows rough agreement with Mount Wilson observations of the solar constant, in the years 1908 to 1920.
- 6. A much more satisfactory agreement is found between this predicted synthetic solar-constant curve and the Mount Wilson determinations of the march of contrast along the east-west diameter of the sun, of 1913 to 1920.
- 7. Higher contrast attends higher solar-constant values.

In several former publications ² I have discussed the periodic changes in observed values of the solar constant of radiation.

For several years I have been investigating the effect on terrestrial weather of these periodic changes in the sun's emission. I had become convinced by the earlier solar-constant studies, just cited, that the sun's radiation varies simultaneously in many regular periods, all

¹ I wish to express my sincere acknowledgments to L. B. Aldrich, Director of the Astrophysical Observatory, who made the data available for this paper and gave highly valuable criticisms; to Frederick E. Fowle, deceased, whose careful measurements of solar contrast appear in table 6; to Mrs. A. M. Bond, deceased, whose critical judgment and accurate computations aided in the preparation of the data; to the many observers on high mountains in distant lands who sacrificially kept up this long campaign of measurement; to Mrs. I. W. Windom, who assisted in preparing this text; and to Miss M. A. Neill, who continuously over many years greatly assisted me in keeping the observing stations in operation.

² Annals Astrophys. Obs., Smithsonian Inst., vol. 5, p. 250 et seq., 1932; vol. 6, p. 178 et seq., 1942. Smithsonian Misc. Coll., vol. 111, No. 7, 1949.

aliquot parts of $22\frac{3}{4}$ years. I hoped, by using a long interval of scores of years of an unbroken series of monthly weather records, that I could discover from them all the submultiples of $22\frac{3}{4}$ years which yield effective periodic variations of the solar radiation.

But I found that the variations of the atmospheric conditions from time to time, some associated with the seasons and some with the sunspot cycle, so badly confuse the phases of responses to solar variation that I could not be certain that all the suspected solar periodicities, inferred from weather records, are real. Hence I felt constrained to reinvestigate the observed fluctuations of the solar constant, to determine directly which of the submultiples of $22\frac{3}{4}$ years are truly periods in solar variation.

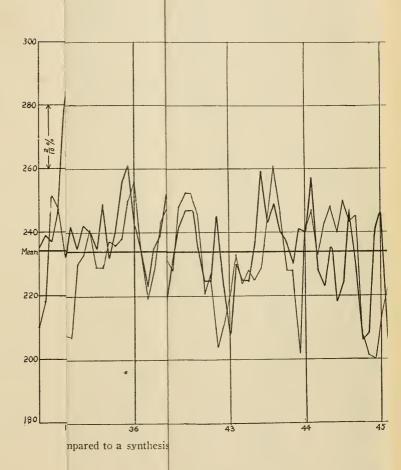
In former papers I have used 273 months as the master period, of which the others are integral submultiples. My present work leads me to prefer 272 months. All the periods which I have found lie within less than I percent of being integral fractions of 272 months.

ADVANTAGES OF METHOD

Some investigators would prefer to submit the available solarconstant data to a Fourier analysis based on 272 months. I prefer to tabulate the data according to each suspected possible period. There are several advantages in this method. In so doing, I divide the total interval covered by the data into several parts, if periods are short enough to furnish a large number of repetitions. In this way the phases of features may be compared in the several independent tabulations of one period. Graphs showing this procedure are given in figure 1. Slight shifts,3 from one to another of the successive tabulations, indicate small corrections to the assumed period. The form of the curve of fluctuation is determined by the tabulations. Also the amplitude of the periodic variation is found. If it is too small to be certainly exceeding the probable error, then the periodicity is to be rejected altogether. Proceeding in this way, I found 23 periodicities in solar-constant results which meet the tests of veridity just indicated. Fifteen other periods were tabulated, but rejected. Each search involved tabulating more than a thousand decade mean values of the solar constant. The results appear in table 1.4

³ See the curves, 6 1/30, of figure 1, in comparison with table 1C, below.

^{*} In tabulating any one periodicity, all the others exercise confusing influences, which are not wholly eliminated, because of the small numbers of repetitive columns going to make up the tables. Hence, irregularities in the curves of figure I are caused by conflicting periodicities, in addition to the effects of accidental errors of observation.





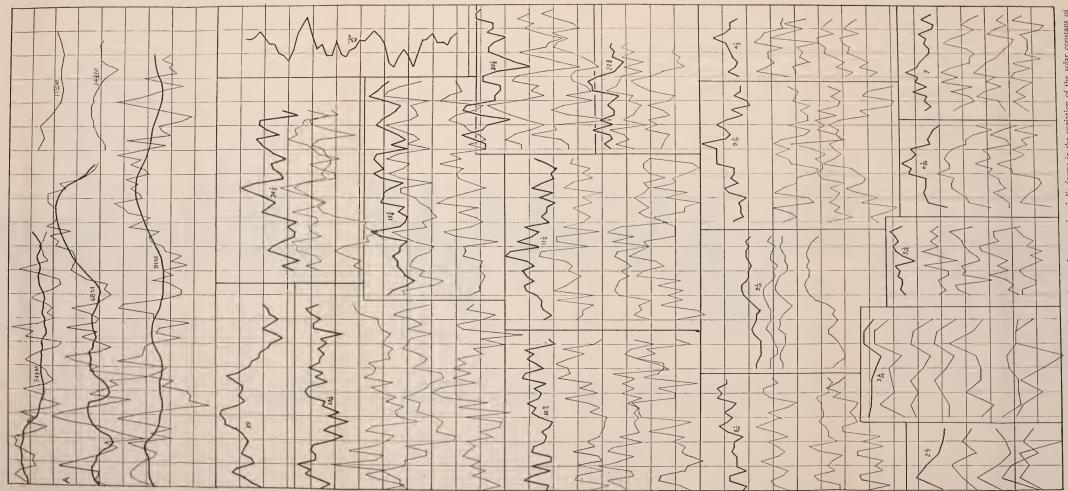
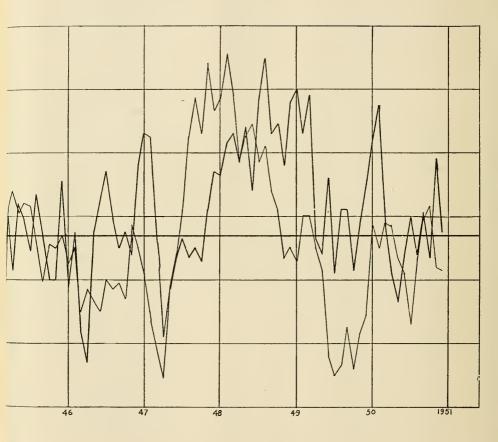


Fig. 1.—Consecutive partial determinations and general means of periodic forms in the variation of the solar constant of radiation as observed 1920 to 1950. Periods are indicated in months. Spaces in ordinates represent 1/10 percent variation in solar radiation.



It may aid to fix ideas on the method of tabulation to give an example. Table 1C is a facsimile of the computation for the period 6 1/30 months. I select it as indicating how fractional parts of months and of 10-day means are treated, so as to preserve the exact average period. I had at first assumed that 6 1/15 months was the proper length of period. The data were separated into three groups. The assumed period corresponds with 18 1/5 10-day intervals. When the mean values for the three groups were computed, they were plotted, superposed. It was then apparent that the maximum ordinates shifted progressively toward earlier dates, as time went on. This indicated that the assumed period is too long by 4/700 of itself. Making this correction, the true period is 6 1/30 months.

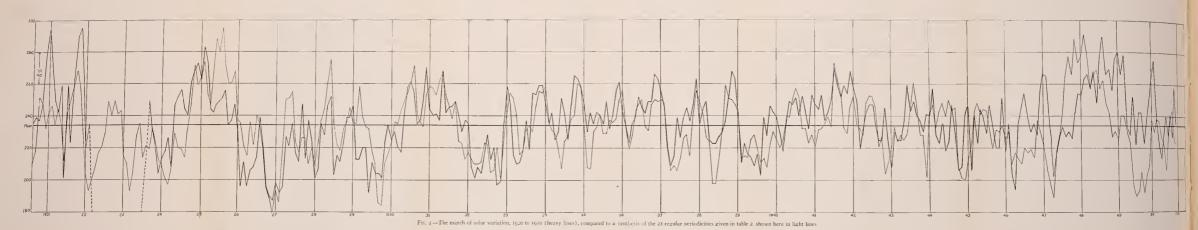
PREPARATION OF DATA

L. B. Aldrich, Director of the Astrophysical Observatory, and his associates had painstakingly considered every circumstance affecting every daily solar-constant observation, at all the Smithsonian mountain stations in various lands. By consensus of three individual opinions, they had assigned to every observed day its most probable solar-constant value, as indicated by the checked results of all stations. Many days were not observed at all. However, there was no decade of any month, from 1920 to 1950, which did not have at least more than one observation.

Mr. Aldrich having been good enough to place these daily solar-constant results in my hands, I computed 10-day and monthly mean values from them for the 31 years 1920 to 1950. To have them in most convenient form for my use, I took their departures from the value 1.900 calories per square centimeter per minute and divided these departures by 1.940. Thus the results became expressed in percentage departures of the solar constant from 1.900 calories. In that form any well-evidenced periodic change resulting from a tabulation shows at once its amplitude in percentage of the solar constant. All values are positive as thus treated, which is convenient in tabulation. These data are given in table 4, appendix I.

PERIODS FOUND AND NOT FOUND

With these clarifying remarks, I now introduce the results. The following periodic changes in the solar constant were found well evidenced. Their approximate relation to 272 months and their amplitudes in percentage of the solar constant are given in table 1A.





91

272

The following periodic changes, given in table 1B, if real, are too small in percentage to be verified.

