# THE DETERMINATION OF METEOR-ORBITS IN THE SOLAR SYSTEM

BY

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(Authorized Translation by Cleveland Abbe)

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## GENERAL THEORY

The orbits of those bodies which, by their entrance into the atmosphere, become incandescent meteors, have a planetary character in the solar system. They are conic-sections with the sun as one focus. That portion of the path revealed to us by observations is so short, that it must be assumed to be a straight line even in the most accurate determinations. Irregular movements that are conditioned by the shape of the body occasionally occur, but are here left out of consideration.

For all orbits that thus become visible, the smallest distance from the earth's center, or in geocentric phraseology, the perigee, is very small in comparison with the distance from the sun or the corresponding radius vector of the orbit of the earth, and can be neglected in comparison therewith. Hence, for the duration of fall the heliocentric radius vector of the meteor's path will be assumed as identical with that of the earth's orbit. Furthermore, since for the same reason, the collisions with the earth can be considered as identical with the passage of the meteor through one of the nodes of its path, therefore from the solar ephemeris for the date of collision we find directly the longitude and the radius vector of that node of its path.

The apparent location of the visible path of the meteor in the sky, depends parallactically on the location of the station of observation. It will, therefore, for the near-by meteors show greater apparent dislocation than for those more distant. Any point of the rectilinear path or its prolongation backwards, that is so far removed from all observing stations that their respective mutual distances no longer come into consideration, will be seen at the same point in the sky from all stations. This perspective point of divergence of the apparent paths instead of the true paths as seen from different places on the earth is the point of apparent radiation or convergence. This deter-

mines the geocentric direction of the observed rectilinear portion of the path. This is, however, except for the influence of the attraction of the earth, which can easily be allowed for, determined by the resultant of the velocity of the earth reversed and the true helio-centric velocity of the meteor. The projective direction of the latter motion as seen on the sky is the true radiation point or radiant which thus determines the true direction of the path of the meteor at the node.

Since the sun must lie in the plane of the orbit of the meteor this plane is determined by the radius vector to the sun and the direction of the orbit at its node, or by the great circle on the sphere that contains the points representing the sun and the true radiant.

But this latter also gives the angle that the true direction of the path makes with the radius vector at the node and thus all further elements of the orbit depend upon the velocity at this portion of the orbit, which must be determined either from the observations or from theoretical considerations.

The problem now before us consists, therefore, of two very different portions sharply separated from each other, of which the first is a geocentric problem, the second concerns the heliocentric conditions. These portions are:

1. The determination of the radiant point and of those quantities which are necessary for estimating the relative velocity.

2. The location of the orbit in the solar system.

With reference to the first part which contains the more difficult and complicated work, but whose results must be decisive as to the further results of the second part, the starting point and the aim can be different. The most important cases of the determination of the radiant are the following:

1. Examination of the path relative to the earth and of the radiant point by means of numerous observations of the same meteor from different places on the earth's surface (corresponding observations in the most general sense of the word).

2. Determination of the radiant by observations of several apparent paths belonging to the same radiant and seen from one station (one-place observations).

## I. DETERMINATION OF THE RADIANT AND THE GEOCENTRIC VELOCITY

### 1. NUMEROUS OBSERVATIONS AT DIFFERENT PLACES

Abundant experiences teach us that, depending upon the location of the orbit with reference to each individual place of observation,

the beginning of the observed luminous streak is ordinarily not the same as observed everywhere, but that in this respect differences up to large angular arcs and up to hundreds of kilometers in linear measure can occur. We must except the cases with long enduring, sharply marked traces when the observations relate especially to these and not to the whole orbit.

In general the orbit may not be deduced by assuming the identity of the indicated initial points of visibility.

Since the true orbit does not differ appreciably from a straight line, the eye of the observer determines with it a plane which is a section or great circle of the celestial sphere. The so-called apparent orbits observed at different places can therefore be considered as arcs of great circles which, on being produced backwards (in the direction opposite to the course of the meteor), will indicate by their intersection the apparent radiant.

The determination of this apparent radiation point depends thus upon the fact that one intersection is given by many great circles that do not coincide but are given by the observations of the same meteor made at many places.

If now two sets of arcs of orbits are at hand the solution, whether by computation or by graphics, is so simple that it does not require further explanation. The problem is more complicated when numerous observations are to be united, and yet it is especially desirable, in fact important, to use a great excess of observations in order to determine the orbits of the great fire balls and meteorites for which we often have less accurate and partly also incomplete material. These are the typical cases to which the following explanations will principally relate.

The positions at which the planetary path of a large meteor is checked by the resistance of the air (the end of the orbit) is almost always so clearly shown by special phenomena that there is seldom any doubt as to the identity of this point. Ordinarily it is the one single point in the orbit of which this can be assumed with equal probability. Moreover, in reference to it, the near agreement among the observers is practically certain and occasionally the research is favored by observations of sound or even by the finding of pieces of fallen fragments of the meteorite.

The most accurate possible determination of the location and altitude of the end of the meteor's track forms in general a very important and generally indispensable preliminary to all further investigations.

There is no great geometrical difficulty in so solving the general problem that the three co-ordinates in space of the end-point, together with the spherical co-ordinates of the radiants, are obtained by the method of least squares from one system of equations. But such a solution, independently of the fact that it would be extremely tedious and have little importance, would only seldom lead to any useful result. It will therefore not be further considered here, but the determination of the end-point will be separately treated as a necessary preliminary work.

### A. DETERMINATION OF THE GEOGRAPHIC CO-ORDINATES AND THE ALTITUDE OF THE END-POINT

There are good reasons why it will generally be useful to divide this problem into two parts, determining first the geographical locations and afterwards the linear altitudes. In certain cases the general treatment of the problem is necessary or at least appropriate. In order to determine all three unknown quantities the observations of three appropriate elements, including one apparent altitude angle, must be furnished by at least two places. The other two elements may be directions or azimuths and it is generally best that this should be the case. Evidently three azimuths are not sufficient if we wish to find the linear altitude, but on the other hand, the problem can generally be solved with three apparent altitudes. The determination of the geographical location merely by means of parallax in altitude will indeed be the only possibility if the parallax in azimuth is not given, for example, if the locations of the places of observation fall in the same vertical plane as the terminal point of the meteor's path.

Sometimes the observed altitudes are so uncertain that the accuracy of the result is diminished by combining them with the observed directions. If we have reliable azimuths with a sufficiently accurate parallax, then it is preferable to determine the location of the terminal point from these alone, whereby we assume that the spheroidal earth can be replaced with sufficient accuracy by a sphere of corresponding radius. We can with advantage make use of the following approximate method.

At the locations $O_1$, $O_2$.... on the earth's surface specified by the geographic co-ordinates $L$ and $\phi$, let observations be made of the azimuth $A^*_{ie}$ of the terminal point $E$ ($L_e$, $\phi_e$) of the meteor's path.

First of all let approximate values $L_o$, $\phi_o$ of $L$, $\phi$ be found by some short, perhaps graphical, method, also let an approximate value $H_v$ for the linear altitude $H_e$ be sought out. Ordinarily these come, so to speak, of their own accord before the commencement of the real

calculations, since in the gradual carrying out of the observations an approximate judgment must be formed of the location of the terminal points. In all important cases that are worthy of a more thorough treatment of the observations a preliminary idea of these quantities will soon be obtained.

With these approximate values we now compute approximate azimuths from $O_i$ to $E$ which will be denoted here by $A^{\circ}_{ie}$ and also those for the readily deducible directions from $E$ to $O_i$ which we shall denote by $A^{\circ}_{ei}$. Furthermore, we find the approximate distances $O_iE$ or the spherical amplitudes $\epsilon_i$ and approximately the apparent altitudes $h_{ie}$. The latter influence this determination only in so far as even quite crude approximations suffice when we do not desire to use observed quantities.

Finally we must form the numerical values of all the differences $A^{\circ}_{ie} - A^{*}_{ie} = \omega_i$. If now we assume $L_e = L_o + \Delta L_o$, $\phi_e = \phi_o + \Delta \phi_o$, in which $\Delta L_o$ and $\Delta \phi_o$ are the corrections to be determined for the preliminary co-ordinates of the terminal point, then if $A_{ie}$ denotes the actual value of the azimuth we shall obtain from the equations

$$\tan \phi_e \cos \phi_i = \cot A_{ie} \sin(L_e - L_i) + \sin \phi_i \cos(L_e - L_i),$$
$$\tan \phi_o \cos \phi_i = \cot A^{\circ}_{ie} \sin(L_o - L_i) + \sin \phi_i \cos(L_o - L_i),$$
$$A_{ie} = A^{*}_{ie} + v_i \sec h_{ie},$$
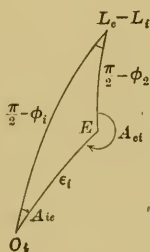


in the usual way, the following set of equations for the errors:

$$v_i = o_i + a_i x + b_i y, \tag{1}$$

where $\quad x = \Delta L_o \cos \phi_o, \quad y = \Delta \phi_o,$

$$\left.\begin{aligned} o_i &= \omega_i \cos h_{ie}, \\ a_i &= \frac{\cos h_{ie}}{\sin \epsilon_i} \cos A^{\circ}_{ei}, \\ b_i &= -\frac{\cos h_{ie}}{\sin \epsilon_i} \cdot \sin A^{\circ}_{ei} = -a_i \tan A^{\circ}_{ei}. \end{aligned}\right\} \tag{2}$$

The general factor $\cos h_{ie}$ is more important the more unequal the distances are, so also are the $h_{ie}$'s because the errors of direction

usually attendant on such observations are not properly differences of azimuth, but small spherical arcs which correspond to very different azimuths according to the zenith distance.

