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THE SUNSPOT PERIOD

(WITH ONE PLATE)

BY
H. HELM CLAYTON

(PUBLICATION 3526)

CITY OF WASHINGTON
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PREFACE

The objects of this paper are:

1. To show that the smoothed plus and minus annual departures from normal pressure observed in the earth's atmosphere are displaced in position in unison with variations in intensity of sunspot maxima.

2. To show that the annual sunspot numbers may be resolved into a number of regular periods of constant length and amplitude which, when combined, make the irregular sunspot period and permit forecasting, with reasonable accuracy, the dates of maxima and minima of sunspots and the intensity of the maxima.

ATMOSPHERIC PRESSURE AND THE SUNSPOT PERIOD

Atmospheric pressure is subject to large fluctuations of short period, especially in high latitudes. In order to study long-period oscillations like those of sunspot numbers, it is necessary to smooth out the short-period fluctuations, just as it is necessary to smooth out the wind-driven waves in the ocean in order to study the tides.

This smoothing is frequently done by numerical averaging. Such averages block out the oscillations of short period but do not eliminate long-period trends in the data nor oscillations much longer than the ones it is desired to study. Smoothing by harmonic formulas is therefore believed to be the best method, because by such smoothing one can eliminate oscillations which are much longer as well as those which are much shorter than those which it is desired to study.

The type of harmonic formula used for this purpose is as follows: Let $l_0, l_1, l_2, l_3 \ldots\ldots l_{n-1}$ be observed values which are associated with equidistant values of some argument, say time; then the single periodic terms, namely, coefficients of a sine curve drawn through the observations, may be represented by the trigonometrical formulas:

$$L = A_0 + A_1 \cos \phi + B_1 \sin \phi, \tag{1}$$

in which

$$A_0 = \frac{\Sigma l}{n}, \tag{2}$$

$$A_1 = \frac{\Sigma l \cos \phi}{\frac{1}{2} n}, \tag{3}$$

$$B_1 = \frac{\Sigma l \sin \phi}{\frac{1}{2} n}, \tag{4}$$

$$\frac{A_1}{B_1} = \tan \theta, \tag{5}$$

$$a = \sqrt{A_1^2 + B_1^2} = \frac{A_1}{\sin \theta} \tag{6}$$

$$\phi = \frac{360°}{n}; \tag{7}$$

where θ=angle of the epoch, namely, the angular distance from zero to the part of the sine curve at the beginning of the period, and a= amplitude, while n=number of terms used.

The method of computation is shown in table 1. In this table the normal monthly temperatures at New York, derived from 50 years of observations, are used, and the coefficients of a sine curve passing through them are computed. From these coefficients, monthly values are then computed and are given at the bottom of the table. It is seen that these differ very little from the observed values, showing that these observed values follow very nearly a sine curve.

The computed values for each month may, however, be obtained in a different way, as shown in table 2. In this table the normal monthly temperatures at New York are multiplied by the cosine values given in column 3 of table 1. The cosine values are slipped down 1 month at a time, and the sum of the products in each case divided by 6 gives the value on a sine curve for the month in which the cosine value is unity.

For example, in the first column of products the cosine is unity in January and the sum of all the products divided by 6 is −21.7, the same as the computed value in table 1 when A_0=0. In the second column of products the cosine unity is placed in February, and the sum of the products divided by 6 is −20.7; and so on successively for each month. These are nearly identical with the computed values in table 1 when $A_0 = 0$. The small differences that exist are accounted for by the fact that the cosine factors were only taken to two or three decimals instead of to four or more. The successive values are thus equivalent to those of moving means except that the smoothing is done by harmonic terms instead of numerically.

TABLE 1.—*Example computation by harmonic formula*

Cycle of 360° divided into 12 parts (1)	Sine values (2)	Cosine values (3)	Normal monthly temperatures, New York [a] (4)	(5) °F.	Temperatures— By sine values (6) °F.	Temperatures— By cosine values (7) °F.
0°	0.00	1.00	January	30.6	0.0	30.6
30°	0.50	0.866	February	30.5	15.3	26.4
60°	0.866	0.50	March	38.0	32.9	19.0
90°	1.00	0.00	April	48.5	48.5	0.0
120°	0.866	−0.50	May	59.4	51.4	−29.7
150°	0.50	−0.866	June	68.5	34.3	−59.3
180°	0.00	−1.00	July	73.5	0.0	−73.5
210°	−0.50	−0.866	August	72.1	−36.1	−62.4
240°	−0.866	−0.50	September	66.4	−57.5	−33.2
270°	−1.00	−0.00	October	55.8	−55.8	0.0
300°	−0.866	0.50	November	44.1	−38.2	22.1
330°	−0.50	0.866	December	34.3	−17.2	29.7
Sum	0.00	0.00		...	−22.4	−130.3

Monthly values computed from θ and a

	Jan.	Feb.	Mar.	Apr.	May	June
For $A_0 = 0$	−21.7	−20.7	−14.1	−3.8	7.6	17.0
For $A_0 = 51.8$	30.1	31.1	37.7	48.0	59.4	68.8
	July	**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**
For $A_0 = 0$	21.7	20.7	14.1	3.8	−7.6	−17.0
For $A_0 = 51.8$	73.5	72.5	65.9	55.6	44.2	34.8

[a] Mean of 51 years, 1873-1923.

$a = \frac{1}{6}\sqrt{(22.4)^2 + (130.3)^2} = 22.04$; $\tan\theta = \frac{130.3}{22.4} = 5.81$; $\theta = 260°$.

θ = epoch; a = amplitude; $A_0 = 51.8$ = Mean for year.

The same results are obtained by the correlation formulas as is shown by the computations in table 3.