Tables 1A, 1B.—Periodicities in solar-constant observations

A. Periodicities confirmed *

В.	Perio	odici	ties	sought
	but	not	fou	nd

Period Months	Amplitude Percent	Period Fraction of 272	Period Months	Period Fraction of 272
21/7	0.05	1/127	41/2	1/60
3 1/20	0.05	1/90	5 1/2	1/50
4 1/3	0.06	1/63	6 1/2	1/42
5 1/18	0.05	1/54	7 5/6	1/35
6 1/30	0.12	1/45	8 1/2	1/32
7	0.08	1/39	10 1/9	1/27
8 1/14	0.06	1/34	10 9/10	1/25
9 1/10	0.08	1/30	136/10†	1/20
97/10	0.10	1/28	14 4/10	1/19
106/10	0.06	1/26	17	1/16
11 1/5	0.17	1/24	18 1/5	1/15
11.43	0.11	1/24	19 1/2	1/14
12.0	0.20	• • • •	21	1/13
13 1/10	0.11	1/21	24 8/10	1/11
15 1/6	0.09	1/18	136	1/2
22 3/4	0.07	1/12		
243/4	0.12	1/11		
30 1/3	0.13	1/9		
34 1/2	0.15	1/8		
39	0.20	1/7		
45 1/2	0.13	1/6		
54 1/2 †	0.13	1/5		
68	0.25	1/4		

^{*}The periodicities of 11.43, 12.0 (the periodicity of 12 months is not used in preparing figure 4; if it were, that figure would present closer accord between the curves), and 24½ months were added to the list after search among the departures of the synthetic values, found by summing 21 periodicities, from the observed solar-constant values. It is indeed curious to find two periodicities both within 1 percent of 1/24 of 272 months. Both of them are excellently evidenced and of good amplitude. The 12-month period is of terrestrial, not solar, causation. When one reflects that the pyrheliometer observes only about 70 percent of the solar constant, the remaining 30 percent being supplied by our estimates of atmospheric transmission, it is perhaps not surprising that the yearly (terrestrial) periodic error in the solar-constant values is as large as 0.2 percent in amplitude. The periodicity of 24¾ months was the only other one which could be discerned in a residual plot of differences, smoothed by 7-month running means.

1/3

was the only other one which could be discerned in a residual plot of discerned by 7-month running means.

† After this work was done, I computed a table of the periodicity of 54 8/10 months in the precipitation of Peoria, III., 1856 to 1939. It showed no periodicity of 54 8/10 months, but four strong, well-shaped periodicities of 54 8/10 † 4 = 13 7/10 months. Hence I think the sun's radiation has a periodic variation of one-twentieth of 22¾ years, though it did not impress me as real in the tabulation of the solar constant.

0.12

All periods of these two lists were separately sought for by tabulating over 1,000 solar-constant 10-day means for each suspected periodicity. The investigation does not cover entirely the years 1922 and

1923. I have elsewhere discussed the large solar change observed in those years.⁵ I still think it was a real one. But it may be either a very unusual sporadic solar change, or it may be a periodic change related to a longer period than 272 months.

CONCERNING DOUBTS OF SOLAR VARIATION

For those who do not have intimate association with the Smithsonian observations of the solar constant of radiation, it seems difficult to accept the results as having the high degree of accuracy claimed for them. Observers, familiar with the clouds, dust, and water-vapor load which the lower atmosphere bears to make it milky, do not readily visualize a sky so clear that, if one holds his little finger at arm's length before the sun, the sky seems deep blue right down to the sun's edge. But even if the superior excellence of stations like Montezuma, Table Mountain, and St. Katherine be granted, it still seems incredible to many that the fraction, amounting to about 30 percent of the solar constant, cut off by the atmosphere, can be so correctly estimated that variations of the order of 1/10 percent of the solar constant can be evaluated.

Still more doubtful does it appear to many that, lacking any theoretical support, it can be proved from the observations that the solar variation consists of 23 simultaneously operating regular periodicities, all aliquot parts of $22\frac{3}{4}$ years. Yet it seems to me this cannot longer be doubted. I have tried to demonstrate by a couple of examples that it is necessary to use integral fractions of $22\frac{3}{4}$ years, rather than any other intervals, to represent the the sun's periodic variation. The two periods I have chosen to experiment upon are those which are 1/7 and 1/45 of $22\frac{3}{4}$ months. In figure I the longer period is plotted as 39 months.

I made a new tabulation in four parts for a period lying between 1/45 and 1/44 of $22\frac{3}{4}$ years. It was assumed to be $6\frac{1}{6}$ months, or 19 10-day intervals. In each of the four groups tabulated there are 14 columns. Taking the mean values, they are as plotted in figure 2,A. Evidently, if the four mean results were combined directly, they would so contradict each other that the general mean would show no periodicity at all. But the principal feature, marked A at its right-hand edge in each plot, is equally displaced from curve to curve toward the left by about 6 10-day intervals. The displacement is 19

⁵ Monthly Weather Rev., U. S. Weather Bureau, February 1923. Proc. Nat. Acad. Sci., vol. 9, No. 6, pp. 194-198, 1923. Smithsonian Misc. Coll. vol. 77, No. 5, 1925 (see fig. 11); vol. 80, No. 2, 1927.

10-day intervals, in all, from curve I to curve IV. Between these curves I and IV lies a stretch of time of about 800 10-day intervals. Hence the period should have been taken less than $6\frac{1}{6}$ months by $19/800 \times 6\frac{1}{6} = 0.146$. Subtracting from 6.163, this yields a corrected period of 6.017 months. Within the error of determination, this checks

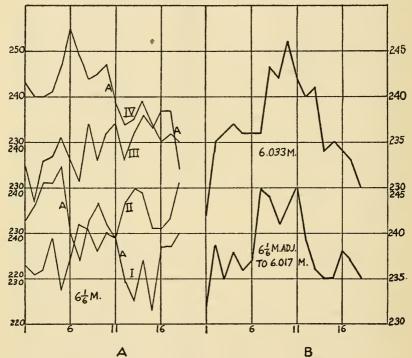


Fig. 2.—The periodicity 6.033 months, confirmed by the displacement of the feature A gradually from I to IV, when the period is assumed to be 6½ months, as shown in figure A. In figure B this displacement is adjusted to a period of 6.017 months, which nearly agrees with the true period, 6.033 months.

with 6.003, which is the period given in table IC. Having displaced curves II, III, and IV by 6, 12, and 19 10-day intervals respectively, and having taken the general mean of the four and plotted it, the result appears in figure 2,B. It is to be compared with the curve of 6.033 months above it, representing the mean value as given in table IC. It must be admitted that the agreement is striking.

Proceeding similarly, I computed two curves 6 for the seventh of

⁶There being but four columns in these part computations for 39 and 37 months, the plots of the results are very ragged, owing to the disturbing influences of 22 other periodic factors superposed.

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	211	216	221	227	232	252	242	273	25.8	232	278	273	242	268	252	247		227	201	211	22 I	211	211	190	221	221	227	201	201	100	227	000	23.2	221	237		263	283	273	200	288	283	283	278	25.2	27.00	278	258	258	233	268		4
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 $22\frac{3}{4}$ years, assumed as 39 months. In this new tabulation I used monthly mean values, instead of 10-day means, as had been done in computing for the curve shown in figure 1. I also computed two curves for a period of 37 months. They show opposition rather than similarity. It now appeared that in both the 39-month and the 37month computations, the principal features were displaced toward the right in the second half of the 31-year interval. The corrected interval from the 39-month tabulation is 39\frac{1}{2} months. Plots of the 37-month tabulation shown in figure 3,A indicated a displacement toward the right of 8 months in an interval of 180 months of time. This gives a positive correction of $\frac{8}{180} \times 37 = 1.6$ months. Thus combined, the contrary curves of figure 3,A yield the lower curve of figure 3,B. Thus the 37-month tabulation yields an adjusted period of 39.6 months, closely agreeing with that yielded by the adjusted 30-month tabulation which was 39.5 months. This later period agrees within slightly more than I percent of being $\frac{273}{7}$, or 39.0 months. (See figure 3,B.)

If critics feel that still more evidence is needed to prove that only integral fractions of $22\frac{3}{4}$ years are to be found in the solar variation, I will remind them that many of the periodicities plotted in figure I show integral fractions of the periods in question superposed upon them. Conspicuous examples in figure I are periodicities of $15\frac{1}{6}$, $34\frac{1}{2}$, 39, $45\frac{1}{2}$, and $54\frac{1}{2}$ months.

ACCURACY OF DATA

As shown in Annals of the Astrophysical Observatory of the Smith-

sonian Institution (vol. 6, p. 163), the comparison of daily solar-constant values, independently measured at stations thousands of miles apart, in opposite hemispheres of the earth, extending over many years, yields a probable error for a well-observed solar-constant value, resulting from work of two stations on a single day, of $\frac{0.164}{\sqrt{2}}$ percent or $\frac{1}{8}$ percent. Using the familiar relation (the probable error of a mean is that of the individual divided by the square root of the number of values), this indicates that a 10-day mean of good quality should be assigned a probable error of 1/25 percent. Then if nine such 10-day means are tabulated in searching for a solar periodicity, the probable error of their mean becomes only 1/75 percent. These considerations indicate not only that real solar variations of 1/10 percent of the solar constant might be detected, but that the features

of the march of a periodic variation of this small amplitude would appear well delineated from a tabulation.

To be sure, these optimum conditions do not always prevail. Not infrequently no more than three or five days of a decade yielded solar-constant observations. Often no more than one station reported. Dur-

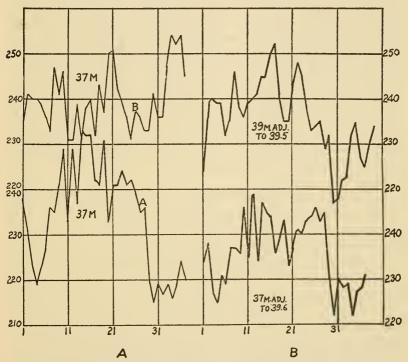


Fig. 3.—The periodicity of approximately $\frac{1}{7} \times 272$ months, tested just as the periodicity of approximately $1/45 \times 272$ months was tested in figure 2.

ing parts of the year less favorable conditions prevailed at one or other of the stations. Such is the case at Table Mountain from March through June, and at Montezuma from November through January. (See figs. 7, 8, pp. 70, 71, Annals, vol. 5.)