From the equations (1), allowing different weights to the individual observations according to the method of least squares, we get $\Delta L_o$ and $\Delta \phi_o$ and therefore also $L_e$ and $\phi_e$ with their mean errors.

The final azimuths are, therefore,

$$A_{ie} = A^*{}_{ie} + v_i \sec h_{ie}.$$

In case the $v_i$'s are too large it is to be recommended that $A_{ie}$ should be computed again directly from the final $L_e$ and $\phi_e$.

Of course we determine $x$ and $y$ in the units of the absolute term $o_i$. In this respect we can introduce various modifications according to circumstances.

If we designate by $\mu_i$ the spherical distance of the approximate terminal point $E_o(L_o, \phi_o)$ from the observed direction designated by $A^*{}_{ie}$, then we have approximately $\omega_i = \mu_i \operatorname{cosec} \epsilon_i$.

If we then put

$$\left.\begin{array}{c} \dfrac{\cos h_{ie}}{\sin \epsilon_i} = \surd \gamma_i, \\[2mm] \cos A^\circ{}_{ei} = a_i, \quad \sin A^\circ{}_{ei} = -b_i, \end{array}\right\} \tag{3}$$

we get

$$v_i = \surd \gamma_i (\mu_i + a_i x + b_i y), \tag{4}$$

and $\surd \gamma_i$ is a weight factor for the whole equation.

If we desire, for example, when reading from the chart for $\mu_i$, as our unit a measure of length which shall then also hold good for $x$ and $y$ while the $v$'s shall be given in degrees, so that in general the units may be taken to be one kilometer for altitude and one degree for azimuth, then the above-given equation for $\surd \gamma_i$ will still have to be multiplied by the following factor

$$\frac{1}{R \cdot \operatorname{arc} 1^\circ} = \frac{1}{111 \cdot 35} = 0.0089807,$$

where the earth's radius is assumed to be $R = 6{,}380$ km.

It is easy to see that the coefficients $a_i$ and $b_i$ can, therefore, with advantage, be read off graphically, especially when the observations do not deserve the most accurate computational methods.

If we also find it advisable to use the parallax in altitude in the determination of the location of the terminal point $(L_e, \phi_e)$, which as we have already mentioned, is scarcely avoidable, then the approximate value $H_o$ must be deduced more accurately and the consequent apparent altitude $h_{ie}$ corresponding to the preliminary distance $D_{ie}$ must be computed more carefully.

If $H_e = H_0 + \Delta H_0$, and $\Delta L_0 \cos \phi_0 = x$, $\Delta \phi_0 = y$, $\Delta H_0 = z$ in longitude measure, while the corrections of the observed directions and the observed altitudes $h_{ie}$ are given in degree measure and if we put

$$
\left.
\begin{aligned}
(\tan h_{ie} + \sin \epsilon_i) \cos \epsilon_i &= K_i, \\
\frac{\cos^2 h_{ic}}{R \cdot \text{arc } 1^\circ \cdot \sin \epsilon_i} &= \sqrt{\gamma_i} \cdot \cos h_{ic} = F_i, \\
h_{ie} - h_{ic} &= n_i, \\
-F_i K_i \sin A^\circ_{ei} &= a_i, \\
-F_i K_i \cos A^\circ_{ei} &= b_i, \\
F_i &= c_i,
\end{aligned}
\right\}
\tag{5}
$$

then the error equations which we determine from the directions become

$$
v_i = n_i + a_i x + b_i y + c_i z.
\tag{6}
$$

From these results again we obtain the normal equations and the unknown quantities.

This method should only be used with reliable altitudes and should only be applied when those deducted from stations in the neighborhood of the terminal point are to be compared with those further removed.

If we desire to determine the linear altitude of the terminal point separately on the basis of the geographical location of this point as already determined, then we usually combine the individual distances $D_{ic}$ of the corrected terminal point with the corresponding observed apparent altitudes $h_{ie}$ in order to obtain individual values of $H_e$ from which an average value may then be determined. The distances do not need to be newly computed when they are known from previous work with reference to the preliminary terminal point, since they can easily be corrected from the corrections already determined, viz.: $\Delta L_0 \cos \phi_0$ and $\Delta \phi_0$, and in fact with the help of the azimuth $A_{ie}$ they can be easily reduced to the definitive point $L_e$, $\phi_e$, for

$$
\Delta D = -\frac{\Delta L \cos \phi_0}{\sin A^\circ_{ie}} = -\frac{\Delta \phi_0}{\cos A^\circ_{ic}}.
$$

Since the reduction of the distance to the chord for such determinations of altitude is generally unnecessary, we can adopt

$$
H_i = D_{ie} \frac{\sin\left(h_i + \dfrac{\epsilon_i}{2}\right)}{\cos(h_i + \epsilon_i)}.
\tag{7}
$$

If we put $h_i + \dfrac{\epsilon_i}{2} = h_i'$ then up to 10,000 kilometers the approximate height will be

$$
H_i = D_{ie} \tan h_i' + \frac{1}{2R}(D_{ie} \tan h_i')^2,
\tag{8}
$$

even the first term is sufficient for several hundred kilometers. In consideration of the various uncertainties, the application of a correction for refraction will seldom be of any advantage, except that it may be done in cases of very large distances.

If we desire to consider the weight when we unite the individual determinations of the $H_i's$ to an average value, then we must consider carefully the weights of the observed values of the $h_i's$ that have been adopted, generally these weights cannot easily be estimated.

Large differences in the altitude above sea level of the individual places of observation are easily allowed for.

## B. THE DETERMINATION OF THE APPARENT POSITION OF THE RADIANT

The data hitherto useful can be given in different forms, viz.:

1. The co-ordinates ($A$, $h$ or $a$, $\delta$) of the beginning and end of the apparent path.

2. Any two other points of the path, or even one point only, when it does not lie too near the assumed terminal point already well established. To this also belongs the indication of the direction of motion by means of some distant stars.

3. The apparent inclination to the vertical of a portion of the orbit at the terminal point, or the position angle with respect to the vertical. This I think can also be given for any other well-established portion of the path. In the case of paths that have a culminating point, the apparent altitude of the culmination is often sufficient, but the azimuth less frequently. Very often it is possible to utilize a determination, that is better than a mere estimate, of the apparent node of the orbit at the horizon or the apparent direction of motion with reference to some point in the horizon.

The utilization of the items mentioned under this latter subdivision No. 3, assumes that the terminal point has already been determined from other data. Sometimes, for instance, in the case of very short paths, these data are more useful than the co-ordinates of the beginning and end of the path. In such cases graphic sketches of the observed path are always a desirable addendum.

At first from the known position and altitude of the terminal point its apparent equatorial co-ordinates for the individual places or observations are calculated and in case 1 are taken instead of the observed, in case 2 they are taken for the co-ordinates of the unspecified terminal point.

In the cases mentioned under item 3, these computed co-ordinates serve for the two ends of the given apparent arc of the orbit. If $N'$

is the inclination of the apparent orbit to the vertical through the terminal point $(A_e, h_e)$ in the incomplete observation as given, while $N$ is the inclination to the horizon at the apparent node of the orbit and $A_k$, the azimuth of this node, then we have

$$\left.\begin{aligned}\cos N &= \sin N' \cos h_e, \\ \tan f &= \tan N' \sin h_e, \\ A_k &= A_e \pm f,\end{aligned}\right\} \tag{9}$$

where the plus or minus sign is taken according as the movement of the meteor is in the direction of increasing azimuth or in the opposite direction.

In a similar way the relations are obtained when the angle with the vertical at any other point of the orbital path has been observed, and for which either the azimuth or altitude or distance from the terminal point along the path must be given.

In cases 1 and 2 the two given points on the path are computed from the co-ordinates $a_i'$, $\delta_i'$, and $a_i''$, $\delta_i''$, and then for the great circle thus determined is computed the right ascension $a_k$ of the ascending node on the equator, and the inclination $J$ to the equator.

With $\dfrac{\tan \delta'}{\tan \delta''} = s$, we have

$$\left.\begin{aligned}\tan (a' - a_k) &= \frac{s \sin (a'' - a')}{1 - s \cos (a'' - a')}, \\ \tan J &= \frac{\tan \delta'}{\sin (a' - a_k)} = \frac{\tan \delta''}{\sin (a'' - a_k)};\end{aligned}\right\} \tag{10}$$

$a_k$ must thus be chosen so that the two equations for $J$ are satisfied.