A cosine curve has the form exhibited in figure 1. Observed data are multiplied by a cosine series representing such a curve and the process repeated step by step, adding one unit of time and dropping one. If the length of the cosine series is near the length of any period which may exist in the observed data, that period will stand out prominently. The process eliminates periods of much smaller and

TABLE 2.—*Temperatures multiplied by cosine values*

	Normal monthly temperatures at New York [a]	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
January	30.6	30.6	...	...	...	...	...	...	...	...	...	...	...
February	30.5	26.4	30.5	...	...	...	...	...	...	...	...	...	...
March	38.0	19.0	32.9	38.0	...	...	...	...	...	...	...	...	...
April	48.5	0.0	24.3	42.0	48.5	...	...	...	...	...	...	...	...
May	59.4	−29.7	0.0	29.7	51.4	59.4	...	...	...	...	...	...	...
June	68.5	−59.3	−34.3	0.0	34.3	59.3	68.5	...	...	...	...	...	...
July	73.5	−73.5	−63.7	−36.8	0.0	36.8	63.7	73.5	...	...	...	...	...
August	72.1	−62.4	−72.1	−62.4	−36.1	0.0	36.1	62.4	72.1	...	...	...	...
September	66.4	−33.2	−57.5	−66.4	−57.5	−33.2	0.0	33.2	57.6	66.4	...	...	...
October	55.8	0.0	−27.9	−48.3	−55.8	−48.3	−27.9	0.0	27.9	48.3	55.8	...	...
November	44.1	22.1	0.0	−22.1	−38.2	−44.1	−38.2	−22.1	0.0	22.1	38.1	44.1	...
December	34.3	29.7	17.2	0.0	−17.2	−29.7	−34.3	−29.7	−17.2	0.0	17.2	29.7	34.3
January	30.6	...	26.5	15.3	0.0	−15.3	−26.5	−30.6	−26.5	−15.3	0.0	15.3	26.5
February	30.5	...	...	26.4	15.3	0.0	−15.3	−26.4	−30.5	−26.4	−15.3	0.0	15.3
March	38.0	...	...	...	32.9	19.0	0.0	−19.0	−32.9	−38.0	−32.9	−19.0	0.0
April	48.5	...	...	...	...	42.0	24.3	0.0	−24.3	−42.0	−48.5	−42.0	−24.3
May	59.4	...	...	...	...	...	51.4	29.7	0.0	−29.7	−51.4	−59.4	−51.4
June	68.5	...	...	...	...	...	...	59.3	34.3	0.0	−34.3	−59.3	−68.5
July	73.5	...	...	...	...	...	...	...	63.7	36.8	0.0	−36.8	−63.7
August	72.1	...	...	...	...	...	...	...	...	62.4	36.1	0.0	−36.1
September	66.4	...	...	...	...	...	...	...	...	...	57.5	33.2	0.0
October	55.8	...	...	...	...	...	...	...	...	...	...	48.3	27.9
November	44.1	...	...	...	...	...	...	...	...	...	...	...	38.2
December	34.3	...	...	...	...	...	...	...	...	...	...	...	...
Sums	...	−130.3	−124.3	−84.6	−22.4	45.9	101.8	130.3	124.2	84.6	22.3	−45.9	−101.8
Means, $\frac{1}{6}$ of sums	...	−21.7	−20.7	−14.1	−3.7	7.6	17.0	21.7	20.7	14.1	3.7	−7.6	−17.0
Observed	...	−21.2	−21.3	−13.8	−3.3	7.6	16.7	21.7	20.3	14.6	4.0	−7.7	−17.5

[a] Averages of 51 years, 1873-1923.

TABLE 3.—*Harmonic terms computed by correlations*

(1) Month	(2) x = cosine values	(3) y = temperature departures	(4) xy = product	(5) x^2	(6) y^2
July	1.00	21.7	21.7	1.00	471
August	.866	20.3	17.6	.75	412
September	.50	14.6	7.3	.25	213
October	.00	4.0	.0	.00	16
November	−.50	−7.7	3.8	.25	136
December	−.866	−17.5	15.1	.75	306
January	−1.00	−21.2	21.2	1.00	449
February	−.866	−21.3	18.4	.75	454
March	−.50	−13.8	6.9	.25	190
April	.00	−3.3	.0	.00	44
May	.50	7.6	3.8	.25	134
June	.866	16.7	14.5	.75	279
Sums	.00	.0	130.3	6.00	3104

$$\text{Correlation coefficient } r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 . \Sigma y^2}} = \frac{130.3}{\sqrt{6 \times 3104}} = \frac{130.3}{136.8} = 0.95$$

$$a = \frac{\Sigma xy}{\Sigma x^2} = \frac{130.3}{6} = 21.7$$

NOTE.—a = ratio of the observed values to a cosine series having plus unity in July and minus unity in January.

much greater lengths than the cosine series and shows no consistent period of any length when treating random data.

For example, if we multiply the monthly mean temperatures at New York City by a cosine series of 12 terms, beginning with any

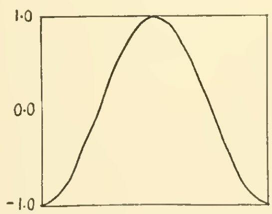


FIG. 1.—A cosine series.

given month, and repeat the process month by month, adding 1 month and dropping 1, taking the mean values in each case, the results will show regular periodic oscillations of 12 months in length. The mean is obtained by dividing the successive sums by $\frac{1}{2}n$, n being the number of terms used. The time is taken at the middle of the series. Figure 2 shows a plot of the annual period in temperature at New York obtained in that manner from 1927-1929. This plot shows that the annual period in temperature at New York is approximately constant in phase from year to year and nearly constant in amplitude.