On these accounts it need not surprise us that, as shown below, while the sum of periodic variations represents the variation of monthly mean solar-constant results to within an average deviation of I/IO percent, much larger departures sometimes occur. However, divergences depend not only on accidental errors of the observations, but, in part also on imperfect determination of the form, amplitude, and period of the periodicities, for reasons explained above.

SUPPORTING EVIDENCES OF VERIDITY OF PERIODICITIES

There are several indications, not flowing from a consideration of probable errors, that strongly support the veridity of periodicities here disclosed:

- I. In tabulating periodicities, the data have been treated independently in several parts. That is to say, there being nearly 1,100 consecutive 10-day means covering an interval of 30 years, it is possible to tabulate in three or more groups, each with numerous columns, all periodicities of less than 20 months in length. For periodicities of between 20 and 40 months I use two tables, covering consecutive intervals of time. (See fig. 1.) Unless these independent part-tabulations agree within their measure of accuracy to indicate continuance of the same form of periodic variations, and with maxima in the same phase throughout the whole time, then such a supposed period is thrown out as nonexisting. For periods exceeding 40 months, the data were not numerous enough to be thus separated into several groups.
- 2. There is an integral relationship between the periods disclosed. All the periods, which the first criterion certifies as veridical, are, to within a deviation of 1 percent, integral submultiples of 272 months. For example, those approximately 91, 68, 54, 45, 39, 34, 30, and a dozen others of shorter period, are all integral fractions, to within 1 percent, of 272 months. We know that a period of about 272 months is related to the average sunspot period of $11\frac{1}{3}$ years, and it was found by G. E. Hale in the behavior of sunspots and magnetism. It is also approximately the period discovered by meteorologists in many climatic phenomena, as well as by Douglass in the growth of trees.

I cannot but think that the fact of the integral relationship, each to each, of the solar-radiation periodicities here disclosed, and the relationship of all of them to a master period of 272 months, well known in other solar and terrestrial phenomena, strengthens the case for validity of these periodicities. If that be granted, surely the existence of these integral solar-radiation relationships, so reminiscent of the overtones of the vibrations of musical instruments, is a phenomenon well worth investigating by astronomers and by students of hydrodynamics.

I have just stated three arguments for the reality of numerous regularly periodic variations of the output of radiation from the sun as follows: A. Measurements whose small probable error is consistent with the amplitudes of the apparent periodicities display them. B. Tabulations of a chosen periodicity, with the data separated into

independent groups, covering successive time intervals, show separately the periodicity in similar amplitudes, forms, and phases. C. The periods are integrally related, each to each, and all are approximately exact integral submultiples of 272 months, itself a well-known period in other solar and terrestrial phenomena. A fourth supporting evidence is to be referred to later.

The argument B is undoubtedly the most telling. In order to display its full weight, I give, in figure 1, a résumé of all the periodicities which I consider real. It is my firm expectation that scientists who examine without bias the arguments A, B, and C and carefully scan figure I and table IC, will yield to the conviction that the sun's contribution of radiation that warms the earth varies in a complex way. In short, they will admit that, like the overtones of a musical note, the radiation of the sun varies simultaneously in a period of approximately 272 months, and in periods, exceeding 20 in number, which are integral submultiples of approximately 272 months. If scientists go thus far, I cannot but think they will go farther and investigate theoretically the hydrodynamics of the phenomenon.

PERIODICITIES OF 223 AND 113 YEARS

I have not tabulated the data so as to display the periodicity of 272 months, because the values are insufficient. There would be too few repetitions to fairly fix the form of this curve. As for the periodicity of $\frac{272}{2} = 136$ months, though it is the well-known $11\frac{1}{3}$ -year sunspot period, it is inconspicuous in the variation of the solar constant. I have twice sought for it. First, I tabulated the original data in columns of 136 months and smoothed their mean values. Second, I smoothed by 7-month running means the residual departures, which separate the original data from the synthetic reproduction of them in figure 4 by 23 periodic terms. Neither treatment gave conclusively a periodicity of 136 months. Its well-evidenced weather influence, I think, is attributable to fluctuation of the intensity of the bombardment of the atmosphere by electric ions, acting as centers of condensation of water vapor and dust, as sunspot numbers wax and wane.

GRAPHS OF RESULTS

Figure 1 is introduced to emphasize the force of the argument B by a graphical appeal to the eye. The figure shows the mean result of every partial tabulation of the values used to compute table 1A, and also the general mean of these partial tabulations for almost all perio-

dicities included in table 1A. Curves for periodicities of 2 1/7 and 3 1/20 months are given on a scale of abscissae $2\frac{1}{2}$ times as great as the other curves. Horizontal lines in figure 1 are separated by 1/10 percent of the solar constant. The curves for periodicity 2 1/7 months are given on a scale of ordinates twice as great as that used for all others. Up to a periodic length of $22\frac{3}{4}$ months, all the curves are plotted at 10-day intervals. Periodicities of $22\frac{3}{4}$ months and longer are plotted in monthly intervals. Of periodicities less than $22\frac{3}{4}$ months in length, one, that of 9 1/10-months period, is shown smoothed throughout by 5-decade running means. It has a small amplitude and would perhaps have seemed doubtful to many had not running means of 5-decade values been shown, instead of the separate 10-day mean values. This smoothing brings out plainly the similarity of the partial tabulations.

The amplitudes of the 23 periodicities plotted in figure 1 may seem to some critics too small to be of any significance. Not so. For it is shown in figure 4 that the synthesis of these 23 periodic fluctuations produces a curve closely matching, and of the same amplitude of variation as, the curve of original observation. A 12-month period of terrestrial origin with amplitude of 0.2 percent is not introduced into figure 4. Its inclusion would improve the agreement there. No additional regular periodicities were discernible. The analysis appears to be exhaustive.

As the periods grow longer, they are apt to display integral submultiples riding upon the period under examination. This is strongly marked with the period of $15\frac{1}{6}$ months. It shows seven subperiods of 21/7 months very plainly. Similarly the $30\frac{1}{2}$ -month curve shows also the 61/30-month influence. The $34\frac{1}{2}$ -month curve shows influence of the $11\frac{1}{4}$ -month period. Other examples are obvious. Note the curves for periodicities of $54\frac{1}{2}$, 68, and 91 months shown in figure 1. Owing to superposed periods of less length, these long periodicities had to be smoothed by 5- or 7-month running means.

In addition to the direct mean results for each period, I give in a few cases also the smoothed mean, resulting from taking 5-value or 7-value running means for the entire length of the periodicity under consideration. These smooth curves give a more convincing and truer idea of the periodicities, thought to be real, than do the rougher direct means, affected by accidental errors of observation and influences of extraneous periods. Readers should bear in mind that the knicks in the broken lines, which look so large, really average less than 1/10 percent of the solar constant. This bears witness to the high accuracy

of the Smithsonian solar-constant observing. Its probable error has been discussed above.

INTEGRAL RELATIONSHIPS

I had long been of the opinion that the regular periodicities of solar variation are all integrally related to approximately 272 months. This impression is supported by the fact, so obvious in figure 1, that the longer periods shown, themselves being integrally related to 272 months, have in several instances shorter periodicities riding on their backs, which are integral submultiples of them. Further proof of the integral relationships is shown in figures 2 and 3, already described.

Assuming that this integral relationship to 272 months is a condition necessary to the real existence of a regular period in solar variation, the number of such periods that are of considerable amplitudes seems not to exceed 23. At least a rather extensive search has not yielded others strong enough to be certainly real. If these be all, and their forms and amplitudes are as shown in figure 1, then a synthesis of them ought to represent the march of solar variation from 1920 to 1950, except for the interval of 1922 and 1923, when exceptionally large solar variations were observed and which is excluded from this analysis. I have made such a synthesis, and compare it with the march of the solar variation in figure 4.

SYNTHESIS OF PERIODICITIES

To determine the quantities plotted in figure 4, I have computed the departures, plus and minus, from the mean ordinate for each smoothed periodicity, as expressed monthly, which together fix the form of its curve. This gives, in each case, a short series of small monthly departures suitable to the form of each periodicity. All the tabulations begin with August 1920 as zero time. In table 2 they are all tabulated in the smoothed form actually used in preparing the synthetic curve shown in figure 4. In computing the mean periodic forms, and afterward in using them for synthesizing the solar-constant values, I allow for fractions of a decade, or of a month, by adding or withdrawing a value from certain columns, or at appropriate intervals in synthesizing, so as to preserve the correct period.

I tabulate these series, end to end, over the whole interval of more than 30 years. Thus I make a great table of 23 columns and 367 lines. Adding algebraically the plus and minus values of the lines across the table, I find the total synthesized monthly departures, in ten-

thousandths of the solar constant, from the mean solar constant 1.94 calories. The results, covering 367 months, are compared in figure 4 with the monthly observational values recorded in table 4.

CLOSE AGREEMENT BETWEEN SYNTHESIS AND OBSERVATION

Table 3, below, shows the high degree of accuracy with which the synthesis of the original 21 periodicities (before those of 11.43 and $24\frac{3}{4}$ months were found) corresponded to the observations.

These results came from the comparison of observation with the synthesis of 21 periodicities. The average departures are reduced below these figures when periodicities of 11.43, 12.0,⁷ and 24³/₄ months are introduced. The value for the best 233 months then becomes 1.00-tenths percent. The larger average departures prior to July 1926 are attributable to the then imperfect development of the "short method" of solar-constant work. The larger departures after 1945 are thought by Mr. Aldrich to be caused by temporary errors in the scales of pyrheliometers used in the field. He hopes to correct this discrepancy.