These determinations can be made rapidly and with sufficient accuracy by the use of a spherical net work or chart on the gnomonic projection and a correspondingly large scale.

This method is also to be used in the third case. Finally, every station of observation that furnishes usable data should employ a great circle drawn through $a_k$ and $J$ with the appropriate direction of motion to determine the radiant. If these apparent orbits were free from error, then by extending them backward, they would all intersect at the radiation point $(a, d)$, that is to say the equation

$$\sin (a - a_{ki}) \tan J_i = \tan d \tag{11}$$

would then be satisfied for each value of $i$ from $l$ to $n$.

But since this cannot be the case because of the errors of observation, we must apply corresponding corrections to the observed orbit paths. In doing this and ignoring certain exceptional cases, it will be assumed that the co-ordinates $a''$, $\delta''$ remain unchanged and there-

fore that the corrections belong only to $a'$, $\delta'$ (the initial point), or to $a_k$ and $J$, which is brought about by a rotation around the apparent terminal point, $a''$, $\delta''$. This rotation must be so made that

$$[ p(\Delta a'^2 \cdot \cos^2 \delta' + \Delta \delta'^2)]$$

is a minimum for the necessary corrections $\cos \delta_i' \cdot \Delta a_i'$ and $\Delta \delta_i'$, taking into consideration their respective weights $p_i$.

To this end, again just as in the case of the determination of the geographic co-ordinates of the terminal point, we proceed best by graphic methods. We adopt an approximate position for the point $(a, d)$ which we can here designate by $(a_0, d_0)$ and compute or graphically measure its normal distance $\Delta_i$ from each apparent orbital path $a_{ki}$, $J_i$.

If we regard $\Delta$ as positive, when $(a_0, d_0)$ lies within the northern polar region of $a_{ki}$, $J_i$, and if we wish to find this distance as correctly computed, we use the following formula:

$$\sin \Delta_i = - \cos J_i \sin d_0 + \sin J_i \cos d_0 \sin(a_0 - a_{ki}). \qquad (12)$$

Now since the $\Delta_i$'s represent discrepancies, the corrections

$$\Delta a_0 \cos d_0, \quad \Delta d_0$$

must be so determined that those shall be zero.

Putting

$$\left.\begin{array}{r} \sin J_i \cos(a_0 - a_{ki}) = \cos P_i, \\ \cos J_i \sec d_0 = \sin P_i', \\ \Delta a_0 \cos d_0 = x, \quad \Delta d_0 = y, \\ \sin \Delta_i = \Delta_i, \end{array}\right\} \qquad (13)$$

the condition for this is

$$o = \Delta_i - x \cos P_i + y \sin P_i'. \qquad (14)$$

In order to properly express the equations of error, we must still consider the fact that $\Delta$, or the change of location that the provisional radiant must experience in order to fall in with the observed orbital path, *is not an observed quantity* since the corrected place to be obtained by the rotation is $(a', \delta')$. Let $\zeta$ be the change that must be produced by the rotation, then we have

$$\frac{\sin \zeta}{\sin \Delta} \doteq \frac{\zeta}{\Delta} = \frac{\sin l}{\sin l'}, \qquad (15)$$

where $l$ denotes the length of the arc from $(a', \delta')$ to $(a'', \delta'')$, $l'$ that from $(a_0, d_0)$ to $(a'', \delta'')$.

Therefore the factor $\dfrac{\sin l}{\sin l'} = \sqrt{g}$ is to be applied to the whole equation as the weight. If we omit this, then the short orbital paths have

much too heavy a weight, since for these this ratio of the sines is a very small fraction.

If the apparent path be computed not by $a'$, $\delta'$, but by the apparent inclination at the terminal point, we shall come nearest to the correct values of the weights if we assume the numerator as unity.

It is best moreover as a matter of course in all special cases to apply the indicated weight of the observed quantities.

The method here explained is based, as already stated, on the assumption that the terminal point can be located with sufficient accuracy to justify the assumption that the apparent positions $a_i''$, $\delta_i''$ computed therefrom remain unchanged in determining the radiant. If this is not allowable, then the weight of the individual $a_i''$, $\delta_i''$'s must be deduced from the average error in the determination of the location and altitude of the terminal point, and then an estimate must be made of the weight to be given the first point $a_i'$, $\delta_i'$ of the orbit. The corrections for both points must then be determined as inversely proportional to the weights $p'$ and $p''$, that is to say, the rotation of the apparent orbit is not around the point $a''$, $\delta''$, but around that point in the orbit which is distant from this by the quantity $\dfrac{p'}{p'+p''} \cdot l$ and from $a'$, $\delta'$ by a quantity $\dfrac{p''}{p'+p''} \cdot l$. If $l'$ represents the spherical distance of this division point from the assumed radiant $a_0$, $d_0$ then the factor becomes

$$\sqrt{g} = \frac{\sin \dfrac{p''l}{p'+p''}}{\sin l''}. \tag{16}$$

If we cannot or do not wish to determine the terminal point of the orbit beforehand, then in general the two points $a'$, $\delta'$ and $a''$, $\delta''$ must receive equal weights so that we must assume [1]

$$\sqrt{g} = \frac{\sin \dfrac{l}{2}}{\sin l''}. \tag{17}$$

If the radiant is determined from the final co-ordinates $a$, $d$, then for each place of observation the arcs from $a$, $d$ to $a_i''$, $\delta_i''$ will indicate, according to their positions, the improved apparent orbit. The normal projections of the point $a'$, $\delta'$ on this arc afford us on one hand the improved apparent arc, and on the other hand the normal components of the errors of observation projected on the path. Those that lay in

[1] On the determination of the radiant from numerous observations without a previous determination of the terminal points, see also R. Lehmann Filhés " On the theory of the shooting stars, Berlin, 1878."

the direction of the path are of course indeterminate because of the inequality of the aspect. The correcting of $a'$, $\delta'$ according to these suggestions is easily done, but is only necessary when the problem is sufficiently trustworthy to allow it to be applied to the determination of the altitude of the initial visibility and of the real length of the path.

### C. THE LOCATION OF THE ORBIT RELATIVE TO THE EARTH ; THE LENGTH OF THE PATH ; THE ALTITUDE OF THE INITIAL VISIBILITY

If for the terminal point $(L_e, \phi_e)$, finally settled upon, the values for the azimuth and altitude of the radiant, referred to the horizon of the terminal point, are calculated from the equatorial co-ordinates of the determined radiant $(a, d)$ with the aid of the meteor's time of fall corresponding to this meridian, then the first corresponds also to the azimuth of the linear meteor path, and the latter corresponds to its inclination relative to the horizon of the terminal point. The azimuth, which specifies the projection of the orbit upon the surface of the earth, can be taken from the chart or computed from the places above whose zenith the meteor passed, and the inclination in connection with the altitude of the terminal point gives the linear altitude which the meteor had at any point in its orbit above the earth's surface. In general we can neither speak of the true length of the meteor's path in the atmosphere, nor of the altitude of the beginning, because as has been remarked, observers very frequently catch different phases of motion. Since the length of the linear path in connection with the estimated duration allows us to deduce approximately the geocentric velocity, therefore the real orbital length must be obtained especially for those observations that give estimates of the duration. The altitude of the first visibility can then always be assumed to be that which results from the longest well-observed path.

In order to determine these quantities, however, all data are given as soon as the above-mentioned projection of the true orbit on the earth's surface and the inclination are well established. For each of the places of observation the corrected position $(a', \delta')$ of the first visible point in the orbit will now come into consideration. The corresponding azimuth then gives us the initial direction in the trajectory of the orbit, whence by the use of the inclination of the orbit to the horizon we get both the corresponding length of the orbit and the altitude of the meteor above this point.

### D. THE GEOCENTRIC OR RELATIVE VELOCITY

If $L_i$ and $t_i$ are associated values of the length of the true orbit as computed from the observations and of the thence estimated

interval of time in seconds required for the passage along this portion, then we have

$$\mathfrak{B} = \frac{L_i}{t_i},$$

which is the average relative velocity of the meteor as deduced from this particular observation, but affected with the great uncertainty that the estimate of duration almost unavoidably brings with it. An average velocity, because we must assume that the velocity cannot remain quite constant during the motion through the atmospheric strata.

If many associated data of $L$ and $t$ are at hand, we have to carefully consider whether, say, a single apparently very reliable estimate of $t$ may be retained alone in connection with the value $L$ corresponding to this observation, or whether an average value should be deduced, but of course always only in reference to associated data. We do not recommend combining the greatest length of orbit deduced from the earliest apparent visibility with the average of all the estimates of duration as is sometimes done, because generally many estimates occur among these that relate only to very short bits of the path.

In general it is best to determine the velocity for each pair $(L, t)$ by themselves and then to take the average with or without giving attention to weight, but even the results of careful efforts to determine the weight are very often doubtful because of the frequent large overestimates of the duration, so that the sources of error are almost invariably on one side.

If there is no estimate of duration belonging to any of the segments of the path, then perhaps the average of the times may be combined with the average of all the $L$'s.