If next a trial for shorter periods is made, it is found that when a cosine series of 24 terms is multiplied by the observed hourly temperatures, a period of 24 hours stands out prominently. Figure 3 shows the daily period in temperature obtained from hourly observations of temperature at Blue Hill Observatory, near Boston, Mass. This plot shows that the maxima and also the minima of temperature

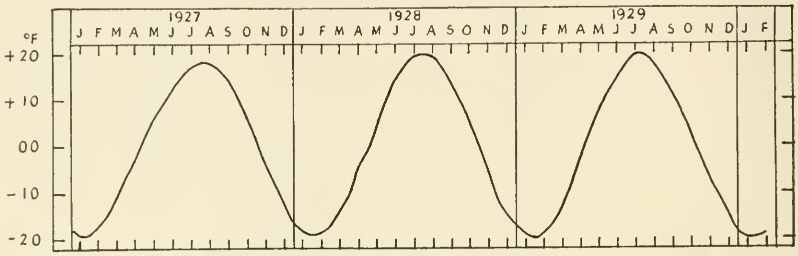


FIG. 2.—Monthly temperatures at New York, smoothed harmonically.

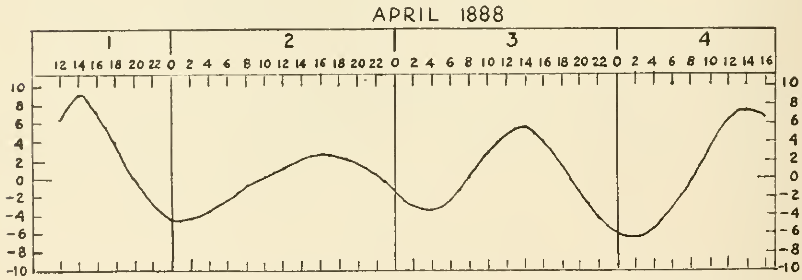


FIG. 3.—Hourly temperatures at Blue Hill, smoothed harmonically.

occur near the same hours every day, but that the period is very variable in amplitude. Further research shows that this period occurs only in the lower atmosphere and that the oscillations are large when the sky is clear and small when there are dense clouds. Hence, its variability in amplitude is closely related to moisture and cloudiness.

Yearly means of atmospheric pressure, when treated in the same manner, show clearly defined oscillations of 10 to 12 years. In figure 4, the result of analyzing the pressure at Stykkisholm, Iceland, with a 12-term series is shown by a continuous line and is compared with a plot of yearly sunspot numbers shown by a dotted line.

The sunspot curve shows a fairly regular period which averages about 11 years, but varies in length and amplitude.

The curve for Stykkisholm is an interesting type. In 1870 and again in 1917, it shows maxima which coincide with the sunspot maxima while from 1875 to 1909 the oscillations are the reverse of the sunspot oscillations. Here then is a period whose maxima and minima coincide approximately in time with those of the sunspots but invert in phase from time to time. In the Bulletin of the American Meteorological Society for July 1938, page 218, it was shown that this inversion in phase was caused by a change in latitude of the departures from normal pressure with an increase of solar activity (see fig. 5). To obtain the lines of equal departure shown on these charts the pressures at more than 200 stations were smoothed in the same way as those of Stykkisholm.

However, the atmosphere does not oscillate back and forth between the centers of plus and minus departures with a fixed zero line

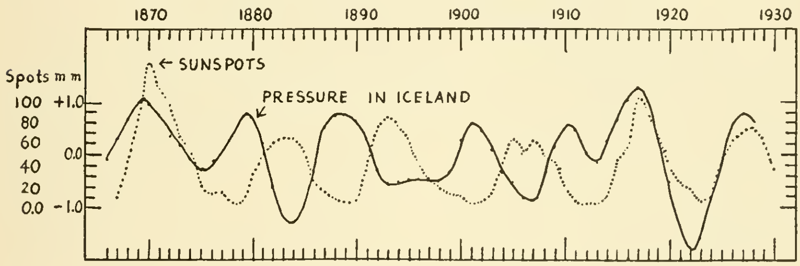


FIG. 4.—Smoothed annual pressures and sunspot numbers.

between them, as might be thought on examining the charts in figure 5, but there occurs a gradual shift of the centers of excess and defect of pressure. This process is illustrated by the curves in figures 6 and 7. It is seen from the first series of curves in figure 6 that, at the time of sunspot maximum in 1917, a barometric maximum existed in high latitudes over the North Atlantic, as shown by the pressures at Jacobshavn and at Stykkisholm. At stations farther south this maximum occurred successively later, until in the latitude of Madeira the pressure oscillated nearly inversely to that of the sunspots. The same conditions apparently occurred in the North Pacific, as shown by the second series of curves in figure 6. The stations are situated near the Pacific Coast of North America. At Tanana the maximum of pressure occurred nearly at the time of sunspot maximum, but came successively later at lower latitudes, until at San Diego the curve is nearly inverse to the sunspot curve.

On the other hand, over low latitudes on or near the central continental masses of North America and Asia a minimum of pressure