Some minds may still prefer to think that the solar-constant observations do not prove the variability of solar radiation. They may point out that the average deviation of the observations from their mean is 0.15 percent, and the average deviation of the synthetic curve from that of observation is still 0.10 percent. They may urge that this amount of improvement is not sufficient to warrant belief in the thesis that the sun's radiation varies in the discovered 23 regular periods, all integral submultiples of 272 months.

Such critics may be reminded that the "weight" of any measurement, that is, its claim to respectful recognition, is proportional to the number of observations that enter into the result; but the probable error (proportional to the average deviation from the mean) is proportional to the square root of the number of observations. It follows that the "weight," or credibility of a solution, is proportional to the square of the average deviation of its components. Hence the weight of the solution here advocated is $\left(\frac{15}{10}\right)^2 = 2.25$ times the weight of the conclusion of an invariable sun.

But it must also be considered that a certain irreducible minimum of accidental error, comparable in a graph to the teeth of a saw, adheres to the solar-constant observations. Whatever excursions from the mean value may be produced by real solar variations, these acci-

 $^{^7}$ The 12-month period is not used in preparing figure 4; its use would improve the agreement of the curves.

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Table 2.—Twenty-three solar periodicities in ten-thousandths of the solar constant, based on August 1920. Also the 12-month terrestrial period, same unit
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2 \frac{1}{7} M: +2 -2. 3 \frac{1}{20} M: 0 -2 +2. 4 \frac{1}{3} M: -1 -2 +3 \pm 0.
 5 \frac{1}{18} \text{ M}: -1 \pm 0 -2 + 2 + 2. 6 \frac{1}{30} M: -4 - 1 + 3 + 6 \pm 0 - 5.
7 M:
         -1 + 1 + 5 + 2 - 1 - 1 - 2. 81/14 M: -2 - 2 - 1 - 1 + 1 + 1
         +3 +2.
9 \text{ I/I0 M}: -2 -4 -3 -1 \pm 0 +2 +3 +1 \pm 0.
97/10 \text{ M}: -4 -3 -1 +1 +5 +5 +2 -1 -4 -3.
106/10 \text{ M}: -1 -1 -1 -1 -3 +1 +1 +2 +3 +1 -1.
II 1/5 M: -4 -2 \pm 0 +3 +1 +9 +3 -1 +4 -2 -8.
11.43 M: +7 + 4 + 6 + 1 - 3 - 4 - 3 - 3 - 4 - 3 - 1.
13 \text{ I/10 M}: +1 +4 +3 -2 -6 -4 +2 +2 +1 \pm 0 -2 +1 +3.
15 \frac{1}{6} M: -3 -6 -6 -1 \pm 0 +2 +1 +2 +3 +2 \pm 0 \pm 0 +2 +1 +1.
223/4 M: -1+1\pm0+1+1+1+1\pm0\pm0+1+2+3+3+2+2
         +1 -1 -2 -3 -3 -2 -1.
24 3/4 M: -2 -2 -1 +1 +2 +3 +3 +4 +4 +4 +3 +3 +2 +1 \pm 0 -2
         -5 -7 -2 \pm 0 \pm 0 \pm 0 \pm 0 -1 -1.
30 \text{ i}/3 \text{ M}: +6 + 5 + 4 + 3 + 3 + 4 + 3 + 1 + 1 \pm 0 \pm 0 \pm 0 - 1 - 3 - 5 - 6
         -6 -5 -5 -6 -6 -4 -3 -2 -1 -1 \pm 0 +3 +3 +4.
34 \frac{1}{2} M: -5 -6 -4 -3 -3 -2 -3 -5 -7 -6 -3 -1 -1 +2 +5 +6
         +8 + 7 + 6 + 4 + 1 - 1 \pm 0 + 1 + 2 + 3 + 3 + 4 + 5 + 5 + 2 + 1
          -1 -3.
         39 M:
         +10 +8 +7 +5 +3 +4 +5 +5 +4 +3 +3 +1 -1 -4 -6
         -8 -10 -10 -10 -9 -9 -8 -6.
45 \frac{1}{2} M: -3 -4 -3 -3 -2 -1 \pm 0 +1 +1 +3 +4 +6 +6 +3 +2 +1
         \pm 0 -2 -3 -1 -1 +1 +1 +2 +3 +2 +2 \pm 0 -1 -3 -4 -5
          -4 -3 -2 \pm 0 +1 +1 \pm 0 -2 -3 -4 -2 -2 -1.
54 \frac{1}{2} M: +4 + 4 + 5 + 6 + 6 + 7 + 7 + 7 + 7 + 6 + 6 + 6 + 5 + 3 \pm 0 - 1
         -I -I -2 -4 -4 -3 -3 -2 -2 -2 -2 -3 -2 -3 -2 -4
          -5 -4 -3 -4 -2 -3 -3 -4 -3 -2 -1 -1 -1 -2 -1 \pm 0
         \pm 0 -2 -2 -1 +1 +2.
68 M:
         -7 -5 -4 -4 -4 -6 -6 -8 -12 -13 -12 -9 -5 -4 -2
          -3 -2 -2 -8 -11 -10 -6 -6 -4 -3 -4 -4 -3 -5
          -5 -6 -5 -4 -4 -4 -4 -6 -7 -8 -7 -8 -6 -4 -2 \pm 0
          +2 +4 +5 +6 +7 +8 +9 +10 +10 +11 +11 +12 +12 +11
          +11 + 10 + 8 + 5 + 2 - 2 - 3 - 7
guM:
          ±0 +1 +2 +2 +2 +2 +2 +3 +4 +2 +1 -1 -2 -3 -3 -3
          -4 -4 -4 -3 -3 -3 -3 -4 -4 -4 -4 -3 -2 -1 \pm 0 \pm 0
          -3 -2 -1 \pm 0 +1 +2 +2 +3 +4 +5 +6 +6 +7 +7 +7
```

The 12-month period of terrestrial causation

-4 -4 -4 -4 -3 -2 -1 -1 ± 0 ± 0 ± 0 .

 $+7 + 6 + 5 + 4 + 3 + 2 + 2 + 1 + 1 + 1 \pm 0 \pm 0 - 1 - 2 - 2 - 3$

Jan. Feb. Mar. Apr. May June July Aug. Sept. Oct. Nov. Dec. +0.1 +0.6 -2.1 -6.7 -0.0 +1.7 +1.4 +2.1 +4.3 +6.2 +13.2 +13.5

dental errors of observation will still load the curve with their sawtoothlike vibrations about its true course. No system of periodicities, which may truly represent the true courses of the solar variation, can possibly follow these small accidental errors of observation. It is therefore unreasonable to demand that such a system of periodicities, even though the true one, can be expected to reduce the average deviation of its curve from the curve of observation below the one-tenth

Table 3.—Average departures of synthetic from observational curve

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Aug. 1920—Mar. 1922, 20 months, 2.01 tenths percent.
Aug. 1923—July 1926, 36 months, 1.82 " "
Aug. 1926—Dec. 1945, 233 months, 1.10 " "
Jan. 1945—Dec. 1950, 60 months, 2.38 " "
Aug. 1920—Dec. 1950, 349 months, 1.45 " "
```

of a percent found. For though, as stated, the probable error of first-rate 10-day means, as found by comparing the simultaneous observations of two solar-constant observations, is 1/25 percent, very many 10-day means are not first rate, as explained above. Moreover the "average deviation" is 5/4 of the "probable error," as is well known, raising the figure to 1/19 percent for the average deviation of first-rate 10-day means.

The real crux of the question, as between the hypothesis of constant solar radiation, and solar radiation varying in 23 regular periods, painstakingly determined and tested by several criteria of reality, lies in considering the large excursions of the curve of observation from its mean. Examples of such methodically marching excursions are found from 1924 to 1927, from 1929 to 1933, from 1937 to 1942, and from 1947 to 1949. The hypothesis of a constant solar radiation offers no explanation for them. On the other hand, the synthetic curve follows these large, methodically marching excursions with some fidelity.

Yet notwithstanding this striking harmony in the principal features between the curve of observation and the synthetic curve of regular periodicities, there are limited intervals of substantial disagreement. Among these the major one occurs in 1922 and 1923, regarding which I have already written. The disagreement in 1920 and 1921 may be attributed to the incomplete development of the short method of solar-constant determination in those earliest years. The same perhaps applies to the disagreement in the years 1924 and 1925, for even then the short method was not fully developed, as now used. As for the period 1946 to 1950, Mr. Aldrich inclines to think the scales of pyrheliometry may have varied a little in those years. There is also

a possibility that, in carrying the computations so far forward as 1950 from their base in 1920, slight errors in the length of the periods have accumulated so as to mar the results of synthesis.

Brief intervals of unusually large divergence between the synthetic and the observed curves occur in 1927, 1929, 1934-1935, 1938, 1940-1941, and 1944. Nearly all these cases occur at the times of the year when sky conditions for observing are inferior at one or both stations, as indicated by figures 7 and 8, pages 70 and 71, Annals, volume 5, already cited. It is not probable, however, that regular periods of variation include *all* the variations of solar radiation. We know, indeed, that outbursts of sunspots and flares cause changes in the sun's output of radiation. Some of the discrepancies referred to are doubtless due to such causes.

I hope the reader will agree that the synthesis of 23 independently and separately computed periodic terms has represented, to within the error of observation, the march of the solar constant as given by the monthly means of the original observations from 1920 to 1950, excluding the extraordinary values of 1922 and 1923. This close agreement in form and amplitude between the observed and the synthetic curve seems to me a fourth kind of evidence supporting the existence of a complex of over 20 regular periods all approximately integral submultiples of 272 months in the observed variation of the sun's output of radiation.

It will occur to the reader that curves of solar observation should tend to repeat their features after 272 months, or approximately 23 years. There is a slight indication that the curve of 1921 in figure 4 is similar to that of 1944, but the work of 1921, as mentioned elsewhere, is too inaccurate to prove it. In the years 1922 and 1923 occurred a unique large depression of the curve of observation. A real test must begin with the year 1924. Unfortunately, as stated elsewhere, there appears to have been a change of scale of about $\frac{1}{3}$ percent in 1948. To correct for it, I subtract 32 units from all the monthly means, July 1948 to February 1950.