Frequently long paths, passing chiefly through the higher layers of the atmosphere and also having their ends at great altitudes, give larger values for the velocity than shorter paths recorded in the lower strata. The answer to the question whether such results also represent quantitatively the retarding influence of the resistance of the atmosphere, or whether they result rather in an overestimate of the duration, which overestimate alters the result for short paths more than for long paths, needs further special study. The fact that individual results of observations certainly refer to particular well-ascertained definite limited higher or lower located parts of the orbit is therefore always very important and consequently must be most thoroughly investigated.

### E. ZENITHAL ATTRACTION

The velocity and the radiant point must for further planetary computation be corrected for the influence of the earth's attraction which first increases and afterwards diminishes the approach to the zenith.

If $z$ is the zenith distance of the estimated apparent radiant at the terminal point $L_e$, $\phi_e$ and $v'$ the relative or geocentric velocity corrected for the attraction of the earth and $z'$ the zenith distance corrected in the same way, then will [1]

$$v' = \sqrt{\mathfrak{B}^2 - 2gR},$$

where $g$ is the acceleration due to gravity and

$$\left.\begin{aligned}
2gR &= 125 \cdot 18, \\
\tan \frac{\Delta_z}{2} &= \frac{\mathfrak{B} - v'}{\mathfrak{B} + v'} \tan \frac{z}{2}, \\
z' &= z + \Delta z,
\end{aligned}\right\} \qquad (18)$$

where all quantities are expressed in kilometers.

The azimuth of the radiant suffers no change on account of zenithal attraction. The equatorial co-ordinates of the radiant are, however, to be newly computed with the help of this azimuth and the corrected zenith distance. When these are used in the following text for the computation of the planetary orbit, they will be designated by $a'$ and $d'$.

### 2. THE OBSERVATIONS OF DIFFERENT METEORS BELONGING TO THE SAME STREAM FROM ONE SINGLE LOCATION ON THE EARTH

When many apparent paths belonging to the same radiant are at hand which have been observed at only one place and within an interval that is not too long, and when the changes in the co-ordinates of the radiant during this interval are unimportant,[2] or have been taken into consideration, then these co-ordinates can also again be determined according to the fundamental theorem that the paths represent the most probable intersections of the respective great

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[1] Schiaparelli, page 251.

[2] The zenith attraction causes a larger or smaller appreciable change in the location of the radiant together with the change in the zenith distance. This may be quite notable in the case of meteors that strike the earth with small velocity. Therefore we should combine into one determination only such observations as those that are not too far apart with respect to zenith distance of the radiant. In general, however, the dislocation in the position of the path increases with the solar longitude, but remains for several days together within the limits that have been indicated by the error of observation. The culmination of observations that cover a period of several weeks can in general only furnish approximate results. [Note.—Such results are of absolutely no value. C. P. Olivier.]

circles.   On the other hand, we have, however, never absolute certainty that the paths taken into consideration do actually belong to the same radiant, wherefore the application of the rigorous method loses in value.

In the case of observations made at one place the geographic location and the altitude of the terminal point of a path cannot be given. Hence we do not have the correction to the terminal position $a''$, $\delta''$ referred to in the appropriate portion of the previous section, which terminal position, therefore, is only of about the same value as $a'$, $\delta'$. Hence it is natural to distribute the corrections uniformly and to undertake the necessary rotation of the arc about the center of the orbit.   The factor for the equation of error will therefore be $\dfrac{\sin\dfrac{l}{2}}{\sin l'}$ where $l'$ denotes the distance of the center of the orbit from the preliminary radiant point.   Otherwise we may apply the same method as in the other case and indeed by the ordinary preferable graphic determination in which the observations are entered on squared paper.

R. Lehmann-Filhés[1] develops the method of deducing the apparent radiant for star shower observations made at one place, from the apparent paths each of which is determined by some point $a$, $\delta$ in the path and its position angle relative to the declination circle through this point.   If the observer has a free choice of this point, then he will frequently fix it more accurately in the neighborhood of a star that is well known to him, and if the attention is not too much taken up by the exact locating of the beginning and ending of the path, then the direction of the path and thus also of the angular position is probably more accurately known than by locating each of the two points which, in observations made at one place, can only be approximately located, if in general we desire to consider them at all.

Equations 14 to 17, in which occurs the position angle $P$ relative to the hour circle of the preliminary radiant $a_0$, $d_0$ and therefore not that determined for any other arbitrary point $a$, $\delta$, can very easily be applied in this method of solution.

Let $W$ be the angle of position determined for $a$, $\delta$ where these co-ordinates, therefore, must be indicated, then $a_k$ can be found at once from

$$\left.\begin{aligned}
\tan(a-a_k) &= \tan W \cdot \sin\delta, \\
\sin P' &= \frac{\cos\delta}{\cos\delta_0}\sin W, \\
\cos P &= \frac{\cos(a_0-a_k)}{\cos(a-a_k)}\cos W.
\end{aligned}\right\} \tag{19}$$

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[1] R. Lehmann-Filhés, Astr. Nachr. 96 (1880), p. 241.

If we substitute this value in the above-mentioned equations, then they become at once modified appropriately for the proposed solution.

It may finally be noted that in the case of observations made at one place the zenithal attraction for the correction of the radiant can only be determined hypothetically, for we must use the zenith of the terminal point which is unknown, and we must know the relative velocity. For this latter the place of observation can be adopted and no great error will result. Instead of the unknown velocity, we generally use the geocentric velocity corresponding to the parabolic orbit.

## II. COMPUTATION OF THE ORBIT RELATIVE TO THE SOLAR SYSTEM

The co-ordinates of the apparent radiant $a'$, $d'$ freed from zenith attraction, and relative to the ecliptic, will be designated by $\lambda'$, $\beta'$. These and the relative geocentric velocity $v'$ are found by the combination of the heliocentric motion of the meteor with the velocity $v$ in the direction indicated by the analogous co-ordinates $\lambda$, $\beta$ combined with the magnitude and direction of the heliocentric motion of the earth at the node of the meteor's orbit. $\lambda$, $\beta$ determine the *true* radiant point and $v$ is the *heliocentric* velocity at this point. The very unimportant influence of the earth's rotation on its axis can here be left out of the consideration.

As the unit for the measurement of $v$ and $v'$ when no other kind is noted, we use the velocity of the earth at its mean distance from the sun. The quantities expressed in kilometers will be reduced to this unit with sufficient accuracy by dividing by 29.59.

For the radius vector, or the distance of the earth from the sun, the unit is the average distance or the semi-major axis of the earth's orbit. For that position of the earth's orbit at which the longitude of the sun is $\odot$ the corresponding distance of the earth expressed in this unit is

$$r = 1 + e' \cos(\odot - \pi).$$

In this expression we may for a long time adopt with sufficient accuracy $e' = 0.01676$ and if $T$ denotes a year, we have

$$\pi = 101° \ 12.8' + 1.03'(T - 1900).$$

The radius vector $r$ is given for every day in every astronomical ephemeris.

If $\odot'$ indicates the longitude of the direction of the normal at that point of the earth's orbit for which the longitude of the sun is $\odot$, then we have

$$\odot' = \odot + \frac{e'}{\text{arc } 1'} \sin(\odot - \pi) = \odot + 57.6 \cdot \sin(\odot - \pi),$$

and therefore the longitude of the radiant point in the ecliptic or the apex of the earth's annual motion is $\odot' - 90°$.

Finally the *heliocentric* velocity of the earth at the distance $r$ from the sun is given by the expression $\sqrt{\dfrac{2}{r} - 1}$.

The quantities $\lambda'$, $\beta'$, $v'$, $\lambda$, $\beta$, $v$ are connected by the three equations

$$\left.\begin{aligned}
v \cos \beta \sin(\odot' - \lambda) &= v' \cos \beta' \sin(\odot' - \lambda') - \sqrt{\dfrac{2}{r} - 1}, \\
v \cos \beta \cos(\odot' - \lambda) &= v' \cos \beta' \cos(\odot' - \lambda'), \\
v \sin \beta &= v' \sin \beta',
\end{aligned}\right\} \quad (20)$$

from which $\lambda$, $\beta$ and $v$ can be found.

If we desire only the magnitude of the heliocentric velocity, we obtain that from the equation

$$v^2 = v'^2 + \left(\dfrac{2}{r} - 1\right) - 2v' \sqrt{\dfrac{2}{r} - 1} \cdot \cos \beta' \cdot \sin(\odot' - \lambda'), \quad (21)$$

and we obtain the semi-major axis of the respective meteor orbits from equation

$$a = \dfrac{r}{2 - rv^2}. \quad (22)$$

This orbit, therefore, either an ellipse or hyperbola according as $a$ is either positive or negative, or according as $v^2 \gtrless \dfrac{2}{r}$.

The parabola corresponds to $a = \infty$, and hence we obtain the special limiting case $v^2 = \dfrac{2}{r}$.

