RECENT STUDIES OF THE SOLAR CONSTANT OF RADIATION

By C. G. ABBOT

INTRODUCTION

Within the last two years the observations of the Smithsonian Astrophysical Observatory under the direction of the Secretary, Mr. Langley, have been largely for the purpose of measuring the total solar radiation, its distribution in the spectrum, and the losses which it suffers by absorption in the solar and terrestrial gaseous envelopes. In the experimental work and reduction of observations Mr. Langley has been aided by the writer, but chiefly by Mr. F. E. Fowle, Jr., whose able handling of the work I wish particularly to acknowledge and commend. Preliminary notices of this investigation have appeared in the *Smithsonian Report* for 1902, and in an article by the Secretary in *The Astrophysical Journal* for March, 1903, to which sources the reader is referred for additional information in relation to the methods of study. In the present paper will be found a summary of the results thus far reached.

ATMOSPHERIC ABSORPTION

It is well known that the effectiveness of the solar and terrestrial gaseous envelopes to intercept by reflection or absorption and thus diminish the intensity of the solar radiations at the earth's surface, varies greatly for rays of different wave-length. It is customary, speaking of the matter in ready though not strictly accurate terms, to combine these two effects of reflection and absorption under the single head of absorption, but to distinguish two kinds of absorption, namely, general and selective, of which the latter includes such sudden alterations of transmission as are seen in the Fraunhofer lines, while the former denotes merely a general weakening of the radiation extending over wide ranges of wave-length. Using this nomenclature, it appears to be the general absorption of the solar and terrestrial envelopes which chiefly affects the amount of solar radiation at the earth's surface, although the selective absorption of water vapor in the atmosphere is also both very effective and very variable.[1]

The procedure employed here to determine the general absorption of the air consists chiefly in making bolographs—that is, automatic energy spectra—of the solar radiation as often as possible throughout days of uniform and excellent sky without alteration of the sensitiveness of the apparatus. Such energy spectra are altered in appearance from one to another by the varying absorption of the different thicknesses of air, so that at a little after noon the height of the curve

[1] K. Ångström has, however, attributed much importance to the absorption of carbonic acid gas, implying by his computation that not less than 61 percent of the solar radiation which reaches the outer layers of the earth's atmosphere is cut off by the absorption of this gas in a vertical transmission through the air. (See *Annalen der Chemie und Physik*, vol. 39, pp. 309–311, 1890.) He locates the absorption of this gas principally in the bands at 2.6μ and 4.3μ; so that, as he says, its effect is not allowed for in the procedure for obtaning the value of the solar constant of radiation adopted by Mr. Langley in his research on Mount Whitney, and which is essentially that employed here. Ångström, while using the same method in part, adds a second term amounting to more than half the whole in his computation, solely referring to the absorption of carbonic acid gas, and thus he attains his oft-quoted result for the solar constant of radiation of 4.0 calories per square centimeter per minute. For several reasons I am inclined to think Ångström has greatly overestimated the importance of this carbonic acid absorption term: First, as he shows, the selective absorption of carbonic acid gas is, so far as I am aware, almost wholly for wave-lengths greater than 2.5μ and principally in two bands between wave-lengths 2.5μ and 2.85μ and between 4.20μ and 4.50μ respectively, where the total amount of the solar radiation is apparently less than one percent of the whole, as determined not only from the appearance of the observed bolographic solar spectrum energy curve itself, but from a consideration of the probable temperature of the sun and the distribution of energy in the spectra of bodies at high temperature. As a very evidently too great estimate of the energy in these wave-length regions, it may be seen that if the radiation outside the atmosphere (see plate XXII) was of the same intensity throughout these bands as at 2.1μ, the area they would include would be only about one-fiftieth the total area under the curves of plate XXII. It is of course very improbable that the height of the curve at 4.3μ is nearly as great as at 2.1 . Thus it would appear that the selective absorption of this gas for direct solar radiation is almost negligible. Second, if carbonic acid exercised a general absorption through the more intense parts of the solar spectrum, it is not apparent why such a general absorption is not included and allowed for in the coefficients of absorption here determined. Third, values of the solar constant computed here for the same day, but from observations made through very different thicknesses of air, are found to agree excellently, which appears to confirm the accuracy of the method of determining the atmospheric absorption which is here employed.

is found to be a maximum for all wave-lengths, and the height falls off as the sun declines in altitude, slowly in the infra-red region of the spectrum, but more and more rapidly as we examine further and further toward the violet, or still more rapidly if we note the great atmospheric absorption bands due to water vapor.