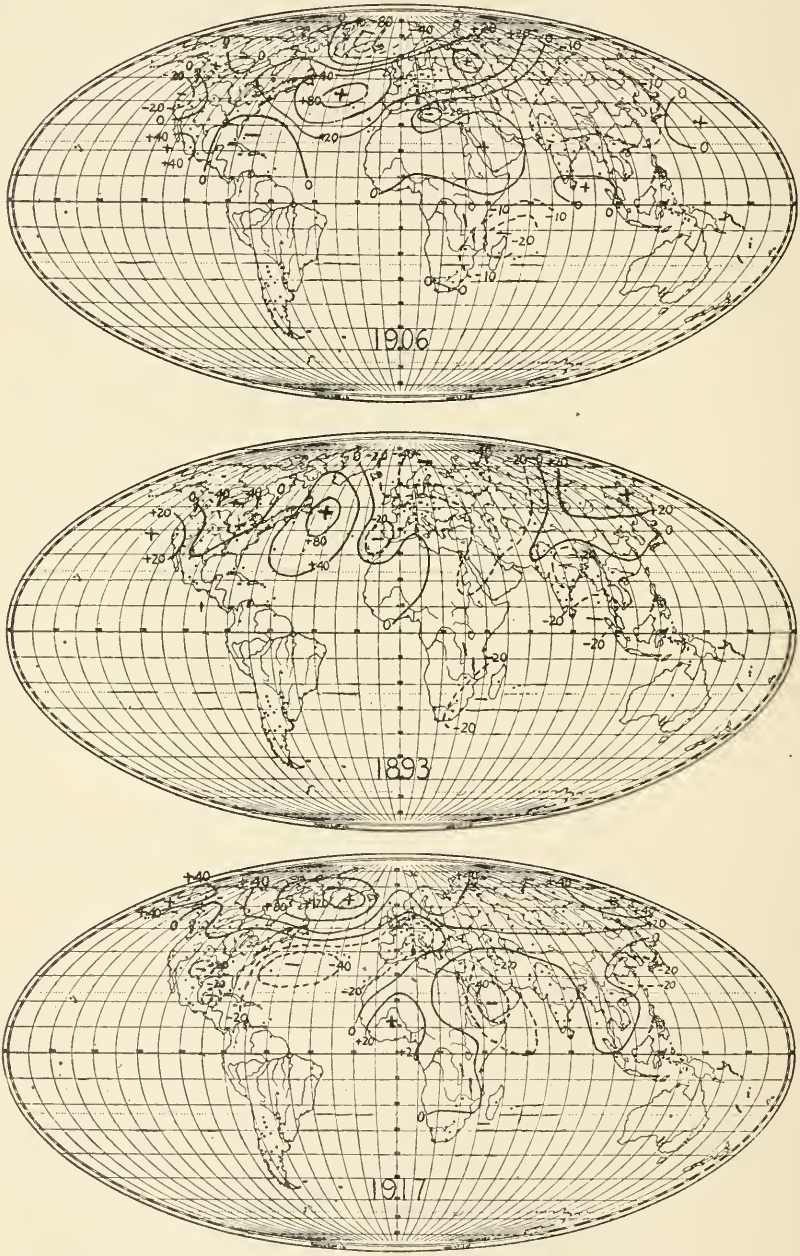


FIG. 5.—Atmospheric pressure at maxima of sunspots, 1906, 1893, 1917. (Units 0.01 mm.)

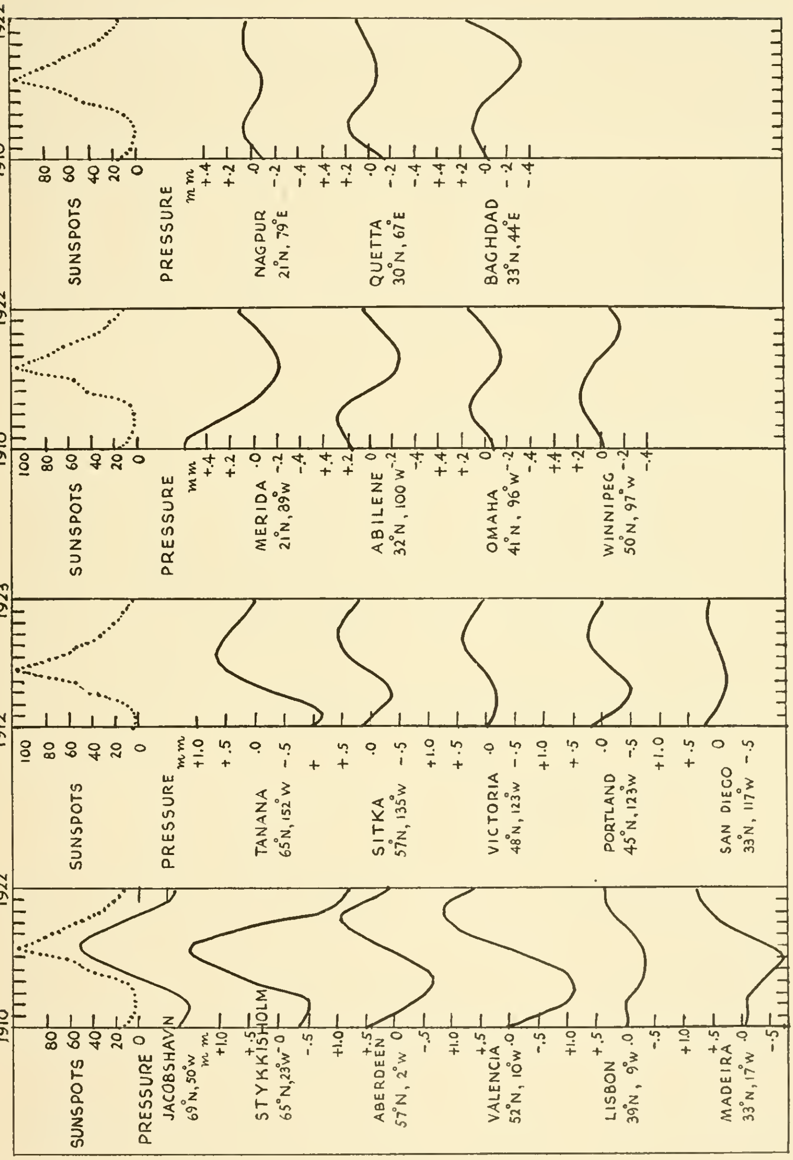


FIG. 6.—Sunspot maximum of 1917 and atmospheric pressure.

occurred about the time of sunspot maximum and apparently moved northward, at least as far as latitude 40°–50°. This condition is shown by the third and fourth series of curves in figure 6.