Sometimes from the periodical return of an especially rich shower of meteors (such as the Leonids), one deduces a well-known radiant $\lambda'$, $\beta'$, and the time of revolution $U$ of a dense swarm of meteors and these can therefore be considered as given or known. In such a case in order to compute the remaining elements of the orbit, we have

$$a = U^{\frac{2}{3}}, \quad (23)$$

and thence also $v$. Then the three equations (20) give us $v'$, $\lambda$ and $\beta$, from which the elements of the orbits of the stream can be deduced according to the following method:

Let $i$ be the inclination of the heliocentric orbit of the meteor to the ecliptic; $\tau$ be the angle of the tangent on this orbit to the radius vector $r$ of the earth's orbit at its node with the meteor orbit; let $p$ be the semi-parameter of the meteor orbit; let $e$ be the excentricity of the meteor orbit; let $q$ be the perihelion distance of the meteor's

orbit; let $w$ be the true anomaly for the radius vector $r$, we then obtain these quantities from the following equations:

$$
\left.\begin{array}{l}
\sin i \sin \tau = \sin \beta, \\
\cos i \sin \tau = -\cos \beta \cdot \sin(\odot - \lambda), \\
\quad \cos \tau = -\cos \beta \cdot \cos(\odot - \lambda),
\end{array}\right\} \tag{24}
$$

$$
\left.\begin{array}{l}
p = r^2 v^2 \sin^2 \tau, \\
e = \sqrt{1 - \dfrac{p}{a}}, \quad q = \dfrac{p}{1+e} = a(1-e), \\
\cos w = -\dfrac{1 - \dfrac{p}{r}}{e}.
\end{array}\right\} \tag{25}
$$

On account of the determination of the remaining elements, we may refer to the computations of orbits and planets since all necessary data have now been given. The quantities $i$ and $\tau$ must always be counted in opposite directions. If we count $\tau$ from the prolongation of the radius vector $r$ toward the point $180° + \odot$ toward the sun, but always in the first or second quadrant, then $\sin i$ always has the sign of $\sin \beta$. For the northern radiant, therefore, the corresponding node is always the descending node, hence we have $\Omega = \odot$. Under this assumption (viz.: positive $\beta$), the movement is toward either the right or left according as $i$ lies in the first or second quadrant. For radiants in the southern latitudes we have $\Omega = 180 + \odot$ and the motion is either toward the right or left according as $i$ results from these equations in the third or fourth quadrant. In the subsequent presentation of the elements, however, $i$ will be given for the corresponding ascending node with the added words right or left motion, viz.: positive or negative, direct or retrograde. Since for a considerable number of the largest meteors (also so-called detonating meteors and even also those associated with the fall of masses), it has been established by the careful use of observations that the heliocentric velocity $v$ for the above-indicated parabolic limits is considerably exceeded so that $a$ may be negative and $e$ greater than one, hence the heliocentric orbit will be a well-marked hyperbola which on account of its position cannot possibly have originated in the solar system, therefore it must be assumed that such bodies have entered the solar system from the outside cosmic space.

Bodies that describe in space nearly parallel orbits at great distances from each other with identical velocities belong therefore to a sidereal stream of considerable extent, and can intersect the earth's orbit in such different nodal lines that excepting $a$ which depends

upon the original cosmic velocity, all other orbital elements of the same type and consequently also the co-ordinates of the apparent radiant can have quite different values.

Under such circumstances a comparison of different cases gives us not *the complex of those elements* that are characteristic of the orbits of planets and comets, but, according to what has already been said, they give exclusively the magnitude and direction of the motion for such large values of the radius vector $\rho$ that relative to these even $r$ may be considered as almost negligibly small. Since such a portion of the orbit is almost the same as the asymptote, we can take $\rho = \infty$ for this distance.

The direction, towards which this asymptote points on the celestial sphere, whose longitude and latitude will be designated by $l$ and $b$, can be called the *sidereal or cosmic point of departure of* the meteor. The determination of this point for different assumptions of $a$ (*i. e.*, for corresponding $v's$) for a given radiant point $\lambda'$, $\beta'$ is really our most important object in reference to hyperbolic meteors.

If $w'$ is the true anomaly of the meteor orbit for the radius vector $\rho = \infty$ and $\sigma$ the angle, which the direction of the asymptote drawn from the point $(l, b)$ makes with the radius vector $r$ at the node of the orbit on the earth's orbit in complete analogy as to $\tau$ in regard to numeration and definition, then we obtain

or if
$$\cos w' = -\frac{1}{e}, \quad \sigma = w' - w, \tag{26}$$

$$\sqrt{\frac{rv^2}{rv^2-2}} = m, \tag{27}$$

$$\tan\frac{\sigma}{2} = \frac{m\sin\tau}{1 + m\cos\tau}, \tag{28}$$

which is more convenient for the inverse determinations, also

$$\cos(2\tau - \sigma) = \frac{2 + (rv^2 - 2)\cos\sigma}{rv^2}, \tag{29}$$

while $l$ and $b$ are then determined from

$$\left.\begin{array}{r}\sin(\odot - l)\cos b = -\sin\sigma\cos i, \\ \cos(\odot - l)\cos b = -\cos\sigma, \\ \sin b = \sin\sigma\sin i.\end{array}\right\} \tag{30}$$

The velocity of entrance into the solar system ($\rho = \infty$) is given by

$$v_0 = \sqrt{\frac{1}{a}} = \sqrt{v^2 - \frac{2}{r}}. \tag{31}$$

For parabolic orbits we should have $rv^2 - 2 = 0$, thus $a = \infty$, accordingly

$$p = 2r \sin^2 \tau, \quad e = 1, \quad w = 180° - 2\tau, \quad q = \frac{p}{2} = r \sin^2 \tau, \quad w' = 180°,$$
$$\sigma = 2\tau, \qquad v_0 = 0, \quad \pi = 180° + \odot \pm w. \tag{32}$$

It can sometimes be of great interest to solve the problem in the inverse order, that is to say, to determine the apparent radiant $\lambda'$, $\beta'$ for a given day of the year when the sun's longitude is $\odot$ and when, therefore, we assume as given:

(a) The necessary elements of the elliptic meteor orbit, or

(b) The parabolic aphelion through $l$ and $b$, or finally

(c) The hyperbolic cosmic starting point, which may also be determined by $l$ and $b$ with $v_0$ or $v$.

Then according to the previous formulæ $a$ is given for the elliptic orbit by the time of rotation, when it is not otherwise known directly by the system of elements, for the hyperbolic orbit $a$ is given either by the equations (31) or (22).

Further

$$v = \sqrt{\frac{2}{r} - \frac{1}{a}}. \tag{33}$$

Since $e^2$ and $i$ are given among the elliptic elements, we can, therefore, also compute

$$p = a(1 - e^2), \quad \sin \tau = \frac{\sqrt{p}}{rv}; \tag{34}$$

and then we can also compute from the equations (24) the position $(\lambda, \beta)$ of the true radiant and from equation (20) $\lambda'$, $\beta'$, $v'$ for the apparent radiant. The zenith attraction can only be applied for a definite latitude and siderial time.

In hyperbolic orbits $a$ is negative in the equation for $v$, but in parabolic orbits it is assumed to be infinite.

The system of equations (30) gives $i$ and $\sigma$ when we know $\odot$, $l$, and $b$. Furthermore we obtain $\tau$ from equation (29), $\lambda$ and $\beta$ from equations (24) and finally, also, $\lambda'$, $\beta'$, $v'$.

Equation (29) gives an associated *pair of radiants* for each initial direction $(l, b)$. Inversely, therefore, it always gives for each $\odot$ two associated values of $(l, b)$ and with *one* position of $(l, b)$ it gives the associated apparent radiants for two far distant points on the apparent celestial sphere. We usually find these two when we reflect that $\cos(2\tau - \sigma)$, even for a specified sign, always gives two values for $2\tau - \sigma$, therefore for a given value of $\sigma$ there are two values of $\tau$ corresponding to the two possible hyperbolas in each plane with a

common focus which possess parallel asymptotes on one side. In this case one orbit is direct, the other is retrograde, and their perihelia lie on opposite sides of the radius vector.

Since one of the two perihelion distances is ordinarily very small, the attendant radiant is generally in that portion of the sky covered with sunshine. For the parabolic orbits the true positions of the two associated radiants lie diametrically opposite one another.

The expressions above given for the determination of the orbit in the present condition of the art of observing meteors go far beyond the needs of the accurate computations. They can therefore only be appropriately applied when we are concerned with the further development of definite theoretical views. In the working up of observations we attain our object sooner and without appreciable loss in accuracy when we assume in general that for the earth's orbit we have $e = 0$, $r = 1$, $\odot' = \odot$, and also that the velocity of the earth is always equal to unity, in short, we take the orbit of the earth as circular.