In figure 4Å, I superpose the corrected curve 1947 to 1950 (light line) upon the observed curve of observation 1924 to 1927 (heavy line). The similarity is striking. During 48 months there are five large divergencies: 0.55, 0.50, and three of 0.45 percent. The extreme range of the great feature shown in figure 4Å is 0.9 percent, and the average deviation between the curves is but 0.19 percent—less than the expected combined probable errors of observing. One regrets that the interval, 276 months, exceeds the expected interval,

272 months. But as solar conditions modify the lengths of the sunspot cycles, they may also slightly modify that of the 272-month cycle from time to time.

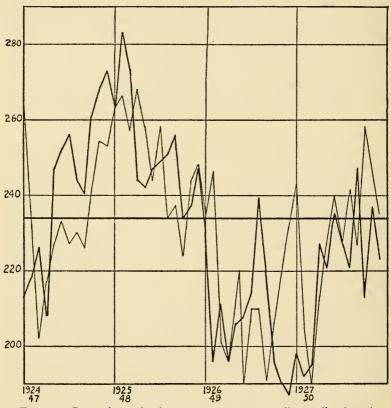


Fig. 4A.—Comparison of solar constants 1924-1927 (heavy lines) and 1947-1950 (light lines).

SCALE OF SOLAR CONSTANT NEARLY UNCHANGED IN 30 YEARS

It is very pleasing that the comparison of synthesized and original curves shows the features generally with equal amplitudes in the two curves. The comparison gives no indication that the scale of observation has changed in 30 years, except perhaps for a rise of 3/10 percent from June 1948 to January 1950. This is remarkable in view of many changes of instruments and of procedures that have taken place meanwhile.

APPENDIX 1

SOLAR-CONSTANT MONTHLY AND 10-DAY MEANS, 1920-1950

Doubtless there are those who are engaged in research on cycles in various lines who may wish to know the Smithsonian results on solar variability as nearly as possible up to date. Mr. Aldrich kindly permits me to publish the following table (table 4) giving the percentage excesses of solar-constant values above 1.900 calories from 1920 to 1950. These percentage excesses are in the form of means of 10 days (i.e., decades of months) and means of months. Taking the first trio of values, given here for illustration, the table may be explained as follows. We have:

2, 8, I, 0, 1, 154 2, 8, II, 0, 2, 139, 153 ⁸ 2, 8, III, 0, 3, 165

The above figure 2, with the figure 0, makes 20, meaning the year 1920. The figure 8 means August, the eighth month of 1920. The Roman numerals I, II, III stand for the first, second, and third decades of August. That is: August 1-9, 10-19, 20-31. The values 154, 139, 165 represent decade-means of the daily excesses of the solar constant by which these observations exceeded in ten-thousandth parts of the mean solar constant (taken as 1.94 calories) the value 1.9000 calories. Thus the value 154 signifies that the mean solar constant for the first decade of August 1920 was 1.54 percent of 1.94 or 0.0299 calorie above 1.90 calories. Finally, the value 153 is the mean of the three decade values and signifies that the average solar constant for August 1920 was 1.90+1.53 percent of 1.94 calories, or 1.930 calories.⁸ As stated above, the percentages of excess over 1.90 calories was chosen to suit my investigation because, first, all values are positive, and second, results come out in percentages of the solar constant.

APPENDIX 2

PROBABLE SOLAR-CONSTANT VALUES BEFORE 1920

Smithsonian solar-constant observations were made in the summers on Mount Wilson, Calif., in most years from 1905 to 1920. But partly because of experimental crudity, and partly from the variability of sky transparency, and mainly because those measurements were all made by the fundamental "long method," which requires constant sky transparency for hours, the results were wide-ranging, from about

⁸ This result is far out of line, and indicates experimental error. In drawing figure 4 I have assumed, instead, 235, given in parenthesis in table 4.

TABLE 4.—Ten-day and monthly means

				_					
2, 8	Į, o	1 154	2, 11	I, 2 III III	82 134		2, 2	I, 5	163 278
	III	2 139(235)		II	83 154			TT.	164 283
	TĨĨ	3 165 153		TTT	84 98	120		ΙĨĨ	165 288 283
		3 103 133		11 a	04 90	129			105 200 203
2, 9	Į, o	4 263	2, 12	Į, 2	85 124		2, 3	Į, 5	166 299
	ΙΙ	5 227 6 227 239		II	86 113			II	167 263
	III	6 227 239		III	87 118	118		III	168 258 273
2, 10	I, o		2, I	1. 3	88 232		2, 4	I, 5	169 263
2, 10	TÎ,	7 227 8 278	-, -	ΙĨ	89 185		-, -	ΙĨ' '	
	717			7 T T					170 252
	III	9 206 237		ΙΙΪ	90 154	190	2, 4	III, 5	171 216 244
2, 11	Į, o	10 278	2, 2	I, 3	91 160		2, 5	I, 5	172 221
-	11	11 258		II	92 142			II	173 258
	III	12 201 246		III	93 77	126		III	174 247 242
			2, 3		94 160	120	2, 6		1/4 24/ 242
2, 12	I, o	13 294	2, 3	_I, 3			2, 0	Į, 5	175 237
	II	14 263		11	95 175			II	176 247
	III	14 263 15 278 278		III	95 175 96 160	165		III	176 247 177 258 247 178 263
2, I	_I, 1	16 299	2, 4	I. 3	97 175		2, 7	Ĭ, 5	178 263
-, -	ΙΪ			I, 3	97 175 98 134		-, ,	IĪ, 2	178 263 179 278
	ΙΪΪ	17 304 18 278 294		ΙΪΪ	90 154	0		ΙΪΪ	180 206 249
	114	10 2/0 294			99 165	158	- 0		180 206 249
2, 2	_I, 1	19 237 20 288	2, 5	_ <u>I</u> , 3	100 175		2, 8	_I, 5	181 258 182 221
	ΙΙ	20 288		II	101 180			II	182 221
	III	21 278 268		III	102 191	182		III	183 273 251
2, 3	Ι, 1	22 299	2, 6	I, 3	103 118		2, 9	Į, 5	184 263
~, 3	ΙÎ,	23 206	-, -	ΙÎ, ,	704 770		~, 9	IĨ, 2	204 203
	7 7 1 1			777	104 170			7 ± ±	185 263
	III	24 242 249		III	105 165	151		III	186 242 256
2, 4	<u>I</u> , 1	25 242	2, 7	I, 3	106 180		2, 10	Į, 5	187 232 188 237
	ΙΙ	26 242		II	107 144 108 227			ΙΙ΄	188 237
	III			III	108 227	184		III	189 232 234
	Ť,	27 242 242 28 267	2, 8	Ť	700 22/	104	2 77		190 216
2, 5	Į, 1		2, 0	I, 3	109 216		2, 11	I, 5	
	ΙΪ	29 247		11	110 206			II	191 252
	III	30 263 259		III	111 211	211		III	192 242 237
2, 6	Ι, 1	31 185	2, 9	<u>I</u> , 3	112 252		2, 12	Į, 5	193 247
-/ -	II	32 206	, ,	III	113 252			II '	194 237
	ΙΪΪ	33 211 201		TÎÎ	113 232	0.10		ΙΪΪ	194 237 195 258 247
		33 211 201			114 242	249			195 258 247
2, 7	_l, 1	34 258	2, 10	_I, 3	115 237 116 221		2, I	_ <u>I</u> , 6	196 237
	ΙΙ	35 268		II	116 221			H	197 258
	III	36 252 259		III	II7 237	232		III	197 258 198 196 230
2, 8	T. t	37 211	2, 11	Į, 3	118 221		2, 2	_I, 6	199 201
, .	ΙÎ,	37 211 38 263	-,	ΙĨ' '	119 227		-, -	ΙΪ	200 206
					119 227				
	IIÎ	39 196 223		IIÎ	120 211	220		IIÎ	201 180 196
2, 9	_I, 1	40 227	2, 12	I, 3	121 221		2, 3	I, 6	202 211
	II	41 263 42 268 253		II	122 216			II.	203 232
	III	42 268 253		III	123 175	204		III	204 191 211
2, 10	Ĭ, 1	43 268	2. 1	Ť	124 216	204	2, 4		204 170
2, 10	11, ,		2, 1	I, 4	124 216		2, 4	I, 6 II	205 170
		44 294		11	125 211 126 211			11	206 201
	III	45 309 290		III	126 211	213		III	207 216 196
2, 11	Ι, 1	46 294	2, 2	Ï, 4	127 221		2, 5	I, 6	208 201
	II.	47 283		ΙĪ'	128 201			II.	209 206
	III	48 309 295		III	129 232	218		III	210 211 206
2 72	Ĭ, ı				129 232	210	2, 6		
2, 12	11, ,	49 273	2, 3	I, 4	130 252		2, 6	I, 6	211 196
		50 247 51 268 263		II'	131 211			II.	212 216
	III	51 268 263		III	132 216	226		III	213 211 208
2, I	I, 2	52 165	2, 4	Į, 4	133 196		2, 7	I, 6	214 221
		53 247		II,	134 206			II	215 211
	ΙΪΪ			ΙΪΪ		000		III	216 211 214
	Ť	0		411 T		208	. 0		216 211 214
2, 2	I, 2	55 206	2, 5	Ţ, 4	130 237		2, 8	<u>I</u> , 6	217 232
	11	56 252		11	137 252 138 252			H	218 232
	III	57 247 235		III	138 252	247		III	219 252 239
2, 3	Ĭ, 2	58 221	2, 6	Ι. 4	139 247		2, 9	Ĭ, 6	220 216
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	III	63 139 156		III	144 252	256		IÎÎ	225 180 196
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	II	65 160		11	146 221			II	227 185 228 201 190
	III	66 165 156		III	147 232	244		III	228 201 190
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	III	72 72 60		III		261		III	234 191 198
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	III	75 103 108		III		268		III	237 160 192
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	ΙĨ	77 52	,	ΙΪ' T	158 263			ΙÎ΄΄	239 216
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	III	81 93 122		III	162 283	263		III	243 201 227