At the time of the less intense solar maximum of 1906 the maxima of pressure were found in latitudes 40°–50° N. over the oceans as shown in figure 5 and apparently moved from west to east, figure 7.

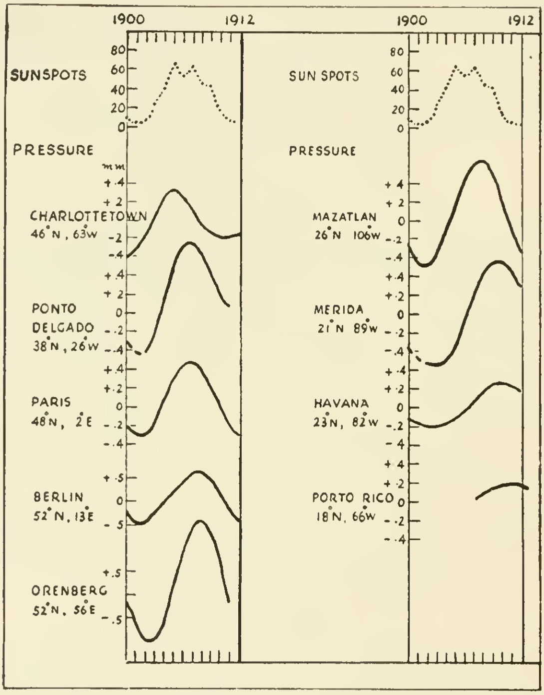


FIG. 7.—Sunspot maximum of 1906 and atmospheric pressure.

Hence, not only the positions of the areas of excess and defect of pressure are determined by the intensity of solar activity, but also the direction of the subsequent movement of these areas. The names *baroplions* and *baromions* have been suggested for these areas.

A cosine series used in the way described above does not necessarily give a period of its own length. If the monthly mean temperatures at New York had been analyzed by a cosine series of 11 terms covering 11 months instead of 12, the resulting curve would have been exactly the same as that shown in figure 2, except that the amplitude would have been somewhat reduced. The same is true had the cosine series contained 13 terms covering 13 months instead of 12. In each case the period shown would have been exactly the 12-months' period which exists in nature. The ordinary process of harmonic analysis gives the elements of a fixed period which is assumed in advance, but the process described above gives any period which may exist in the data near the length of the assumed period. The true length of the period is determined by the average length of the interval between the maxima and minima shown by the analysis.

In the case of cosine analysis of random data the maxima and minima occur at irregular intervals and the average amplitude approxi-

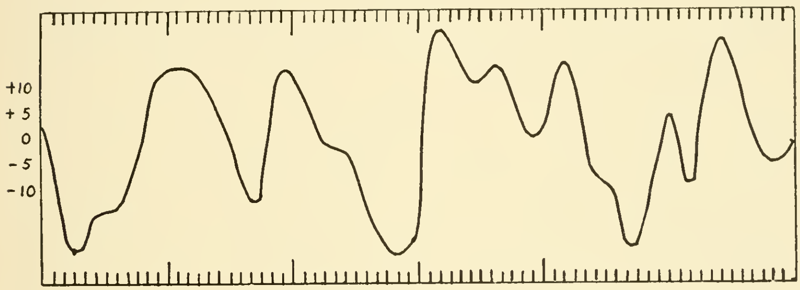


FIG. 8.—Some random data, smoothed harmonically.

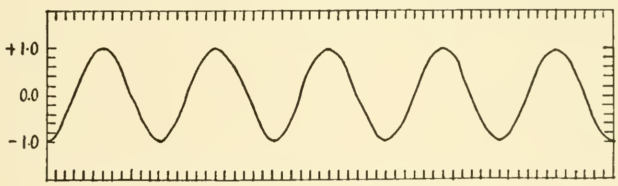


FIG. 9.—A sequence of 5 cosine curves.

mates zero. A plot of a series of random numbers smoothed harmonically is given in figure 8. These were treated by the same method as were the monthly temperatures at New York (see fig. 2) and the yearly pressures at Stykkisholm (see fig. 4). The curve in figure 8 shows large oscillations but no evidence of periodicity.

When evidence of periodicity is found it can usually be separated from random oscillations by obtaining means of three, five, or more successive periods.

In case the amplitudes of the periodic terms are relatively small, it is desirable, in order to bring out the periodic term clearly, to repeat the cosine series several times consecutively. When plotted, these successive series have the form of waves, as shown in figure 9.

Observed data are multiplied by such a series of 3, 5, or 10 waves, mean values are obtained, and the process is repeated by dropping one unit of time and adding another. In this way, whenever there is a period in the observed data near the period of the cosine waves, it is separated from chance variations and from periods of other length so as to stand out prominently in a plot of the results of the computations. The numerical work is like that shown in column 7 of table 1 repeated several times consecutively, or else the values are treated as shown in table 3. In this table column 1 contains the numbers which, when plotted, form a curve like that in figure 9. These numbers are then correlated with numbers representing sunspots, solar radiation, atmospheric pressure, or other physical quantity. If the correlation is sufficiently high, a period of the approximate length of the cosine

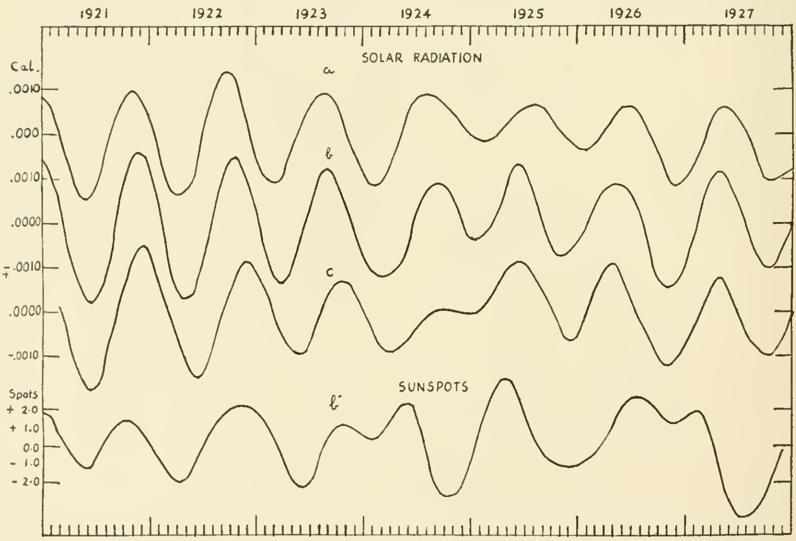


FIG. 10.—Eleven-month period in solar radiation and sunspots.