It is easy to simplify all the previous expressions by means of these substitutions, only we must say that it is not important to seek for the true radiant when we can attain the ordinary elements in the shortest way. We find, in fact,

$$\left.\begin{array}{l} v^2 = v'^2 + 1 - 2v' \cos \beta' \sin(\odot - \lambda'), \\ \cos \tau = -\dfrac{v'}{v} \cos \beta' \cos(\odot - \lambda'), \end{array}\right\} \quad (35)$$

$$\left.\begin{array}{l} \sin i = -\tan \beta' \cdot \cot \tau \cdot \sec(\odot - \lambda'), \\ \cos i = \dfrac{1 + v \cos \tau \tan(\odot - \lambda')}{v \sin \tau}, \end{array}\right\} \quad (36)$$

and the calculations may be carried further by making the indicated substitutions in the known formulæ. For the determination of the sidereal starting point $l$, $b$, we have

$$\left.\begin{array}{l} \tan \dfrac{\sigma}{2} = \dfrac{m \sin \tau}{1 + m \cos \tau}, \quad \text{where } \dfrac{v}{\sqrt{v^2 - 2}} = m, \\ \cos(2\tau - \sigma) = \dfrac{2 + (v^2 - 2) \cos \sigma}{v^2}. \end{array}\right\} \quad (37)$$

whereupon $l$ and $b$ are found from (30). The velocity for $\rho = \infty$ is $v_0 = \sqrt{v^2 - 2}$.

The simplification of the inverse problem to find $l$ and $b$ for the radiants from $\lambda'$, $\beta'$ does not here need any further explanation. Computational results from the elements of the orbit from the velocity determined by the observations are almost useless because the basis

for this work is too uncertain. The most careful deduction of the velocity is certainly very important in order to at least make known the degree of probability that the corresponding planetary orbit departs to one side or the other from the parabola. The computation of the elements, if we really wish to attempt it, should be carried out with various assumed and appropriate values of the velocities, whereby the general result can be better arranged for comparison with other cases.

## III. RESULTS OF OBSERVATIONS AND COMPUTATIONS RELATIVE TO METEORS

### 1. AVERAGE ACCURACY OF OBSERVATIONS AND COMPUTATIONS

#### A. AT THE END OF THE ORBIT OR POINT OF ARREST

The average error of a given direction (of an azimuth) as the result of 351 observations is found to be ±5.8 degrees.

These determinations were made in 12 per cent of the cases by simultaneous reference to stars, and in about 20 per cent of the cases on the basis of subsequent measurements, in the remaining cases by reference to terrestrial objects in the neighborhood by means of plans and charts.

The average error of any indication or description of the apparent altitude or zenith distance may be taken as ±4.1 degrees from 235 cases.

As a general rule in these cases we have considered only direct references to stars or subsequent measurements. Crude estimates, as is well known, give almost always apparent altitudes that are far too large. When such cases have been used in exceptional cases, we have in general reduced them to $\frac{2}{3}$ or $\frac{3}{5}$ of the given estimates.

The azimuths were not examined so closely since in such cases errors in one direction are less to be feared, which is also easily to be seen by reason of the larger average error of a single observation.

The average error in the determination of the *geographic position* of the terminal point as deduced from 42 cases amounts to ±8.3 kilometers. The very best determinations are uncertain by $3\frac{1}{2}$ or 4 kilometers. The average error of the computed linear altitudes amounted to ±3.4 kilometers.

#### B. AT THE INITIAL POINT OF THE OBSERVED PATH

The nature of the observations brings it about that a meteor can be first observed by many observers at very different points of its path. These are differences that do not possess any of the characteristics

of errors of observation. These characteristics belong only to the transverse departures from the corrected apparent path.

These departures deduced from 217 cases for the initial point average ±4.2. They are deduced for the initial point in a similar way as for the terminal point, but by excluding rough data more frequently than in the latter case.

*The apparent relations to the stars* when they are determined immediately after the observations resulted in giving the average error of such a relation as ±3.5 degrees for the initial or the terminal point respectively.

## C. ESTIMATES OF THE APPARENT INCLINATIONS OF THE OBSERVED PATH

These estimates show from an average of 250 cases a mean error of ±6.5 degrees. Such estimates generally refer to the vertical through the observed terminal point, or some other specific point in the path and were generally obtained by graphic sketches. Radiants that do not lie far above the horizon were generally affected by only a small part of this uncertainty.

## D. THE ACCURACY ATTAINED IN THE DETERMINATION OF THE RADIANTS

Of large and generally detonating meteorites, 43 reliable determinations for 537 apparent paths, therefore on the average 12 or 13 observations for each case, gave the average error in the location of these points on the sky at ±3.3 degrees.

The number of the orbital paths in the individual cases was very uneven, exceeding 40 in many cases, but was often only three or four.

At the present time more than 420 radiants of detonating meteors have been found, of which, however, about 30 per cent are identical with others. Much larger is the number of the radiants of shooting-stars listed in the last catalog. Denning[1] compiled from the appropriate literature as well as from his own observations 4,367[2] such radiant points of which, however, probably more than half coincide with others.

When he remarks[3] that on the average, during every night, more than 50 radiants are in action, this is quite correct, but on account of the lesser frequency of individual meteors, many of these radiants cannot be demonstrated every night at the same place and during the same year.

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[1] W. F. Denning, General Catalogue (1899).

[2] Large numbers of these radiants have no proved existence and are probably fictitious.   C. P. Olivier.

[3] W. F. Denning, *ibid.,* p. 203.

The rich streams of shooting-stars afford so much observational material that their radiant points can generally be determined more accurately than those of the fire balls, as the following examples may show in which the average errors are shown:

LOCATION OF APPARENT RADIANTS

|  | Right Ascension | Declination | Maximum | |
|---|---|---|---|---|
| Leonids .............. | 150.1° (± 0.3°) | + 23.0° (± 0.2°) | November | 14 |
| Perseids .............. | 44.0°   .... | + 56.9°   .... | August | 11 |
| Lyrids ................ | 271.5° (± 0.7°) | + 33.4° (± 0.4°) | April | 20 |
| Andromedids ........ | 23.8° (± 0.9°) | + 44.0° (± 0.2°) | November | 26-28 |
| Quadrantids .......... | 230.9° (± 0.7°) | + 51.3° (± 0.4°) | January | 2 |
| Geminids ............ | 108.3° (± 0.5°) | + 33.6° (± 0.4°) | December | 10-12 |
| Orionids ............. | 89.7° (± 0.5°) | + 15.6° (± 0.3°) | October | 10-16 |
| Orionids ............. | 91.5° (± 0.3°) | + 15.7° (± 0.3°) | October | 16-22 |

Denning believes[1] that the activity of the Perseids should be assumed to extend from July 11 to August 19 by reason of which the radiant point should experience a change from

$$a = 11.5°, \ \delta = 47.7° \text{ to } a = 56.6°, \ \delta = 59.1°.$$

All these streams consist of more or less densely collected particles along the whole extent of their paths as far as yet known, but only the Leonids consist of a specially rich swarm returning to the perihelion in a well-established period of 33¼ years.

## 2. THE RESULTS AS TO THE ALTITUDE OF THE LUMINOSITY AND OF THE TERMINAL POINTS AND THEIR RELATIONS TO OTHER OF THE FACTORS

In this respect we must distinguish between the small phenomena which we know as shooting-stars and the large meteors known as fire balls which frequently exhibit during their path through the atmosphere a remarkable development of light and often produce great noise and sometimes well-established falls of meteorites.

With regard to the *shooting-stars* of the so-called *Perseids* in August, Weiss,[2] in Vienna, from the discussion of 49 reliable corresponding observations, found on the average for the first luminosity 115 km., for the extinction 88 km.

Independent of this H. A. Newton[3] also found for the *Perseids* from 38 observations the following altitudes, viz.: luminosity 112 km. and extinction 90 km. in excellent agreement with the preceding.

[1] W. F. Denning, Astr. Nachr. 148 (1899), p. 283.
[2] E. Weiss, Wien. Ber., 1868.
[3] H. A. Newton, American Journal of Science and Arts, 2 Series, 40.

For the star shower of the Leonids in November, Newton [1] computed from 78 determinations the average altitudes of luminosity 155 km. and extinction 98 km. According to this the whole path of light for the November Leonids was higher than for the August Perseids which is evidently in connection with the fact that the Leonids entered into the atmosphere with a relative velocity of about 70 km. and the Perseids with only 60 km. Moreover the masses of the November stream seem also to have special chemical characters. [2]

The determination of 159 altitudes of shooting-stars from the fifth to the first magnitude, and of the most varied *radiants*, [3] therefore also including some very slight velocities, gives in general for the luminosity 108.5 km. and for the extinction 86.3 km. which is therefore only a slight variation from the values found for the August Perseids.

For the large meteors, including those with detonations and those with falling meteorites, and from determinations that are especially reliable, but without selection of the greatest phenomena, I found for the luminosity from 121 cases an altitude of 138.6 km.

For the extinction from 213 cases an altitude of 49.7 km.

This collection of data shows the influence of the larger masses, especially because of the comparatively slight altitude of the stopping point and therefore because of the deeper penetration into the atmosphere. This is shown still more plainly by a further classification. For the altitude of the stopping point I found [4] on the average 60 km. for 147 fire balls without detonations; 31 km. for 57 meteors with detonations; 22 km. for 16 falls of meteorites, thus it may well be proper to explain these different types of shooting-stars as due to a gradual increase in mass, since larger masses experience a relatively smaller resistance in the atmosphere and thus can penetrate deeper than the smaller masses.

Still more important are the relations of the altitude of the terminal point to the value of the geocentric velocities that we obtain from the observations. In order to avoid as far as possible any one-sided

[1] H. A. Newton, *ibid*.