It is assumed that the atmospheric transmission for a very narrow portion[1] of spectrum may be expressed by the relation—

$$e = e_0 a^{m\frac{\beta}{\beta_0}} \tag{1}$$

where e and e_0 are the intensities of light of this wave-length at the earth's surface and outside the atmospheric respectively, a the fraction transmitted by the atmosphere for zenith sun, m the air mass, or ratio of the length of the transmitting column of air to that for zenith sun, and β and β_0 the observed and standard barometer readings respectively. Upon the bolograph the height d corresponding to any given wave-length is directly proportional to the amount of energy of that wave-length. Accordingly we may introduce a factor k constant for the single wave-length in question

$$d = ke = ke_0 a^{m\frac{\beta}{\beta_0}} \tag{2}$$

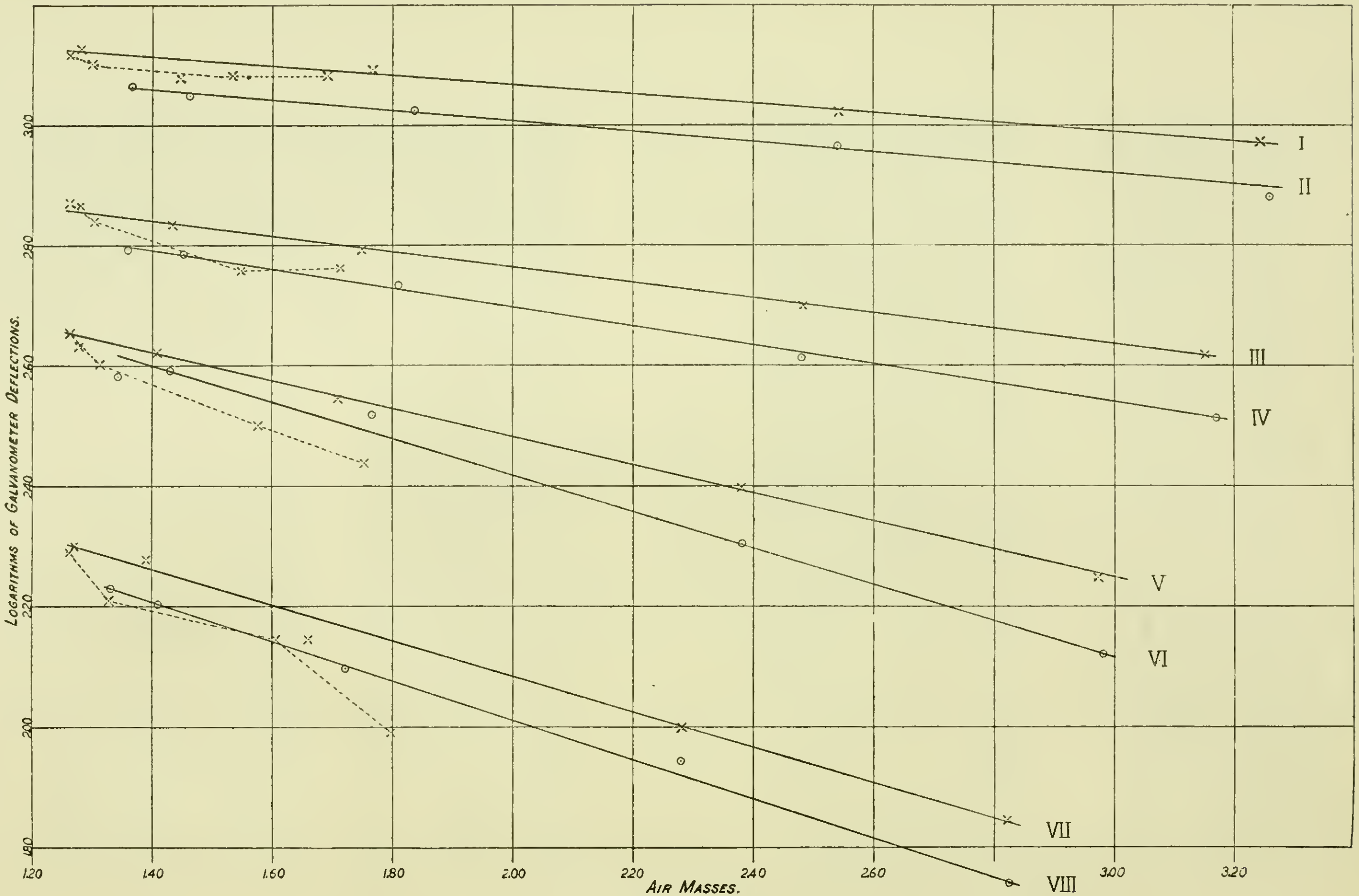
and hence

$$\log d = m\frac{\beta}{\beta_0} \log a + \log (ke_0) \tag{3}$$

As the last term of equation (3) is to be supposed constant during the day's observations, the expression is in the form of the equation of a straight line, and if the logarithms of the deflections at the given wave-length on the successive bolographs be plotted as ordinates with the quantities $(m\beta/\beta_0)$ as abscissæ, the several points so determined should fall on a straight line of which the tangent of the inclination is the logarithm of the transmission coefficient (a) for the given wave-length.