waves in column 1 is indicated. Then, if successive values of *a* (commonly called the regression coefficient) are computed, the numbers when plotted give curves like that shown in figures 10 and 11. In other words, whether the data are treated by the harmonic formulas or by the correlation formulas, the same results are obtained. Since the correlation formulas are in general use, it will perhaps be easier for most readers to understand the problem as one of correlation. Physicists to whom I presented the method understood it better in that way.

The monthly mean values of solar radiation obtained by the Astrophysical Observatory of the Smithsonian Institution were treated in this manner, and the results are plotted in three curves *a, b,* and *c* in figure 10.

Curve *a* was obtained from 5 consecutive cosine waves, each covering 10 units of time, curve *b* from 5 consecutive cosine waves, each covering 11 units of time and curve *c* from 5 waves each covering 12 units of time.

Each curve shows recurrent maxima and minima at intervals of about 11 months. Hence, it is evident that the length of the cosine series does not have to be exactly of the same length as the hidden

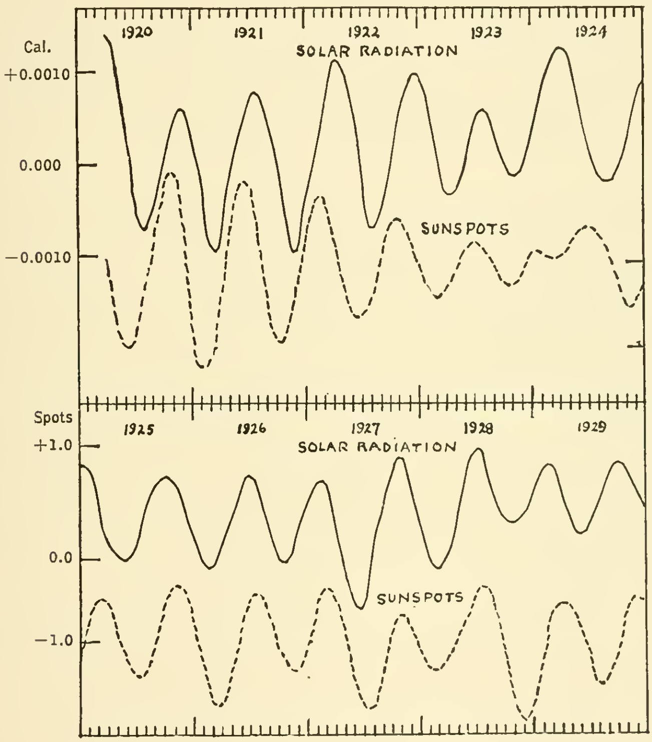


FIG. 11.—Eight-month period in solar radiation and sunspots.

period in order to disclose it, but only approximately of the same length. The cosine waves of 10 units of time do not produce a 10-months' period, and the series of 12 does not produce a 12-months' period.

When the monthly sunspot numbers from Zurich are treated in this same manner with an 11-term series, there are found oscillations similar in length and form to those found for solar radiation (see curve *b*′), but the oscillations are not quite so regular. The maxima of spots occur nearly at the same time as the maxima of solar radiation.

The monthly means of solar radiation and the monthly sunspot numbers were next multiplied by a cosine series of five waves each of eight terms; the resulting means, when plotted, give curves like those shown in figure 11. Both curves show a period of about 8 months during the years 1920-1929. The maxima in the two curves differ slightly in time, the solar radiation maxima being occasionally slightly behind and at other times slightly ahead of the spot maxima; but in the average the two are simultaneous, the greater amount of solar radiation coming at the same time as the maxima of spots.

These periods of about 11 months and 8 months are the same as those found by Dr. C. G. Abbot in the monthly means of solar radiation. In addition figures 10 and 11 show that there exist similar periods in the monthly sunspot numbers coinciding in phase with the solar radiation.

These periods are not exactly 11 months and 8 months in length. They appear to be about one-twelfth and one-sixteenth respectively of the sunspot period of 11.17 years or 134 months.

The analogy nearest to the process described is found in the phenomena of sound. Certain physical objects, as for example, a taut wire, respond only to sound waves of one rate of vibration or to waves near that length. If waves of increasing length or rates of vibration are sounded, the object will begin to respond with vibrations when the key note of the object is approached and will show strong response when the key note is reached. Changes in physical state may thus be studied; a glass tumbler will emit one key note when it is empty and a different note when it is partially filled with water.

In a similar manner changes in atmospheric conditions may be studied by a succession of cosine waves, such for example as the diurnal waves of temperature which have a large amplitude when the sky is clear and a lesser amplitude when it is cloudy.

The method of computation described here in the search for hidden periodicities is slow and tedious, but efforts are in progress to perfect a machine which will do the work more rapidly and more accurately. The machinery consists of a Coradi harmonic analyzer, and a Fergusson universal pantograph which permits changing the horizontal time scale to fit the analyzer without changing the vertical scale of quantity. In this manner any series of harmonic waves of whatever length may be mechanically integrated and the plot advanced step by step until the entire curve is covered.