[2] The Leonids among all meteors provide the longest enduring luminous tails in the atmosphere.

[3] Memoirs of the British Astronomical Association, Vols. 9, 12. Part 1 (1900-1903).

[4] G. v. Niessl, "Über die Periheldistanzen und andere Bahnelemente jener Meteoriten, deren Fallerscheinungen mit einiger Sicherheit beobachtet werden konnten," Brünn (1891).

views I have investigated very various material and I submit the most important results in the following tables 1, 2, 3, 4, with the preliminary remark that the groups, under tables 1 and 2, relate principally to small phenomena (star showers), but those under tables 3 and 4 relate principally to fire balls.

TABLE 1 (NOTE 1).

| Limiting altitudes | No. of cases | Average observed geocentric velocity | Average terminal altitude |
|---|---|---|---|
| Above 100 km. | 23 | 67.7 km. | 106.6 km. |
| 80-100 " | 48 | 51.5 " | 88.8 " |
| 60- 80 " | 33 | 35.5 " | 73.0 " |
| 30- 60 " | 17 | 30.1 " | 46.2 " |
| below 30 " | 2 | 23.2 " | 28.8 " |

TABLE 2 (NOTE 2).

| Above 100 km. | 10 | 72.3 km. | 112.2 km. |
|---|---|---|---|
| 80-100 " | 11 | 43.0 " | 88.3 " |
| 60- 80 " | 13 | 40.5 " | 74.2 " |
| 50- 60 " | 6 | 35.4 " | 59.2 " |
| below 50 " | 14 | 27.4 " | 33.9 " |

TABLE 3 (NOTE 3).

| Above 60 km. | 12 ( 1 deton.) | 51.8 km. | 86.4 km. |
|---|---|---|---|
| 50- 60 " | 19 ( 3 deton.) | 55.0 " | 54.2 " |
| 30- 50 " | 43 (16 deton.) | 40.6 " | 39.0 " |
| below 30 " | 28 (13 deton.) | 37.6 " | 24.0 " |

[1] The cases under 1 are taken from the above mentioned "Memoirs of the Br. Assoc." (Foot-note 3, p. 25). 78% of them are shooting-stars, 11% meteors from one to four times the magnitude of Venus, 11% fire balls up to the magnitude of the moon without records of detonations.

[2] 2 refers to the older work published by Denning (W. F. Denning, "107 Real paths of Fireballs and Shooting-stars observed in England from 1886 to 1896." Lond. Astr. Soc. Monthly Not. 57, p. 161) wherein 74% are shooting-stars, as above, 6% meteors of the magnitude of Venus or somewhat more, 20% fire balls including *one* which detonated.

[3] 3 is taken from a list of mostly very large meteors which I published in the same number of the Monthly Notices, p. 170. Of the 100 cases given there the velocity could not always be determined; with later additions they gave, however, 102 coherent determinations of the terminal altitude and velocity. This material contains only 8% of shooting-stars of from the first to the fourth magnitude, 27% meteors from one to several times the magnitude of Venus, while 65% are large fire balls comparable in magnitude with the moon or sun, of these 30% detonated and seven were accompanied by a fall of a meteorite.

TABLE 4 (NOTE 1).

| Apparent elongation of the radiants | | No. of cases | Average altitude of the terminal point |
|---|---|---|---|
| Above 100 km. | 9 | 76.8 km. | 116.8 km. |
| 80-100 " | 9 | 72.0 " | 89.3 " |
| 60- 80 " | 16 | 49.4 " | 72.5 " |
| 50- 60 " | 21 | 49.1 " | 58.9 " |
| 30- 50 " | 35 | 42.7 " | 39.0 " |
| below 30 " | 21 | 36.5 " | 22.1 " |

All four of these collocations show a perfectly definite regular connection between the geocentric velocity deduced from observations and the altitude of the stopping point since they diminish together.

It is natural to conclude that a meteor can penetrate into the atmosphere deeper in proportion as it moves with a low velocity. That this conclusion is correct is also enforced by the following facts:

1. The more rapid Leonids are extinguished at greater altitudes than the slow-moving Perseids as mentioned above.

2. Under the assumption of equal heliocentric velocity, the meteors that meet the earth in its direct course (from the apex) enter the atmosphere with a relative velocity that is about 56 km. greater than that of the meteors that enter the earth from the opposite side (or the anti-apex). If we arrange the meteors according to the distance of the radiant from the apex of the earth, we find the following results:

1. From the material in foot-note 2, page 26.

| Between | 0 and | 40 | 12 | 95.6 km. |
|---|---|---|---|---|
| " | 40 " | 70 | 12 . | 84.5 " |
| " | 70 " | 90 | 12 | 61.6 " |
| " | 90 " | 110 | 10 | 59.8 " |
| " | 110 " | 180 | 10 | 52.1 " |

2. From the material in foot-note 3, page 26.

| Between | 0 and | 80 | 13 | 54.2 km. |
|---|---|---|---|---|
| " | 80 " | 90 | 9 | 50.5 " |
| " | 90 " | 100 | 10 | 44.5 " |
| " | 100 " | 110 | 11 | 40.2 " |
| " | 110 " | 120 | 7 | 38.6 " |
| " | 120 " | 150 | 13 | 38.6 " |
| " | 150 " | 180 | 7 | 36.4 " |

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[1] 4 arises from an analysis of 111 cases which I have worked up from material in older literature. Since the notation of magnitude was formerly less determinate, I can only say that these data likewise refer chiefly to large meteors (22% detonating).

Apart from the details that are unimportant in the establishment
of this point, both these series show the expected connection between
the elongation and the terminal altitude. In the first list the fact that
the latter terminal altitudes are in general greater than in the second
list is certainly because the meteors in the former list are principally
the smaller kind that burn up at great altitudes, whereas mine or the
second list relates mostly to the large meteors. In this respect, there-
fore, the latter is to a certain extent a supplementary continuation of
the first. In part, also, the smaller observed velocity of the deep-
penetrating meteors must be the result of the diminution of the
velocity during the path through the atmosphere. The final stoppage
of the meteor is almost instantaneous, still there are certain phe-
nomena that result from observations that relate only to the lowest
portion of the path in the deeper layers of the atmosphere that show
slighter velocities than the average velocity that results from a con-
sideration of the whole path of the meteor in the same case.[1] But of
course the error in the estimation of short intervals of time renders
difficult any decision on this point.

## COMPARISON WITH THE THEORY OF THE RESISTANCE OF THE ATMOSPHERE

In order to compute the resistance of the air experienced by the
meteor in its individual phases, one has attempted to utilize the expe-
riences which have been deduced from experiments with the spherical
balls of artillery. The following collection and summary is based
upon the formula for resistance given by *Robert*.[2] It assumes a
vertical movement of a spherical body weighing 118 grams having a
density of 3.5 and a radius of 2 cm. The two assumed velocities
with which the ball enters the atmosphere relate to the extreme values
of a parabolic orbit, viz.: at the apex and the anti-apex. If for the
same density we have another radius of the sphere, viz.: $2r$ cm., and
if the inclination of the path toward the horizon be $h$, then in the
column for the velocity the values will remain unchanged if the
corresponding atmospheric pressure be multiplied by $r \sin h$.

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[1] *Cf.* G. v. Niessl, Wein. Ber., 114 (1905), p. 1511.
[2] Schiaparelli, pp. 23 and 24 after S. Robert, "Del moto dei projettili nei
mezzi resistenti." Torino, 1855. (Memorie dell' Academia delle scienze di
Torino, II, 9, 16.)

| I | | | II | | |
|---|---|---|---|---|---|
| The entrance velocity 72 km. | | | The entrance velocity 16 km. | | |
| Pressure mm. | Altitude km. | Velocity km. | Pressure mm. | Altitude km. | Velocity km. |
| .... | .. | 72 | .... | .. | 16 |
| 0.00007 | 129 | 70 | 0.006 | 94 | 14 |
| 0.00014 | 125 | 68 | 0.016 | 86 | 12 |
| 0.0005 | 114 | 60 | 0.032 | 80 | 10 |
| 0.0013 | 106 | 48 | 0.062 | 75 | 8 |
| 0.0031 | 99 | 36 | 0.128 | 69 | 6 |
| 0.0082 | 91 | 24 | 0.305 | 62 | 4 |
| 0.036 | 80 | 12 | 1.1229 | 51 | 2 |
| 0.082 | 73 | 8 | 4.299 | 41 | 1 |
| 0.315 | 62 . | 4 | 11.619 | 33 | 0.5 |
| 1.249 | 51 | 2 | .... | .. | .. |
| 4.318 | 41 | 1 | .... | .. | .. |
| 11.639 | 32 | 0.5 | .... | .. | .. |

In both these tables the altitude 33 km., at which the planetary velocity is almost completely neutralized, is almost exactly equal to the average altitude of stoppage given for detonating meteors on page 26. But the method of the diminution of the velocity is contrary to all other experiences. Thus, for instance, in the first table the estimated duration for the reduction from 129 km. down to 33 km. velocity should be only 1/18 part of the entrance velocity and for meteors that are first observed in lower regions, for instance between 80 km. and 33 km. which frequently occurs the reduction should indeed amount to only 2/36. Practically for meteors that have been observed only in the lowest part of the path, we obtain almost always much lower velocities, but certainly not to the extent here given.