TABLE 4.—Continued

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2, 6 I, 7	247 258	2, 9 I, 9 II		3, 12 I, 1 _II	409 221
11	248 221	11	329 191	11	410 247
III	249 227 235	III	330 211 204	ΙΪΪ	411 242 237
2, 7 <u>I</u> , 7	250 232	2, 10 I, 9	331 211	3, 1 I, 2	412 242
II	251 216	II	332 216	II	413 242
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2, 8 I, 7	253 211	2, 11 I, 9	334 206	3, 2 I, 2	415 227
II	254 221	II	335 227	II	416 232
ΙΪΪ	234 221	ΙΪΪ	335 227 336 237 223	ΙÎÎ	410 232
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2, 9 I, 7	256 237	2, 12 <u>I</u> , 9	337 237 338 237	3, 3 <u>I</u> , 2	418 175
II	257 258 258 247 247	II	338 237	II	419 221
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ΙΙΪ	258 247 247	III	339 227 234	III	420 206 201
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II	260 206	II	34 I 232	II,	
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III	261 211 213	III	342 232 225	III	423 211 208
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111	263 232	-11	344 232	-11	425 227
III	264 247 237	III	345 247 230	ΙΪΪ	426 154 204
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_11	266 227	_11	347 211	_11	428 221
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II	272 211	1.1	353 252	11	434 227
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11	275 247	II	356 273	II	437 232
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II	277 216 278 227	11	359 278		440 180
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III	282 247 246	III	363 252 254	III	444 201 199
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_II	284 278	II	365 227	-11	446 237
III	285 232 252 286 232	III	365 232 235	III	447 211 235 448 258
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I I	287 221	II	368 237	11	449 247 450 268 258
III	288 216 223	III	369 247 237	III	450 268 258
				3, 2 I, 3	451 258
2, 8 I, 8 II		3, 11 <u>I</u> , o	370 242	3, 2 I, 3	451 250
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III	489 227 235	111	570 247 239	III	651 232 230
3, 3 I,	4 490 247	3, 6 <u>I</u> , 6	571 258	3, 9 I, 8	652 237
II	491 221	11	572 252	11	653 232
III	492 258 242	III	573 247 252	III	654 242 237
3, 4 I,	4 493 232	3, 7 I, 6	574 252	3, 10 I, 8	655 247
11	494 221	II	575 242	ĨĨ	656 247
III	495 221 225	III	576 242 245	III	657 263 252
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ίίι	497 242 498 221 230	ıii	578 252	III	659 268 660 268 268
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III	504 232 241	III	585 242 249	III	000 242 240
3, 8 I,	4 505 211	3, 11 I, 6	586 268	3. 2 I. 0	667 216 668 185
11	500 237	11	587 273 588 258 266		668 185
III	507 227 225	III	588 258 266	111	669 232 211
3, 9 I, II	4 508 232	3, 12 I, 6 II	589 278	3, 3 I, 9	670 221
111			590 263	111	671 216
III	510 263 247	III	591 247 263	III	672 232 223
3, 10 I,	4 511 263 512 268	3, 1 I, 7	592 247	3, 4 I, 9	673 221 674 227
iii	513 263 265	ıii	593 273 594 242 254	ıii	675 201 216
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111	516 258 263	III	597 252 245	III	677 221 678 211 214
3, 12 I,	4 517 268	3, 3 I, 7	598 211	3, 6 I, 9	679 211
11	518 258	11	599 221	11	680 196
III	519 247 258	ΙΙΪ	600 227 220	III	681 221 209
3, I <u>I</u> ,	5 520 242	3, 4 <u>I</u> , 7	601 201	3, 7 <u>I</u> , 9	682 221
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III	522 232 247	III	603 216 209	III	684 201 216
3, 2 I,	5 523 237	3, 5 I, 7	604 180	3, 8 I, 9	685 201
ווֹן	524 237	111	605 227	111	686 180
	525 216 230 5 526 221	3, 6 I, 7	606 237 215 607 237		687 227 203 688 252
3, 3 1,	5 526 221 527 242	3, 6 I, 7	607 237 608 237	3, 9 I, 9	688 252 689 232
ΙΪΪ	528 263 242	iiı	609 242 239	iiı	690 232 239
3. 4 I.	5 529 237	1. 7 I. 7	610 221	3, 10 I, 9	691 221
II'	530 242	"" ′ IĪ' ′	611 227	II	692 237
111	531 227 235	III	612 232 227	III	693 227 228
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II	533 232	II	614 232	11	695 258
IIÎ	534 247 242	III	615 242 239	III	696 221 246
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II	536 237	II	617 247 618 237 245	II	698 258
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3, 8 I,	5 541 247	3, 11 Î, 7	622 247	4, 2 Î, o	703 227
1.1	542 263	11	623 247	11	704 227
III	543 237 249	III	624 247 247	IĨĨ	705 232 229
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11	545 237	II	626 252	11	707 211
IIĪ	546 227 232	III	627 278 263	III	708 211 218
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		111	629 268	II	710 211
III	549 242 240		630 206 235 631 221	III	711 268 235
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3, 12 I,	5 553 247	3, 3 I.8	634 268	4, 6 I, o	
II	554 263	II	635 237	4, 0 II	715 247 716 252
III	555 273 261	III	636 242 249	ıîî	717 242 247
3, I I,	5 556 237	3, 4 I, 8	637 232	4. 7 I. 0	718 258
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IIÎ	558 227 242	III	639 237 225	III	720 242 251
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III	561 196 234	III	642 206 223	IIÎ	723 242 240
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iii	563 237 564 232 223	III	644 227 645 232 223	111	725 252
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3, 4 II	5 565 232 566 237	3, / 11,	647 232	io ii	727 247 728 227
ΙΪÌ	567 237 235	ıìì	647 232 648 221 227	ΙÎÎ	729 252 242

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	II	I	730	252		2	I	811	216		5	I	892	237	
		II	731	227			II	812	242			ΙĪ	893	237	
		III	732	216	232		III	812	232	220		III	804	242	230
	12	Î			-3-	3		814	206	-,,0	6	Ĩ	80.5	232	-39
	12	ΙÎ	733	242		3	ΙÎ	815	200		Ü	ΙÌ	095	232	
			734	250			ΙΪΪ	0.5	247			T T T	896	22/	
		ΙΙΪ	735	258	253			816	221	225		ΙΙĮ	897	227	229
4,	1	Ĭ, 1	736	232		4	Ī	817	2 I I		7	Ĩ	898	227 258	
		11	737	268			II	818	227			II	899	242	
		III	738	242	247		III	819	237	225		III	900	242	247
	2	I	739	216		5	I	820	242		8	I	901	247	
		II	740	247		Ť	II	821	232			ΙÎ		232	
		III	741	273	245		III	822	237	237		III		22 I	222
	-	Ī	742	258	243	6	Î	823	252	2.37	0	Ĭ	004	232	233
	3	ΙÎ				v	ΙÎ	824	252		9	ΙÎ	904	232	
		T 7 7	743	232			ΙΪΪ	024	263				905	216	
		ΙΙΪ	744		253			825		259		ΙΙΪ		211	220
	4	Ĩ		247		7	Ī	826	252		10	Ĩ	907	232	
		II		242			ΙΪ	827	221			ĨĬ		221	
		III	747	221	237		III	828	257	243		III	909	206	220
	5	I	748	247		8	I	829	247		II	I	910	258	
		II	749	216			H	830	242			ΙĪ		242	
		III	750	263	2.1.2		III	831		249		III		252	251
	6	Î	751	263		9		832	232	-49	12	Ī	913		-3-
	U	ΙÎ	751	216		9	ΙÎ	833	2.72			ΙÎ		216	
		ΙΪΪ	752	-60			ΙΪΪ	033	247			ΙΪΪ			218
			753	208	249			834	242					216	210
	7	Ţ	754	258		10	I	835	232		4, I	I, 6	916	22 I	
		ΙĪ	755	268			II	836	247			ΪΪ	917	227 258	
		III	756	283	270		III	837	232	237		III	918	258	235
	8	I	クミク	283		11	Ţ	838	237		2	I	919	211	
		II		252			II	839	247			ΙĪ	920	20 I	
		III	750		259		III	840	206	230		III		201	204
	9	Î	759 760	278	-59	12		841	232	2,50	3			191	204
	9	ΙÌ	761	2/0		1.0	ΙÎ	842	227		.,	ΙÎ		185	
		ΙΪΪ	701				ΙΪΪ	042				ΙΪΪ	923	105	***
			762	210	252		111	843		24 I			924		194
	10	Ţ	763	273		4, I		844	252		4	I	925	252	
		II	764	258			11	845	242				926	242	
		III	765	247	259		ΠĪ	846	227	240		ΙΙΪ	927	2 I I	235
	II	I	766	268		2	I	847	257		5	II	928	252	
		II	767	247			Ħ	848	263			11	929	252	
		III	768		254		III	849	252	257		III	930	227	244
	12	Ī	769	263	- 57-4	3		850	216	-07	6	I	931	258	
		ΙÎ	770	258		.,	Ιİ	851	242			H	932	247	
		ΙΪΪ	770	283	268		ΙΪΪ	8	227	228		III	033	258	254
	1	T		288	200			852		220	7	Ť	024	227	,-
4,		I, 2				4	I	853	216		,	ΙÎ	934 935	25/	
		II	773				ĨĨ	854	227			ΪΪ	935	250	
		III	774	247	257		III	855	227	223			930	221	239
	2	I	775	247		5	I	856	242		4, 8	I, 6	937	221	
		II		252			II	857	237			II		216	
		III	777	247	249		III	858	227	235		ΠĪ	939	252	230
	3	I		22 I		6		859	237	,.,	9	I	940	252	
	.,	ΙÍ	770	227		v	ΙÎ	860	221			H	941	232	
		ΙΪΪ	779 780	276						228		III		221	235
			700	216	219	_	III	861		228	10	I		216	-05
	4	Ţ	781	227		7	- 1	862	237			ΙĪ		232	
		II	782	237			II	863	227			TÎÎ	944	237	228
		III	783	232	232	_	III	864	237	234	11	III	945	23/	220
	5	τ	784	227		4, 8	I, 4	865	263		11	ΤÎ	946	247	
		TÌ	785	242			11	866	216			TTT	947	250	
		III	786	257	242		III	867	206	228		ΙΙΪ	948	203	250
	6	I		257		9	I	868	206		12	Ţ	949	304	Ω
		ΙÎ	788	227		,	ΙĪ		221			ĨĨ	950	258 263 304 273 221	(1)
		IÍÍ	780	247	247		ΙΪΪ	870	191	206		III	951	221	(?) 266
	-	Ĭ	789	24/	24/			8-7	220	200	4, I	I, 7	952	270	
	7	ΙΪ	790			10	ΙΪ	871	232			II	053	258	
		11	791					872	206	0		ΙΪΪ	054	258	265
	0	IIÎ	792		247		ΙΙΪ	873		208		Î	000	258 216	(3)
4.	8	I, 2	793	232		II	I	874	247		-	ΙΪ	955 956	2101	(+)
		II	794	237			H	075	237			117	950	237	
		III		237	235		III	876		239		III	957	242	232
	9	τ	796	247	00	12	I	877			3	_I	958	242 185	
		II	797	196			ΙÎ	877 878	237 268			H	9.59	200	
		III	798		225		ΙΪΪ	870	222	216		III	960	216	202
	10	Ĭ	790		225		T .	879 880	202	246	4	I		191	
	10	TT	799	232		4, I	Ĭ, 5		20 I		4	ΙÎ	962		
		II	800	227			11	881	311			ΙΪΪ	060	232	217
		IIÎ	801		225		ΙΙΙ	882		206					~1/
	ΙI	I	802	237		2		883	242		5	Ţ	904	232	
		H	803	242			H	884	247			ΙΪ	965		
		III	804	257	245		III	885 886	232	240		III		216	227
	12	I	805	232		3		886	232		6	I	967	211	
		ΙÎ	806	211		.3	ΙÎ	887	232			ΙÎ	968	247	
		ΙΪΪ		227	223		ΙΪΪ	887 888		223		III		242	233
1	I		808	206	3	4			232	22,,	7	I	970	237	
4.	1	I, 3 II		200		4	ΙÌ	800	252		/	Ιİ	077	232	
			809	200	0		TIT	90	253	2/1		ΙΪΪ	971	211	227
		III	810	211	208		III	091	247	244		111	972	211	22/