Mr. Langley has stated that the attempted measures of the solar constant from a station near sea-level like Washington are subject to great uncertainty from the necessity of the very large and doubtful extrapolation for atmospheric absorption. Without in the least questioning this, and while calling special attention to the great interest which would attach to a repetition of the experiments at high altitudes, I incline to the belief that the closeness with which the plotted points determined as above described lie upon a straight line for wide ranges of air mass is a reasonably sure criterion of

[1] In our practice less than the width between the D lines.



TRANSPARENCY OF THE ATMOSPHERE FROM BOLOGRAPHIC OBSERVATIONS.

the accuracy of the extrapolation. In order to give an impression of the weight which should be assigned to the solar constant values shortly to be given, I call attention to plate XX, which contains the plots for deducing atmospheric transmission at several wave-lengths for two days, March 25, 1903, and March 26, 1903, observations for the two days being represented by circles and crosses respectively. The tangent of the angle of inclination of the plotted lines is the logarithm of the coefficient of transparency of the atmosphere for vertical transmission of a ray of the given wave-length. Plots I and II represent a wave-length of 1.027 μ; III and IV, 0.656 μ; V and VI, 0.468 μ; and VII and VIII, 0.395 μ. In connection with this branch of the subject it is well to remark what the experience of meteorologists generally no doubt confirms, that the afternoon hours are found far more uniform in transparency of the air than the morning hours, so that the observations of atmospheric transmission for use in computing values of the solar constant are obtained chiefly in the afternoon. Forenoon observations are distinguished in plate XX by being connected by dotted lines.