For an entrance velocity of 16 km. the observed average velocity between 94 and 33 km. in altitude is only $\frac{1}{4}$ of the original, or still too small to observe. Nothing important is changed with respect to these data if we make still other assumptions than those of I and II.

The true geocentric velocity of the Leonids among which there are many particularly bright shooting-stars is given us with some certainty from the known orbital period and amounts to not much less than 72 km. For a small body in this meteoric stream of 2 mm. diameter and 0.12 gram weight, whose orbit intersects the atmosphere at an inclination of 30 degrees to the horizon, we should have to divide by 20 the atmospheric pressures above given under case I, hence the appropriate altitudes will be as follows:

III

| Altitude km. | Velocity km. | Altitude km. | Velocity km. |
|---|---|---|---|
| 153 | 70 | 103 | 12 |
| 147 | 68 | 97 | 8 |
| 138 | 60 | 86 | 4 |
| 130 | 48 | 75 | 2 |
| 123 | 36 | 65 | 1 |
| 115 | 24 | 57 | 0.5 |

For the entire orbit from 153 km. to 57 km., as an average velocity even in this case, we obtain only 4 km. or 1/18 part of the entrance velocity. As before stated, the orbits of the Leonids are supposed to lie between 155 and 98 km. because the particles are entirely dissipated by the very considerable rise in temperature before they can descend any lower. In fact the bodies of this stream, among all the shooting-stars, leave behind them the most enduring tails. It, therefore, appears proper to consider the above given plan as only applicable within similar limits or between 153 and 97 km. Thus we obtain on the average only about 27 km. or ⅔ of the original velocity whereas from the observations the difference relative to the theoretical average is much smaller.

Direct observations, no matter how fragmentary the data, make it in general probable that the diminution of the velocity in the upper atmospheric mediums is slighter, but in the latter or lower part of the orbit or its lowest part which reaches far down must be greater than they would seem to be according to the previous theoretical views. Many optical phenomena that seem to be connected with the checking of the fire balls also seem to agree with this view.

The dependence of the altitude of the terminal point of the path upon the velocity at entrance demands also some explanation. Under probable assumptions (*Cf.* Schiaparelli, p. 231 ffg.) it is proven that the large but very different velocities of two bodies which enter under otherwise equal conditions into the atmosphere and pass through it in straight line orbits are diminished by the resistance in such a way that at any given altitude they attain almost the identical velocity. In the examples I and II this velocity is attained at the altitude 62 km.

We must not however conclude from this that the terminal altitude is independent of the entrance altitude. For the meteor occurring at the greatest altitude should convert much more kinetic energy into heat and would therefore be much more rapidly consumed.

### 3. The Masses of the Shooting-Stars

A. Herschel[1] concluded from the comparison of the light power of the shooting-stars at a given distance with that of a given mass of gas, that the meteor of first magnitude on the average weighs very few grams and the smaller meteors weigh only a fraction of a gram.

By similar comparisons with the Drummond light, V. F. Sands,[2] with reference to the Leonids of 1867, found the following estimates:

| Apparent brightness | Mass or weight |
|---|---|
| Jupiter | 0.67 grams |
| Sirius | 0.45 " |
| First Mag. Star | 0.06 " |
| Second " | 0.02 " |
| Third " | 0.01 " |
| Fourth " | 0.006 " |
| Fifth " | 0.004 " |

Even if such estimates can give only approximate results, still they confirm the assumption that as a substratum for the shooting-star phenomenon only very small masses come into consideration which probably in their original condition could scarcely attain the earth's surface.

According to Buchner[3] the smallest of those aereolites that have been found, of which one can assume that they have descended as individual bodies, is 24 grams. Much smaller particles, which under the protection of preceding larger pieces describe their path through the atmosphere in small aggregates and cause a rain of stones, are in this respect not comparable with the shooting-stars. On the other hand they form, together with the larger pieces, one fire ball.

Experienced observers of shooting-stars frequently expressed themselves that the exterior appearance allows one to infer great original variation in composition.[4] Some points in respect to difference of the velocity might well have been attributed to them. So long as we know nothing more accurate relative to the physical nature of the matter of the shooting-stars, we shall have to consider the difference in the masses between them and the meteorites.

### 4. The Average and the Unusual Lengths of the Paths

Out of 185 computed paths of shooting-stars from the fifth to the first magnitude as given in the last English statistics, I find on the

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[1] A. Herschel, Proc. Brit. Meteor, Soc. II, 19.

[2] F. Sands, Wash. Obs., 1867, pp. 19-30.

[3] Schiaparelli, p. 203.

[4] Weiss, Beiträge, p. 303.

average for the length of the visible path 57 km., on the other hand for 120 large and of these about 30 per cent detonating meteors from my own and other computations I find on the average the length of the path 319 km. In some cases, especially when the radiant point lies in the neighborhood of the horizon, there occur still much longer paths. Such an example is here given.

The meteor of July 7, 1892,[1] was visible when it was 74 km. above the neighborhood of Slobozia in Roumania and could be followed without break until it was 158 km. high about 70 km. WNW. of the mouth of the Tiber above the Tyrrhenian sea. There the observations could no longer be made, not so much because of an explosion, as by reason of the apparently gradual recession. It is not unlikely that at this great altitude the remaining part of this mass actually left the atmosphere. The demonstrated length of the path was 1350 km. This is the first and hitherto also the only case known to me of an undoubted ascending path. Since the point of perigee was undoubtedly at about 74 km., therefore, to the east of Slobozia there must have been a length of path at least as long.

It is worth remarking that in 1892, October 18, in lower Austria and Bohemia, a meteor was observed,[2] especially because of its remarkable long trail which the residuum left behind. So far as the path has been determined with any certainty, this meteor in the course of 1630 km. passed from an altitude of 257 km. above a region 70 km. east of Konigsburg in Prussia, until it was 43 km. high somewhat to the south of the island of Elba. The streak of light or trail, straight and horizontal as if drawn with a ruler, was at least 634 km. long and remained visible for about three minutes, so that its location between the stars could be easily determined.

## 5. THE HELIOCENTRIC VELOCITY

The heliocentric velocity which results from the observed geocentric velocity in most cases far exceeds the limits for a parabolic orbit and leads to the necessary assumption of a decided hyperbolic orbit. It is certainly remarkable that this almost always occurs in the case of those orbits that have the most favorable location for the most accurate determination of the velocity, viz.: for those that are directed from the neighborhood of the anti-apex, as for example in

[1] G. v. Niessl. Wien. Ber., 1893, 102, p. 265.
[2] G. v. Niessl. Verhdl. d. naturf. Vereins in Brüm, 1901, 39, p. 220.

the case of the meteorite of Pultusk for whose path J. G. Galle [1] has shown that the minimum value of the excentricity equals 2.277.

The average of 26 of the best determinations of the paths and mostly large detonating fire balls, gives for the heliocentric velocity 59.05 km., and therefore $a = -\frac{1}{2}$ very nearly. For those few cases of meteors for which sufficient observational material is at hand to enable us to draw any conclusion as to the form of the orbits, the hyperbolic form is beyond all doubt.

For 154 large meteors whose paths are derived from older material or were first computed by myself, there resulted on the average 59.8 km. as the heliocentric velocity.

*Schiaparelli,*[2] *more than 36 years ago, said* " It is in fact remarkable that whenever we have been able to investigate with any approximation the velocity with which a meteorite or a group of meteorites have penetrated into the atmosphere, we have always found that the corresponding absolute velocity is *greater than the parabolic would be*.

Therefore, in general, the large meteors are undoubtedly of interstellar origin. As opposed to this conclusion, we find that streams of shooting-stars pursue the same orbits as those of certain well-known comets of well-known periodicity. They are, therefore, interplanetary shooting-stars. Hence we are inclined to consider the large meteors as interstellar, but the smaller shooting-stars as interplanetary. Still we must call attention to the undeniable fact that most of the radiation points of meteors and detonating fire balls as well as the other large meteors, as far as they can be safely determined, *agree with well-established shooting-star radiants.* It is difficult in such cases to ascribe interplanetary orbits to the corresponding small phenomena when the hyperbola described by the large meteors issues from the same radiant points in the cosmic space.

In attempting to solve this apparent contradiction, one might perhaps assume that we have included under the name of shooting-stars different phenomena that are only superficially similar, but whose dynamic basis and cosmic significance are probably not all similar. From the many experiences of the last ten years we may at least draw the conclusion that in the phenomena as a whole there make themselves felt both the limited and minor interplanetary meteors as well as the extended interstellar meteors having particles of the greatest variety as to size, mass, quality, and velocity.

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[1] J. G. Galle. Über die Bahn des am 30. Januar 1868 beobachteten n. bei Pultusk im Königreiche Polen als Steinregen neidergefallenen Meteors. Breslau, 1868.

[2] Schiaparelli. Pp. 207 and 209.

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