Observed data are analyzed with this mechanism and the results, when plotted, disclose any hidden periodicity near the length of the cosine series of harmonic waves. Plate 1 shows a photograph of these instruments arranged for use.

Analysis by this method brings out the hidden periods both in length and amplitude and discloses any changes which take place in phase or amplitude.

ANALYSIS OF THE SUNSPOT NUMBERS

The sunspot period is one of those periods in nature which vary in length and in intensity. The question arises whether these variations are capable of being analyzed into a number of regular periods which can be extended into the future, as are the tidal fluctuations, or whether the sunspots are due to irregular explosions in the sun which cannot be resolved into regular periods.

To test this question, the sunspot numbers published at Zurich were subjected to an analysis by the method outlined in the preceding pages. The method was applied to the sunspot data in order to see whether they could be analyzed into regular periods of different lengths which could be used to predict the times and intensity of maxima and minima of sunspots. The results are very encouraging.

The first analysis indicated a fundamental sunspot period of 11.35 years which was modified by other periods, several of which tend to coincide with the fundamental period every 68 years.

In obtaining these periods, all the sunspot data from 1749 to 1936 were used. The data preceding 1793 were, however, meager and are given little weight by the directors of the Zurich Observatory, who are responsible for the collection. Beginning the analysis with the more trustworthy data in 1793, the fundamental sunspot period becomes 11.17 years.

The length and amplitude of the secondary periods in sunspots were determined in two ways, first, directly from the observed annual means; and, second, from the residuals after determining the average, or normal value, for each year of the 11.17-year fundamental period and subtracting these normals from the observed data, thus approximately or entirely eliminating the influence of that period.

These secondary periods have an amplitude much less than the fundamental period, but the amplitude increases as the length of the period approaches that of the fundamental period. The periods of 9.93 and 11.9 have amplitudes nearly half that of the fundamental period.

The length and amplitudes of the periods found were as follows:

TABLE 4.—*Sunspot periods by harmonic analysis*

Length	Amplitude
5.56 years	4 spots
8.12 "	6 "
8.94 "	10 "
9.93 "	13 "
11.17 "	35 "
11.90 "	15 "
14.89 "	9 "
19.86 "	4 "

It is believed that this method of analysis gives the lengths and amplitudes of these periods with more accuracy than has been attained heretofore, but it is of interest to compare the results with the periods derived by other methods of analysis by various research workers. This comparison is given in table 5.

TABLE 5.—*Length of sunspot periods in years found by various research workers*

A. Schuster (1906)	4.8	...	...	8.4	...	...	11.13	13.5	...	...
K. Stumpff (1928)	...	5.6	7.3	...	8.8	10.0	11.13	12.9	14.3	20.5
A. E. Douglass (1936)	...	...	...	...	8.5	10.0	11.4	13.5	14.3	...
D. Alter (1928)	...	...	7.6	8.1	8.7	10.0	11.37	...	14.0	21.0
H. H. Clayton (1938)	...	5.6	...	8.1	8.9	9.9	11.17	11.9	14.9	19.9

The analysis of Schuster and Stumpff was made by means of the Schuster periodogram, the analysis by Alter[1] was derived from the correlation periodogram, and the analysis by Douglass[2] was made with an ingenious instrument which he calls a cyclogram.

It is interesting to note how well these periods by different workers agree with each other, especially the periods between 8.1 and 14.9 years. The most marked discrepancy is in the periods of about 12 to 13.5 years, but here there are probably two periods, one of 11.9 years and another of about 13 years. It is also of interest to note that most of them are near submultiples of a period of about 89.36 years, a period which is shown in tables 6 and 7 and in figure 12 to give very closely the sunspot maxima and minima for more than 300 years. It seems clear that if there are a number of periods combining to make

[1] Alter, D., A new analysis of sunspot numbers. Monthly Weather Rev., vol. 56, p. 399, Oct. 1928.

[2] Douglass, A. E., A study of cycles, p. 129. Carnegie Inst. of Washington, 1936.

the yearly sunspot numbers, then these periods are approximately of the length and amplitude given in table 4.

From scattered observations of sunspots in the past three centuries a table was made at the Zurich Observatory by Wolf and Wolfer giving the approximate time of maxima and minima of spots from 1610 to the beginning of regular observations.

In order to test whether a period of 89.36 years could be used in determining the dates of these maxima and minima, going backward from observed maxima and minima between 1856 and 1933, tables 6 and 7 were constructed. Table 6 gives the computed and observed minima of spots, and table 7 gives the computed and observed maxima of spots.

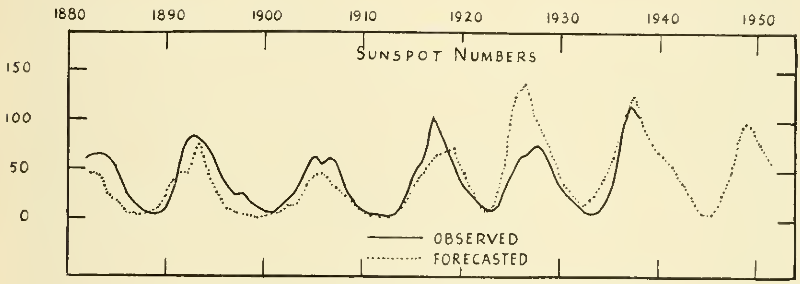


FIG. 12.—Sunspot numbers forecasted and observed.

Only a few of Wolfer's estimated dates of minima and maxima of sunspots from 1610 to 1850 differ greatly from the computed values given in the table, and these few were not assigned a weight exceeding 2 on a scale of 10 in Wolfer's table.