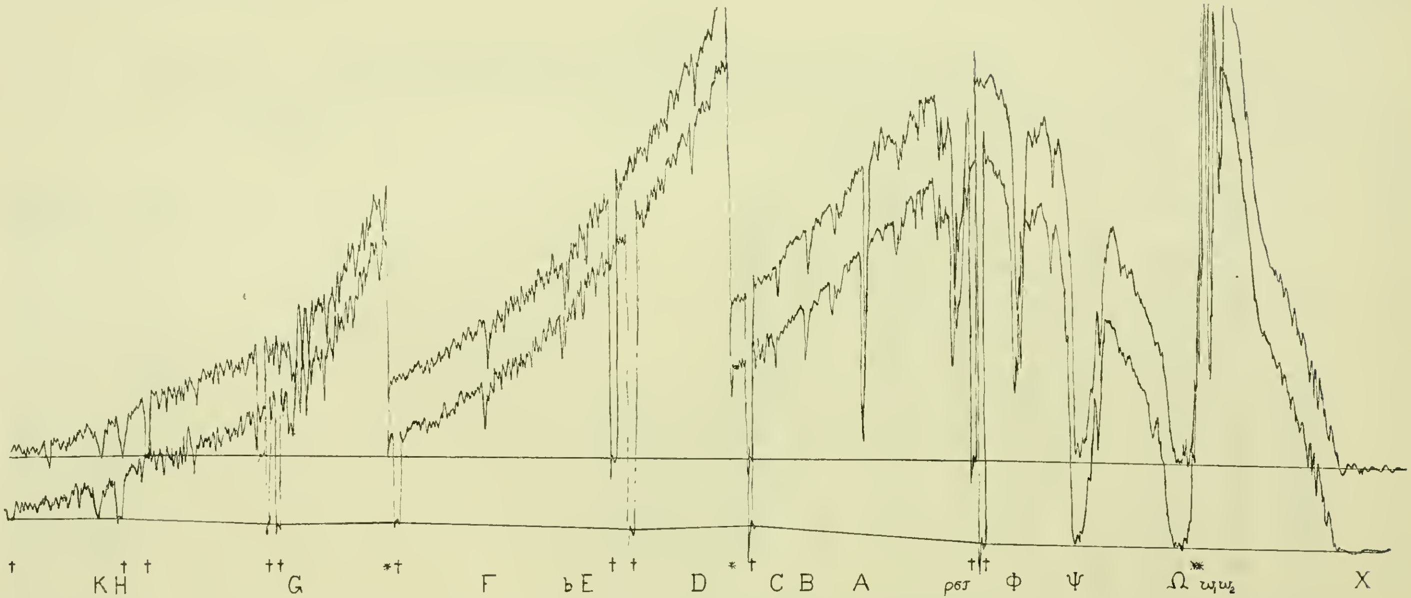
In order to fix our ideas both of the magnitude and the variability of the absorption of the earth's atmosphere, the following table, showing the percentage of transmission at numerous different wave-lengths for the days indicated, is given. The computations upon which the table is based were made at wave-lengths specially selected to avoid large terrestrial absorption bands, and thus the table gives values of the general absorption only. A few reductions have been made to determine the selective absorption within the numerous atmospheric bands of water vapor and oxygen, but while their discussion has gone far enough to show that equation (1) apparently holds good in these bands, these results are not yet far enough advanced to be included in the tables. While, as another criterion of the accuracy of the method of extrapolation, it is found, in accord with what has just been said, that the employment of these observed values of transmission within the water-vapor bands would practically fill up these bands in computations of the form of the solar energy curve outside the atmosphere, yet in determining the solar constant they are smoothed over and the general transmission constants corresponding with the smoothed curves are employed in the computation.

The days included in Table I were all nearly cloudless, and thus the results represent the transmission of the atmosphere in better than average conditions. In order to bring out clearly what seems to be a marked decrease in the transparency of the air for the present

TABLE I.

Coefficient of Atmospheric Transmission for Radiation from Zenith Sun.