	TABLE 4.—Concluded 8															
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	8	Ţ	973	216		10	Ť	1012	258			12	1	1057	263	
		11	979	221			11	1016	258				11	1058	242	
		111	975	252	230		TTŤ	1017	253	350	_	_	111	1059	288	204
	9	TT	970	221		II	7.7	1018	278		5,	I	1, 0	1000	288	
		TIT	977	237	226		111	1019	200	0-6			111	1001	208	
		111	970	221	220		YII	1020	203	270		_	111	1062	200	275
,	. 0	TT	979	227		12	ΥŤ	1021	270			2	7.7	1003	227	
		TIT	081	262	242		TŤŤ	1022	278	280			TŤŤ	1065	237	227
1	т	Ť	082	273	242	4 T	Ť,	1023	278	200		2	Ť	1066	247	237
•	•	ΤÎ	083	2/2		4, *	ŢŢ,	1025	247			.3	ΤÎ	1067	227	
		ΙΪΪ	084	247	254		ΤΪΪ	1026	273	266			ΤĨΪ	1068	211	222
I	2	Ĩ	085	258	-34	2	î	1027	304			d	î	1060	221	
		ΙÏ	986	237			ΙÏ	1028	204			7	ΙĪ	1070	211	
		III	987	263	253		III	1029	232	278			III	1071	206	213
4,	I	1,8	988	278		3	1	1030	242			5	1	1072	232	_
		Π	989	247		_	Π	1031	252			_	Π	1073	206	
		III	990	263	263		III	1032	206	233			III	1074	247	228
	2	_I	991	273		4	I	1033	22 I			6	Ţ	1075	242	
		II	992	268			II	1034	206				II	1076	232	
		ΠÎ	993	258	266		Щ	1035	258	228			Ш	1077	247	240
	3	1 7	994	278		5	-Ţ	1036	242			7	, į	1078	211	
		111	995	247			11	1037	273				11	1079	242	
		111	996	247	257	,	117	1038	242	252		0	111	1080	232	228
	4	τŤ	997	293	(1)	D	7.1	1039	232		5,	δ	1, 0	1001	253	
		111	990	252	268		111	1040	191				111	1083	253	247
		, T	1000	250	200	_	111	1041	242	222			111	1003	206	241
	5	ΤŤ	1000	250		7	7 7	1042	237			9	τÍ	1085	200	
		τii	1001	262	258		111	1043	242	242			TÎÏ	1086	237	227
	6	Ť	1002	283	230	8	117	1044	247	242		τo	Î	1087	242	/
	•	ΤÎ	1004	273		U	ΤÎ	1045	262				ΤÎ	1088	240	
		III	1005	273	276		TĨĨ	1047	221	242			ΙÎÎ	1089	283	258
	7	I	1006	200	-,-	0	Ĩ	1048	227	-4-		11	Ī	1000	263	
		II	1007	288			II	1049	237				II	1001	227	
		III	1008	283	290		III	1050	206	223			III	1092	252	247
4,	8	I, 8	1009	283		10	1	1051	232	Ü		12	1	1093	232	
		II	1010	278			II	1052	221				II	1094	247	
		ΙΙΪ	1011	237	266		III	1053	263	239			III	1095	227	235
	9	1,	1012	252		11	_I	1054	268							
		11	1013	278			II	1055	247							
		111	1014	278	269		111	1056	237	251						

1.9 to 2.0 calories, or even more. Still, by forming these less-accurate solar-constant values into large groups of days, according to magnitude, H. H. Clayton was able to correlate solar changes with weather elements.

It now occurs to me that since the periodicities now discovered in the solar emission have been expressed as to form and amplitude, and since 1920 seem to be permanent as far as known in period, amplitude, and form, it may be worth while to synthesize monthly mean solar variation backward from 1920. This done, it would be possible to compare the values synthesized with monthly mean solar-constant values observed on Mount Wilson. If, on the whole, high, medium, and low solar constants as synthesized correspond to high, medium, and low Mount Wilson values, it will be a confirmatory evidence of the sun's real variability, of the constancy of periodicities, of their comprising nearly the total solar variation, and of the value of Clayton's work on the correlation of solar variation with weather.

Table 5 gives the synthesized monthly solar-constant values from

⁹ Smithsonian Misc. Coll., vol. 68, No. 3, 1917.

August 1908 to December 1920. These results are given graphically in figure 5,C. These are actual estimated solar constants in calories per square centimeter per minute, not, as in table 4, percentage departures from 1.90 calories.

COMPARISON OF SYNTHETIC WITH MOUNT WILSON SOLAR-CONSTANT VALUES

From table 53, page 193, volume 4, Annals of the Astrophysical Observatory, I take monthly solar-constant values determined from Mount Wilson observations in the months May to November, 1908 to 1920. I omit four values, July and August 1912, because the sky was then very much fouled by dust from the volcano, Mount Katmai. I also omit July values of 1910 and 1917 because they are very wild indeed, far beyond the limits of dispersal of the others.

Having plotted the Mount Wilson values and such parts of the synthetic series as corresponded in time with them, I saw that there was a gradual rise in values in both observed and synthetic series from 1908 to 1914. I drew straight lines best following this trend to represent the means of the values over that interval, and read off the departures of the individual solar-constant values on the plot from these lines. For the rest of the total interval, that is 1915 to 1920, I read departures from straight horizontal lines drawn in the mean of ordinates. The plot was in arbitrary units, with the units for ordinates in the synthetic plot twice as large as those for the Mount Wilson data. These departure values follow in table 6.

Taking the sums of the data in the columns of table 6 they yield:

Mount Wilson÷synthetic = $\frac{503}{284}$ = 1.77. Recalling the ratio of units, 2 to 1, it appears that the dispersal of Mount Wilson data is 3.54 times as great as that of the synthetic data. The synthetic curve 1920-1950, however, as plotted in figure 4, shows practically the same range of variation as does the curve of original modern observations. Hence it appears that the Mount Wilson solar-constant observations of 1908 to 1920 are probably $3\frac{1}{2}$ times less accurate than the modern work set forth in table 4.

Taking account of the numbers of departures of the same sign in the columns of table 6, and the numbers of them of opposite signs, the sums are 28 and 21.

Taking the sums of departures that are of the same sign in both columns, the results are 324 for Mount Wilson and 170 for the syn-

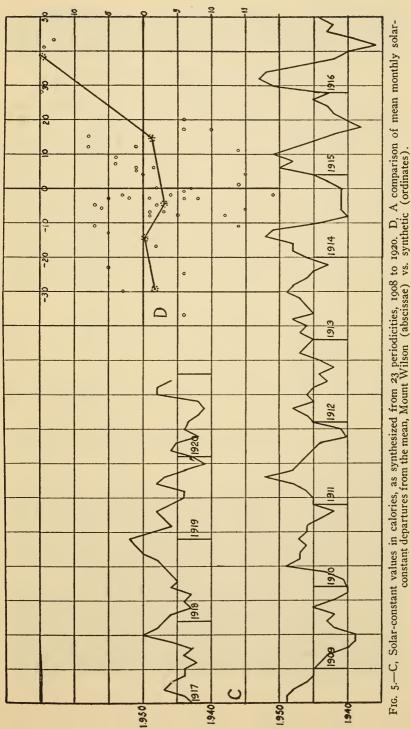
¹⁰ See Smithsonian Misc. Coll., vol. 60, No. 29, 1913.