The next step was to try how nearly the 89.36-year period could be used in forecasting recent sunspot periods. Beginning with the minimum of 1798, for which a weight of 8 on a scale of 10 was assigned in Wolfer's table, the sunspot numbers were projected forward 89.36 years in time and compared with observed values. This comparison is given in figure 12. It will be seen from this figure that the projected curve, shown by the dotted line, gives a good forecast of the dates of observed maxima and minima of sunspots, shown by the full curve, for the interval of 55 years, 1883-1937, and gives an approximation to the intensity of the maxima, excepting the one in 1928. From this showing it is a reasonable inference that a similar accuracy can be obtained for 89.4 years in the future.

The dotted curve is projected forward to 1950 and indicates the next minimum of sunspots in 1945 and the next maximum in 1949.

This forecast is evidently more accurate than simply projecting each maximum and minimum 11 years ahead as it occurs.

A still more accurate forecast probably can be made by a separate combination of all the periods. This combination will be tried when the amplitude and phase of each period are determined with the greatest accuracy possible from existing data.

TABLE 6.—*Sunspot minima*

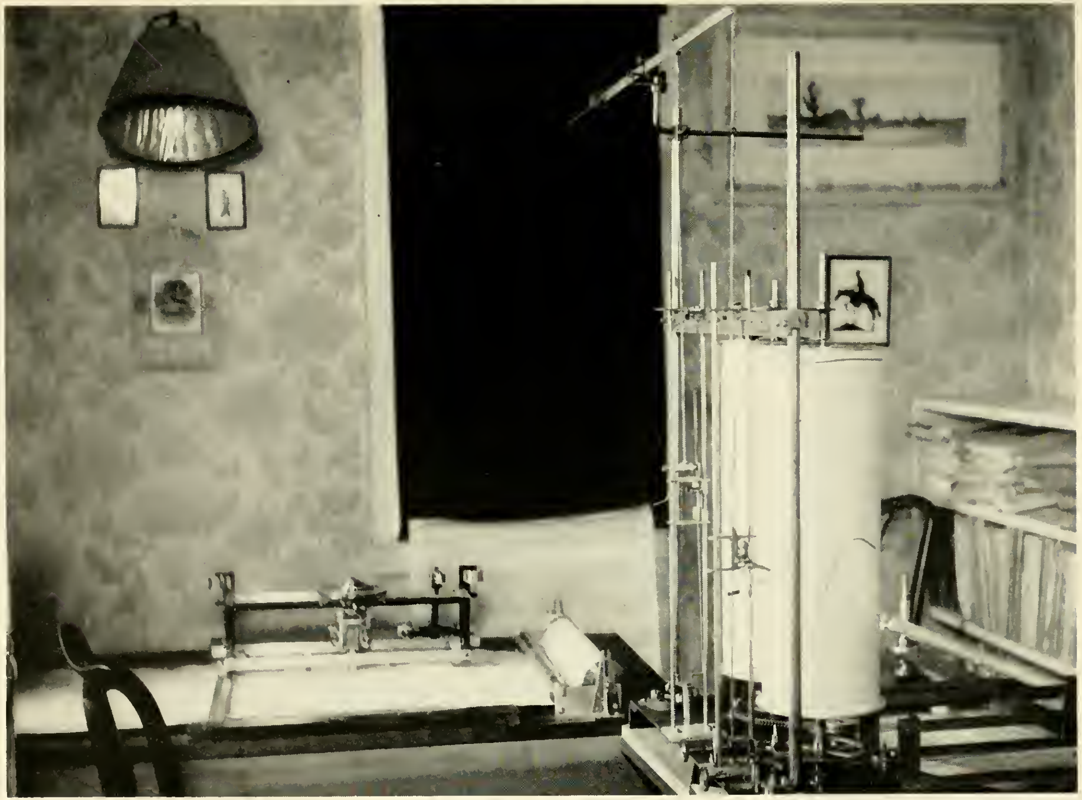
Observed minima	Computed			Wolfer's estimated dates of occurrence		
	89.36 years	178.72 years	268.08 years			
Date	Date	Date	Date	Date	Date	Date
1933.8	1844.4	1755.1	1665.7	1843.5	1755.2	1666.0
1923.6	1834.2	1744.9	1655.5	1833.9	1745.0	1655.0
1913.6	1824.2	1734.9	1645.5	1823.3	1734.0	1645.0
1901.7	1812.3	1723.0	1633.6	1810.6	1723.5	1634.0
1889.6	1800.2	1710.9	1621.5	1798.3	1712.0	1619.0
1878.9	1789.5	1700.2	1610.8	1784.7	1698.0	1610.8
1867.2	1777.8	1688.5	.....	1775.5	1689.5	.....
1856.0	1766.6	1677.3	.....	1766.5	1679.5	.....

TABLE 7.—*Sunspot maxima*

Observed maxima	Computed			Wolfer's estimated dates of occurrence		
	89.36 years	178.72 years	268.08 years			
Date	Date	Date	Date	Date	Date	Date
1928.4	1839.0	1749.7	1660.3	1837.2	1750.3	1660.0
1917.6	1828.2	1738.9	1649.5	1829.9	1738.7	1649.0
1906.4	1817.0	1727.7	1638.3	1816.4	1727.5	1639.5
1894.1	1804.7	1715.4	1626.0	1805.2	1718.2	1626.0
1883.9	1794.5	1705.2	1615.8	1788.1	1705.5	1615.5
1870.6	1781.2	1691.9	.....	1778.4	1693.0	.....
1860.1	1770.7	1681.4	.....	1769.7	1685.0	.....
1848.1	1758.7	1669.4	.....	1761.5	1675.0	.....

In view of the fact that the sunspot period is becoming important in numerous fields of scientific activity, much significance attaches to the forecasting of the time and intensity of the maxima and minima.

Prof. S. P. Fergusson made the necessary adjustments and modifications of the instruments illustrated in plate 1, and Miss M. I. Robinson assisted in the computations needed in the preparation of this treatise.



ANALYZER AND PANTOGRAPH