Wave-Length.	0.40 µ	0.45 µ	0.50 µ	0.60 µ	0.70 µ	0.80 µ	0.90 µ	1.00 µ	1.20 µ	1.60 µ	2.00 µ
Date.	Transmission Coefficients for Unit Air-Mass.										
October 25, 1901			0.81	0.82	0.89	0.94		0.95	0.96	0.95	
November 2, 1901			0.80		0.87	0.92		0.94	0.95	0.94	
March 21, 1902			0.83	0.80	0.84					0.87	
May 8, 1902			0.89	0.77	0.90	0.94		0.95	0.94	0.91	
September 11, 1902			0.80	0.78	0.87	0.89	0.92	0.92	0.94	0.93	
October 9, 1902			0.70	0.78	0.84	0.87	0.89	0.90	0.91	0.93	
October 15, 1902			0.73	0.78	0.86	0.89	0.90	0.91	0.93	0.96	0.94
October 16, 1902			0.50	0.58	0.79	0.82		0.86	0.90	0.91	
October 22, 1902			0.84	0.82	0.88	0.91	0.93	0.94	0.94	0.95	
November 15, 1902			0.75	0.79	0.83	0.89	0.91	0.92	0.93	0.95	0.96
February 19, 1903	0.67	0.64	0.66	0.72	0.76	0.80	0.83	0.85	0.86	0.90	0.92
February 25, 1903	0.48	0.60	0.66	0.68	0.74	0.83	0.88	0.90	0.93	0.93	0.92
March 3, 1903	0.40	0.48	0.66	0.73	0.79	0.84	0.87	0.89	0.92	0.96	0.96
March 25, 1903	0.47	0.50	0.57	0.66	0.72	0.76	0.79	0.81	0.84	[illegible]	0.89
March 26, 1903	0.52	0.58	0.62	0.68	0.77	0.80	0.81	0.83	0.85	0.89	0.90
April 17, 1903	0.55	0.60	0.69	0.77	0.80	0.82	0.87	0.90	0.94	0.97	0.97
April 28, 1903	0.39	0.52	0.56	0.64	0.71	0.74	0.76	0.78	0.82	0.88	0.89
April 29, 1903	0.46	0.49	0.56	0.66	0.72	0.76	0.77	0.80	0.83	0.88	0.90
July 7, 1903	0.42	0.60	0.66	0.69	0.77	0.82	0.85	0.86	0.88	0.89	0.86
General Mean	0.484	0.557	0.700	0.730	0.808	0.847	0.856	0.884	0.903	0.920	0.919
Mean of 1901–2			0.765	0.769	0.857	0.897	0.910	0.921	0.933	0.930	0.950
Mean of 1903	0.484	0.557	0.627	0.692	0.753	0.797	0.825	0.847	0.874	0.909	0.912
Percentage difference between mean of 1903 and that of 1901–2.			20 %	10 %	13 %	12 %	10 %	8.4 %	6.5 %	2.3 %	4.1 %



BOLOGRAPHIC ENERGY CURVES OF THE SOLAR SPECTRUM OF A 60° GLASS PRISM. OBSERVATIONS OF APRIL 17, 1903.

(† Beam cut off by shutter to give position of zero line. * Slit diminished by interposing grill diaphragms. ** Slit increased by removing grill diaphragms.)

calendar year, the means of the general absorption coefficients have been taken for the observations of 1901–02, and for those of 1903 separately. There is an average difference of ten percent in favor of the earlier years, and this cannot, so far as I know, be accounted for in any other way than by recognizing an actual decrease in the transparency of the air, beginning somewhere between November 15, 1902, and February 19, 1903. It might be urged that the change is perhaps an annual one, as most of the results of 1901–02 are in the autumn and those of 1903 in the spring. But in contradiction to this view we find the observations of March and May, 1902, generally above the mean of that year, so that I incline to think the change rather extraordinary than annual in character. Such a change would imply a corresponding reduction in the amount of direct solar radiation at the earth's surface, and if general over a wide area would seem to be likely to occasion some alteration of climate. Recent actinometric observations reported by several observers in this country and in Europe[1] seem to strengthen the probability that the change in transparency of the air is widespread, for their measures of solar radiation at the earth's surface have been appreciably lower of late than for the same months of former years. Several writers have suggested the possibility of the wide dissemination of fine dust clouds from the volcanic eruptions of 1902, in explanation of the lower values. It will be noted from Table I that the differences between the means of 1901–02 and 1903 are largest for short wave-lengths and diminish nearly uniformly toward the infra-red as far as a wave-length of 1.2 μ, which would probably be in harmony with this hypothesis; for such small dust particles might be expected to scatter and absorb the shorter wave-lengths most, not being large enough to act like an opaque screen diminishing all wave-lengths proportionally.