Nov. 40 Dec. 41

Table 5.—Synthesized solar constant, 1908-1920

Values to be prefixed by 1.9

							•				
1908	Aug.	49	1912	Jan.	45	1915	Jan.	45	1918	Jan.	47
	Sept.	49		Feb.	46		Feb.	50		Feb.	46
	Oct.	48		Mar.	48		Mar.	48		Mar.	
	Nov.	46		Apr.	45		Apr.	51		Apr.	44
	Dec.	45		May	46		May	48		May	
1909	Jan.	45		June	45		June	45		June	46
	Feb.	44		July	43		July	42		July	45
	Mar.	43		Aug.			Aug.			Aug.	
	Apr.	40		Sept.	42		Sept.	40		Sept.	47
	May	3 9		Oct.	44		Oct.	42		Oct.	48
	June	39		Nov.	47		Nov.	43		Nov.	50
	July	42		Dec.	46		Dec.	45		Dec.	51
	Aug.	43	1913	Jan.	45	1916	Jan.	43	1919	Jan.	52
	Sept.			Feb.	47		Feb.	51		Feb.	49
	Oct.	45		Mar.	46		Mar.	53		Mar.	46
	Nov.	42		Apr.	48		Apr.	52		Apr.	47
	Dec.	40		May	45		May	47		May	48
1910	Jan.	40		June	46		June	42		June	46
	Feb.	41		July	47		July	40		July	44
	Mar.	43		Aug.	49		Aug.	36		Aug.	44
	Apr.	49		Sept.	48		Sept.	39		Sept.	48
	May	47		Oct.	46		Oct.	43		Oct.	47
	June	47		Nov.	45		Nov.	42		Nov.	44
	July	46		Dec.	43		Dec.	44		Dec.	41
	Aug.	47	1914	Jan.	46	1917	Jan.	43	1920	Jan.	43
	Sept.	46		Feb.	48		Feb.	44		Feb.	46
	Oct.	46		Mar.	48		Mar.	47		Mar.	45
	Nov.	44		Apr.	52		Apr.	46		Apr.	42
	Dec.	42		May	51		May	44		May	44
1911	Jan.	45		June	44		June	44		June	43
	Feb.	45		July	40		July	42		July	42
	Mar.	•		Aug.	•		Aug.			Aug.	-
	Apr.	48		Sept.	4 I		Sept.	43		Sept.	
	May	52		Oct.	4I		Oct.	46		Oct.	48
	June	48		Nov.	•		Nov.	•		Nov.	
	July	47		Dec.	43		Dec.	48		Dec.	46
	Aug.	•									
	Sept.										
	Oct.	44									



thetic data. The corresponding sums for departures of opposite signs are 199 and 135. Thus, according to Mount Wilson, agreeing departures preponderate in total magnitude over disagreeing depar-

Table 6.—Comparison of Mount Wilson and synthetic values

1908	Mount Wilson	Syn.		Mount Wilson	Syn.	1916	Mount Wilson	Syn.
Aug.	+45	+16	May	+ 5	+ I	June	— I	— 6
Sept.	+28	+15	June	— 8	— 1	July	— 3	—10
Oct.	+43	+13	1913			Aug.	+ 2	—ı8
1909			Aug.	- 7	+ 5	Sept.	— 8	—12
June	+17	— 6	Sept.	—3 0	+ 3	1917		
July	— 3	— I	1914			July	+20	 6
Aug	+12	+ 1	June	± o	— 7	Aug.	+ 6	— 2
Sept.	— 7	— I	July	+ 4	-15	Sept.	- 2	- 4
1910			Aug.	+11	-14	1918		
May	+12	+ 8	Sept.	-11	-14	June	- 7	+ 2
June	— 5	+ 7	Oct.	 6	-15	July	+ 4	±ο
July	-24	+ 5	1915			Aug.	— 3	+ 2
Aug.	-11	+ 7	June	— 8	\pm o	Sept.	+9	+ 4
Sept.	-13	+ 5	July	— 3	 6	1919		
Oct.	+ 3	+ 5	Sept.	+ I	-14	June	+ 7	+ 4
1911			Oct.	+17	-10	July	± o	— 2
June	+15	+ 8				Aug.	— 5	— 2
July	-13	+ 5				Sept.	6	+ 6
Aug.	- 2	+ 3				1920		
Sept.	+ 6	+ 1				July	-25	— 6
Oct.	-17	- 2				Aug.	— 3	 8
						Sept.	-37	— 6

tures as $\frac{324}{199}$ = 1.6. Similarly, for synthetic values the results are $\frac{170}{135}$ = 1.3.

Finally, I show in figure 5,D, the Mount Wilson departures as abscissae against the synthetic departures as ordinates. The plotted points are greatly scattered, as the inaccuracy of Mount Wilson solar-constant values would lead us to expect. Yet, on the whole, the comparison indicates that high departures tend to occur simultaneously in both sets of data, and low departures similarly.

Thus four kinds of rough indications agree to confirm the view that the synthetic solar-constant values of 1908 to 1920 are supported as to their validity, at least in some degree, by the evidences from Mount Wilson observations. The four evidences are: 1. Both sets of data yield upward trends from 1908 to 1914. 2. Departures from representative lines have the same signs 28 times, opposite signs, 21.

3. The summation of departures of the same sign exceeds that for those of opposite sign about $1\frac{1}{2}$ times. 4. The plot of departures indicates a positive correlation between Mount Wilson and synthetic solar-constant values.

The great inferiority in accuracy of Mount Wilson values of the solar constant forbids a high degree of correlation, even if the synthetic values are as correct from 1908 to 1920 as they are from 1920 to 1950. This inferiority arises from the fact that all the Mount Wilson values result from observations by the "long method." That method requires for accuracy a sky of constant transparency over several hours. If the sky improves, the solar-constant value is too high, and vice versa. Moreover, only one value was obtained per day with the "long method." In modern solar-constant work by the "short method," several values are obtained and combined on each day of observation. The sky is required to retain uniform transparency only during about 10 minutes of each observation. It might vary decidedly from one determination to another of the day's group, and yet all the solar-constant values of the day be closely agreeing.

SOLAR CONSTANT AND SOLAR CONTRAST

The Mount Wilson work offers another test of the probable validity of the synthetic solar-constant curve of 1908 to 1920. From 1913 to 1920 we were accustomed to produce drift energy curves in several wavelengths, observing intensities along the east-west diameter of an 8-inch solar image, on every day that we observed the solar constant of radiation. These U-shaped curves, which show the contrast in brightness between the center and edges of the sun's disk, were all measured as described in volume 4 of the Annals of the Smithsonian Astrophysical Observatory. We used an empirical formula to obtain a value to represent the average contrast between center and edge of the sun's disk on each day of observation. These data are given in tables 75 to 82 of volume 4 of the Annals.

It was thought probable that the "solar contrast" would be greater on days when the "solar constant" was higher. Some figures, indicating that this is so, are given in volumes 3 and 4 of the Annals.

Table 7, which follows here, is prepared from the "solar contrast" tables of the Annals, volume 4, and from table 6, just given, which presents synthetic solar-constant values of 1908 to 1920. To prepare the solar-contrast values for this use, means of the daily values are taken of every month given in Annals 4. Then, in order to eliminate systematic errors which might introduce inconsistencies, a separate

mean value is computed for the available months of each year, 1913 to 1920. Differences from these yearly means are given in column 2 of table 7. To make the synthetic solar-constant values entirely com-

Table 7.—Comparison of synthetic solar-constant departures with solar-contrast values of 1913-1920

Solar-constant departures in thousandths of a calorie.

Solar constant	Solar contrast
+17	+19
- 3	-32
-13	+14
+36	-35
+ 6	-24
-34	-18
-14	+28
+ 6	+49
+20	+10
0	—29
40	—75
10	0
+30	+ 4
-17	-18
+ 3	+15
- 7	+14
-14	-23
- 4	I2
+ 6	+ 8
+16	+16
+ 5	- 8
-15	-13
-15	+13
+25	+40
+ 3	+46
- 7	+18
+ 3	-7 0
+23	-13
- 4	+ 9
	1 2

parable to these contrast values, separate means of them are taken for each year of the comparison, including only the months used in obtaining the separate contrast means. Differences from these synthetic solar-constant means, expressed in thousandths of a calorie, form column I of table 7.

Counting the numbers of months when values in columns 1 and 2 have the same sign and opposite signs, the numbers (counting zero

values into each group) are 18 and 13, respectively. So here is another straw pointing to the reliability of the synthetic solar-constant values. But more convincing, and more informing, is figure 6. Here the

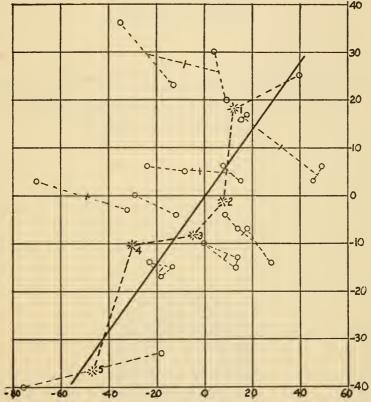


Fig. 6.—Mount Wilson solar contrast (abscissae) vs. synthetic solar constants (ordinates).

values in the columns of table 7 are plotted against each other, solar constants as ordinates, solar contrasts as abscissae. In order to bring out plainly the fact that higher contrast values attend higher synthetic solar-constant values, stars 1, 2, 3, 4, 5, have been plotted to give the centers of gravity of groups of 8, 8, 5, 5, and 2 months, respectively. A full heavy line has been drawn to show the trend of the results.