COMPUTATIONS OF THE SOLAR CONSTANT OF RADIATION

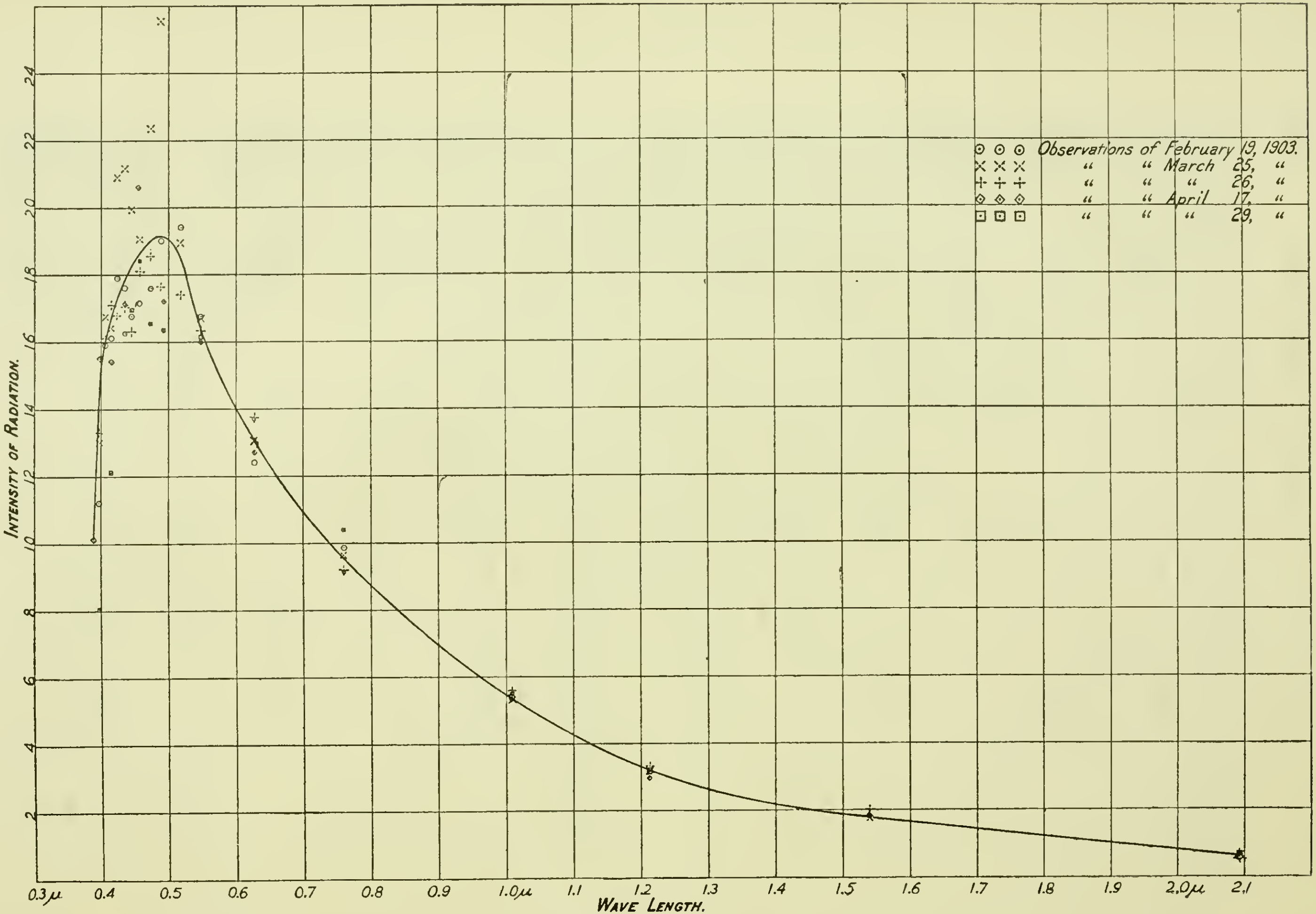
The coefficients of general atmospheric transmission resting upon measures at twenty-four different wave-lengths from 0.37 μ to 2.3 μ on series of bolographic curves have been employed at the Astrophysical Observatory in connection with bolographs and actinometric data to compute the solar constant of radiation outside the atmosphere. Referring to plate XXI, the area included underneath a spectral energy curve is directly proportional to the total radiation absorbed by the bolometer over the range of wave-lengths included

[1] See note by H. H. Kimball, *Monthly Weather Review*, May, 1903.

in the curve. But this area is not strictly proportional to the total solar radiation at the earth's surface, as determined by actinometer observations, for the reason that the radiation has been unequally reduced at different wave-lengths by losses at the siderostat mirror, within the spectroscope, and by selective absorption at the bolometer itself. It is necessary to correct the curve so that it shall as accurately as possible represent the distribution of energy in the solar beam prior to these losses. Inasmuch as the coefficient of total absorption of the lampblacked bolometer strip is upward of 95 percent, it is believed that no considerable error is admitted by neglecting its differences of absorption for different wave-lengths, and no correction is applied for this. The relative absorption of the spectroscope for different wave-lengths is frequently determined, and that of the siderostat mirror still more frequently, for in both these optical parts of the apparatus there is rapid deterioration of the reflecting power of the silvered glass surfaces. At present this indeed forms one of the main difficulties and sources of error of the investigation, for a whole day of observing and several days of computing are required for each determination of the absorption of the apparatus, which would be determined once and for all if constant reflecting surfaces could be employed.

With the coefficients of absorption of the apparatus thus determined, each small area included under the bolographic curve for a very narrow range of wave-lengths is increased so that the total corrected area is then proportional to the solar radiation at the earth's surface as measured with the actinometer or pyrheliometer. Then by the aid of formula 1, given above, and employing the transmission coefficient *a* determined from the series of bolographs of the day, each small area is again corrected till it becomes proportional to the total radiation of that wave-length outside the atmosphere. The ratio of the sum of these finally corrected areas to the total corrected area at the earth's surface is the factor by which the reduced pyrheliometer reading is to be multiplied to give the "solar constant" so-called.

It is evident that these values depend directly upon the pyrheliometer or actinometer readings for their accuracy, so that these instruments become here of major importance. In the work thus far a mercury pyrheliometer has been used as the primary standard, and the daily observations have been taken sometimes with it, sometimes with a Crova alcohol actinometer (specially constructed for the Institution under M. Crova's valued supervision), and sometimes with both instruments simultaneously. It has been shown by re-



DISTRIBUTION OF RADIATION IN THE NORMAL SOLAR SPECTRUM OUTSIDE THE EARTH'S ATMOSPHERE.

peated comparisons of the two instruments and by comparisons of the pyrheliometer with another type that they give proportional results under widely differing conditions of wind and temperature, so that I have no question of the relative accuracy of the actinometric data employed in computing values of the solar constant within two percent. There is, on the other hand, room for question as to the absolute magnitudes of the values given, for these depend on the constants and the theory of the mercury pyrheliometer. Steps are being taken to get further checks on this matter, and in a later publication it is expected to recompute the data in accord with later information. For the present then, the values in the following table are to be held as relatively accurate and consistent among themselves, but subject later to correction by a common multiplying factor.

TABLE II.

Values of the Solar Constant of Radiation. From Bolographic Studies.

Date.			Hour Angle. West.	Air Mass.	Calories per Square Centimeter per Minute.		Solar Constant Corrected for Mean Distance of the Sun.
					At the Earth's Surface.	Outside the Atmosphere.	
			h m		Cal.	Cal.	
1902.	Oct.	9	0 6	1.425	1.42	2.20	2.19
"	"	15	1 31	1.624	1.44	2.21	2.19
"	"	22	3 01	2.415	1.20	2.18	2.16
1903.	Feb.	19	1 01	1.642	1.35	2.34	2.28
"	"	"	2 22	2.003	1.20	2.31	2.25
"	Mar.	3	0 59	1.429	1.34	2.31	2.26
"	"	25	2 01	1.454	1.19	2.29	2.27
"	"	26	1 57	1.438	1.16	2.11	2.10
"	"	"	2 59	1.754	1.05	2.09	2.07
"	Apr.	17	2 45	1.463	1.19	1.97	1.99
"	"	28	1 07	1.145	1.29	2.23	2.27
"	"	29	2 26	1.308	1.05	1.93	1.97
					General Mean.		2.167
					Mean of results prior to March 26, 1903.		2.229
					Mean of results after March 26, 1903.		2.080

The bolographs used in the computations extend from wave-length 0.37μ to wave-length 2.5μ with the exception of those of October, 1902, which reached only to a wave-length of 0.48μ in the violet. For these latter bolographs a correction of about twelve percent was applied, founded on the later work, and thus the results for October, 1902, are entitled to slightly less weight on this account. All the areas have been extrapolated for the radiations lying outside at both ends of the region 0.37μ to 2.5μ, but the corrections so applied amount to less than one percent altogether. Their magnitude was determined by an inspection of the rate of decrease of successive corrected areas approaching the limits of the curves, and the corrections were checked both by computing according to Wien's formula the probable form of the solar energy curve corresponding to the assumed solar temperature of 6000°, and by examination of the normal energy curves outside the atmosphere as computed from bolographs and given in plate XXII.

I have thought it worth while to give in addition to the general mean, the means also of observations before and after March 26, when, for some unexplained reason, a fall of about 10 percent was noted in the computed solar constant. The observations of February 19,[1] March 25, March 26, and April 29, 1903, appear to be entitled to the greatest weight among those given, on account of the regularity of the actinometric curves of those days and the closeness with which the plotted points for determining the atmospheric transmission coefficients lie upon straight lines, as shown for two of the days in question on plate XX. Since May 1 it has been almost impossible to get sufficient observations for computing a solar constant owing to cloudiness, but interest attaches to further determinations and these are to be made when practicable.

FORM OF THE NORMAL SOLAR ENERGY SPECTRUM OUTSIDE THE EARTH'S ATMOSPHERE AND THE PROBABLE TEMPERATURE OF THE SUN

The reader has no doubt noted that, by applying corrections for atmospheric and instrumental absorption, the bolographic spectrum energy curves may be reduced in form as well as in area to represent the distribution of energy in the spectrum of the solar beam outside the atmosphere. This has been done in several instances, and in doing so the curves have been transformed from the prismatic to the normal wave-length scale by taking account of the prismatic dispersion, and several of these curves are platted in plate XXII. No account is taken in the curves, shown in plate XXII, of selective absorption bands whether solar or terrestrial, smoothed curves only being given.

It will be noted that there is a fair agreement in general form between these independently derived curves, and that they unite in

[1] February 19, 1903, was the most extraordinary day as regards absence of water vapor in the atmosphere which has ever been noted here. The great water-vapor bands $\varphi\psi\Omega$ in the infra-red spectrum were nearly filled up, and the long wave-length side of the band Ω presented an almost unrecognizable appearance.

fixing the wave-length of maximum energy at about 0.49μ.[1] Their agreement would be more exact, there can be little doubt, if it were not for the large and variable absorption of the silvered surfaces in the optical apparatus for wave-lengths at and beyond the region of maximum energy. The transmission of the spectroscope at a wave-length of 0.45μ has varied on this account at different times from 33 percent to 15 percent, whereas at wave-lengths of 1μ and thereabouts the transmission always approaches 90 percent. The spectroscope mirrors are resilvered about once in two months and the siderostat mirrors still oftener.

Paschen has empirically derived a law connecting temperature with wave-length of maximum radiation, which is expressed as follows, where T is the absolute temperature and λ_{max} the wave-length of maximum intensity of radiation expressed in microns:

$$\lambda_{max}\, T = \text{constant.}$$

The value of this constant for the radiation of a "black body" or perfect radiator as determined by Paschen,[2] Lummer and Pringsheim,[3] and others is about 2900, while for bright platinum Lummer and Pringsheim give 2630 with values for other substances intermediate between these.

Taking the higher value in connection with the observed position of maximum in the solar energy curve outside the atmosphere, we find that the sun's radiation may be assumed comparable as regards the wave-length of maximum radiation to the emission of a "black body" at 5920° absolute. Readers will draw their own conclusions as to the probability that the solar temperature actually lies near this value, but it may be remarked that a further correction of the energy spectrum curve for the selective absorption of the solar envelope would undoubtedly reduce the wave-length of maximum radiation still further, and would thus incline us to the view that the interior of the sun is at a higher temperature than the above considerations alone would indicate.

[1] The wave-length of maximum energy determined by Mr. Langley on Mount Whitney was about 0.52μ.

[2] *Verhandlungen d. Deutschen Phys. Ges.*, III, 37, 1901.

[3] Paschen, *Astrophysical Journal*, IX, 306, 1899.